THE
Young Gentleman's
TRIGONOMETRY,
MECHANICKS,
AND
OPTICKS.

Containing such

ELEMENTS of the said Arts or Sciences, as are most Useful and Easy to be known.

By *EDWARD WELLS*, D.D.
Rector of *Cotesbach* in *Leicestershire*.

LONDON,
Printed for *James Knapton*, at the *Crown* in St. *Paul*'s Church-Yard. 1714.

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THE PREFACE.

BEING once prevail'd upon to draw up the Young Gentleman's Geometry, *I judg'd it requiſite alſo to draw up this Treatiſe of* Trigonometry; *forasmuch as Trigonometry is really no other than one Branch of Geometry, and that too a Branch, which is of principal Uſe in the common Concerns of Life; at leaſt that Part of it, which is call'd* Plain *Trigonometry. As for* Spherical *Trigonometry, it carrying in it much more of Difficulty, and at the ſame time being of much leſs Uſe in the common Affairs of Life, (as relating chiefly to Aſtronomical Calculations or the like,) I have omitted it,*

as not agreeabls to the Deſign of this Treatiſe.

On the like Account, tho' I judg'd it not proper wholly to omit the Fundamental Theorems *of plain Trigonometry; yet I have plac'd them ſo in* Chap. II. *that the young Student may paſs them over, without any Abruptneſs, if the Demonſtration of the two latter prove too difficult, or at leaſt if he cares not to take the Pains to make himſelf Maſter of the ſaid Demonſtrations.*

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THE

THE Young Gentleman's TRIGONOMETRY.

CHAP. I.

Containing the Explication of Trigonometrical Terms.

1. Trigonometry, what.

EVERY Triangle consists in all of seven Parts, *viz.* three Sides, three Angles, and the Area or Space comprehended by the Sides. Now, although the Word *Trigonometry* do's in the *Greek* Language literally signify *the Measuring of a Triangle*; yet, in its proper Sense, or as it is distinguish'd from *Geometry*, it (do's not denote the Measuring of the *Area* of a Triangle, this being deduced from (*) other than what are

(*) The Measuring of the Area of a plain right-lin'd Triangle, is deduced from the Principles of *Geometry*,

are properly call'd Trigonometrical Principles, but it) denotes only the Measuring, or rather the Art of finding the Measure of, the unknown *Sides and Angles* of a Triangle, by the Help of those that are given or known.

2. *The Solution of a Triangle, what.* And the actual finding of the unknown Sides and Angles of a Triangle, is call'd, in one Word, the *Solution* or *Resolving* of a Triangle.

3. *The Solution of Triangles on what founded.* The Solution of Triangles is founded on that mutual Proportion, which is between the Sides and Angles of any Triangle. And this Proportion is known by finding the Proportion, which the Ray of a Circle has to certain other right Lines apply'd to the same Circle. And these right Lines are distinguish'd in general by the Names of *Chords*, *Sines*, *Tangents*, and *Secants*.

4. *A Chord, what.* A (†) *Chord* is a right Line drawn from one Extremity of any Arch to the other. Thus, *Fig.* 1, CH is the Chord

Fig. 1.

(as distinguish'd from *Trigonometry*,) viz. from *Theorem* [illegible] *Chap.* 1, *of the Young Gentleman's Geometry.* And the Measuring of the Area of a spherical Triangle, is deduced from the Measuring of a spherical Surface.

(†) As an *Arch* is so call'd, because it resembles a bent *Bow*; so a *Chord* is so call'd, because it resembles the *String* of a bent Bow. It is otherwise call'd a *Subtense*, because it *subtends*, (*i. e.* is extended under) the Arch from one End of it to the other.

both

both of the leſſer Arch CTH, and alſo of the greater Arch CDH. And it is to be noted, that a Chord of 60 Degrees, is always equal to the Ray of the ſame Circle.

8

A (‖) *Sine* is either a Right or Verſed Sine. A *right Sine* is one Half CA of a Chord CH, biſected by the Diameter DT; or, it is a Perpendicular CA, let fall from C one End of an Arch CT, upon a Diameter DT, drawn from T the other End of the Arch. A *right Sine* is frequently ſtil'd ſimply *a Sine*; and it is to be obſerv'd, that by the Word *Sine*, put ſimply or by it ſelf, is always to be underſtood a *right Sine*.

5. *A Sine, or right Sine, what.*

It is alſo to be obſerv'd, that, according to the fore-going Definition of a Sine, the Sine BK of an Arch KT of 90 Degrees, is always equal to the Ray of the ſame Circle; and ſo the greateſt Sine that can be. Whence it is call'd, the *total* or *whole* Sine.

6. *A total or whole Sine, what.*

A (*) *vers'd* Sine is the Segment of the Ray intercepted between the right

7. *A vers'd Sine, what.*

(‖) The Uſe of *Sines* is ſaid to be introduced by the *Arabs*; and for what Reaſon a Sine is ſo call'd, I know no good Account.

(*) A vers'd Sine is otherwiſe call'd, at leaſt by ſome ancient Writers, the *Sagitta* or *Arrow*, becauſe it ſtands in its Arch and Chord, as an Arrow do's to its Bow (when bent) and String.

Sine and the Arch. Thus, AT is the vers'd Sine of the Arch CT, and AD the vers'd Sine of the Arch CD.

8. *A Tangent, and Secant, what.*

A (†) *Tangent* and a *Secant* mutually explain one the other. For, if a right Line BN of an indeterminate Length be drawn from the Center B, through C one End of an Arch CT, and from T the End of the Diameter DT, there be erected a Perpendicular TN, extended till it meets with BN; the said Perpendicular TN will be the Tangent of the Arch CT, and the Line BN will be the Secant of the same Arch. And it is to be noted, that the Tangent of 45 Degrees, is always equal to the Ray of the same Circle; as may be seen, *Fig.* 2.

9. *A Cosine, Cotangent, Cosecant, and Covers'd, what.*

As when the Arch CT is less than 90 Degrees, the Arch CK (which, being added to the Arch CT, do's compleat or make it up an Arch of 90 Degrees thence) is call'd the (‖) *Complement* of the Arch CT; so (*Fig.* 1.) the Sine CQ, the Tangent KO, the Secant BO, and the vers'd Sine KQ of the Arch CK, are re-

(†) A *Tangent* is so nam'd, because it *touches* (not cuts) one End T of its Arch CT; and a *Secant* is so nam'd because it *cuts* or crosses the other End C of its Arch CT.

(‖) It is not to be omitted, that the Excess CK of the Arch CD above 90 Degrees, is by some Writers call'd the *Complement* of the said Arch CD.

spective

spectively call'd the *Sine*, the *Tangent*, the *Secant*, and the *vers'd Sine of the Complement* of the Arch CT; or in short, the (*) *Cosine*, the *Cotangent*, the *Cosecant*, and the *Covers'd* of the said Arch CT.

10. *The Supplement or Conjunct of an Arch, what.*

The Arch CD, (which, being added to the Arch CT, makes it up a Semicircle,) is peculiarly (†) stil'd the *Supplement* or *Conjunct* of the Arch CT. And it is to be noted, that both the Arch CT, and its Conjunct CD, have the same Sine CA, the same Tangent TN, and the same Secant BN.

11. *Tables of Sines and Tangents, what.*

Tables of (||) Sines and Tangents are so call'd, because in them is set down the Ratio or Proportion, which Sines and Tangents have to their Ray.

(*) It is of Use to observe, that the Cosine CQ is always equal to BA, the Segment of the Ray between the Center B, and the Sine CA. Wherefore, if the Arch CD be greater than 90 Degrees, then the Sum of the Ray and Cosine is equal to the vers'd Sine; *viz.* DB + BA = DA. But, if the Arch CT be less than 90 Degrees, then the Difference of the Ray and Cosine is equal to the vers'd Sine, *viz.* BT − BA = AT.

(†) Namely, to avoid Confusion by giving the same Name of *Complement*, both to CK and to CD, as is done by some.

(||) It appears from §. 5, of this Chapter, that every Sine, (*i. e.* right Sine) is the Half of a Chord, and that the Arch of the Sine, is the Half of the Arch of the said Chord. Wherefore, because there is the same Proportion between two Halves, as there is between their Wholes, and the said Proportion between the Halves

(as

12. Natural *and* artificial *Sines and Tangents, what.*

And it is to be noted, that, if the ſaid Proportion be expreſs'd in the Tables by natural or common Numbers, then the Table is call'd, a Table of *natural* Sines and Tangents. But, if the ſaid Proportion be expreſs'd by artificial Numbers, *i. e.* Logarithms, then the Table is call'd, a Table of (*) *artificial* Sines and Tangents.

13. *A* Scale or Line *of Chords, Sines, Tangents, and Secants, &c. what.* *Fig.* 2.

If the ſaid Proportion be not expreſs'd by Numbers, but the ſeveral Sines and Tangents, as alſo (†) Chords and Secants, be themſelves actually transferr'd into, (*i. e.* their ſeveral Lengths ſet off upon) ſeveral diſtinct Scales or Lines, (as *Fig.* 2); then the ſaid Scales or Lines are reſpectively

(as being expreſſible in leſs Numbers) is much eaſier to be calculated, than the Proportion between their Whole hence, Sines are uſed rather than Chords in Trigonometrical Calculations, and the Proportion of Sines (not Chords) to the Ray is ſet forth in Trigonometrical Tables.

(*) It has been afore obſerv'd in my *Arithmetic* that the *Multiplication* and *Diviſion* of great Numbers is much eaſier to be perform'd by Logarithms than natural Numbers. Hence it is that Tables of artificial Sines and Tangents, are more uſed than thoſe of natural Sines and Tangents, and conſequently adjoin'd to most Trigonometrical Treatiſes.

(†) The Reaſon, why there are no Tables of Chords has been obſerv'd already in the Note on §. 11. There are however *Scales of Chords*, theſe being as eaſily made as *Scales of Sines*, and as eaſy or eaſier for Uſe in ſome Reſpects. And the like is to be underſtood of *Secants*

reſpectively ſtil'd, one *a Scale* or *Line of Chords*, the other *of Sines*, the other *of Tangents*, and the other *of Secants*. And the like is to be underſtood of the Scales of any other Sort of Trigonometrical Lines, as of vers'd Sines, Coſines, *&c.*

14. *A Scale of* equal Parts, *what.* *Fig.* 3, & 4.

Beſides the fore-mention'd Scales, there is alſo another Sort of Scale, call'd a *Scale of equal Parts*; which, although it

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viz. as to the Reaſon, why there are few or no *Tables* of them, but there are uſually, if not always, *Scales* of them adjoin'd to the other Scales. If it be ask'd, why the Scale of Chords is never continued beyond 90 Degrees, when the greateſt Chord is the Diameter or Chord of 180 Degrees? The Reaſon is, becauſe the Chords *above* 90 Degrees, may be very eaſily ſupply'd by a Scale of Chords *to* 90 Degrees, *viz.* Chord of 105 Degrees = Chord 90 Degrees + Chord 15 Degrees. And ſo of any other to 180 Degrees. The Reaſon, why Tables and Scales of Sines go no further than 90 Degrees, is obvious, *viz.* becauſe a Sine of 90 Degrees, is the greateſt Sine that can be, as appears from §. 5, and 6, of this Chapter. In like manner, the greateſt Tangent and Secant is of 89 Degrees, 59 Minutes. For, at 90 Degrees, Lines drawn in like manner as a Tangent and Secant, become parallel one to the other, and ſo never meet to conſtitute a Tangent and Secant, at leaſt properly ſo call'd. Hence Tables of Tangents proceed only to 89 Degrees, 59 Minutes. And as for Scales of Tangents, and alſo Secants, they are uſually continued no farther than to about 70 or 75 Degrees; becauſe, after that they extend to a vaſt Length, if the Scales be of any conſiderable Size, ſo as to be fit for Uſe. Which may be perceiv'd by *Fig.* 2, where the Scales are of a very ſmall Size, as being deſign'd not ſo much for practical Uſe, as to ſhew only the Manner of Making ſuch Scales. Such Scales, as are fit for Practice, are to be bought ready made of ſuch as ſell Mathematical Inſtruments.

be not deduced from Principles of Trigonometry, yet is of so great Use therein, (as well as in other Parts of the Mathematicks,) that it will be requisite to describe it here, especially, since it has not been afore describ'd. It is then call'd, *a Scale of equal Parts*, because it shews the Measure of any Line or Extent, apply'd thereto, or taken therefrom, in some Number of *equal Parts*, denoting either Inches, or Feet, or Yards, *&c.*

15. *A simple or plain Scale of equal Parts, what.*

It is usually distinguish'd into the *simple* or *plain* Scale, and the *diagonal* Scale. The *simple* or *plain* Scale, is no other than a right Line divided into any Number of equal Segments or greater Parts; One of which greater Parts, (*viz.* at either End of the said Line,) is sub-divided into Ten other lesser and equal Parts, as *Fig.* 3; So, that this Scale represents any two Degrees of decuple Proportion, *viz.* if the Sub-divisions of the Segment BA be taken to denote Units, then the other equal Segments CB, DC, *&c.* will denote Tens. If the aforesaid Sub-divisions denote Tens, then the Segments will denote Hundreds, *&c.* Hence, the Extent from C to 4 will denote either 14 or 140 (*&c.*) Inches, or Feet, or Yards, *&c.*

The

The *diagonal* Scale will be better apprehended by *Fig.* 4, than by Words. It thence appears to differ from the ſimple Scale principally in this, *viz.* that one of its extreme Segments (being like the ſimple Scale) ſubdivided into ten equal Parts, each of the ſaid ten equal Parts, are again ſubdivided into ten other ſtill leſs and proportional Particles, by ten right Lines obliquely croſſing the Scale, which are the *Diagonals* of ſo many (‖) Parallelograms; whence this Scale takes its Name. Hence, by the Diviſions and Sub-diviſions of this Scale, are repreſented three Degrees of decuple Pro-

16. *A diagonal Scale what.*

portion.

(‖) This is illuſtrated, *Fig.* 5, which is divided after the ſame manner, as is the Segment KL of the diagonal Scale, *Fig.* 4; and has withal the *Sides* of each Parallelogram deſcribed by *pricke* Lines, ſo that the ſaid Figure contains five Parallelograms, *viz.* ABCD, and CDEF, and EFGH, and GHIK, and IKLM; which are each croſſed by Diagonals, *viz.* the firſt by the Diagonal CB, the ſecond by the Diagonal ED, the third by the Diagonal GF, the fourth by the Diagonal IH, and the fifth by the Diagonal LK. And theſe Diagonals anſwer the right Lines obliquely croſſing the Segment KL of the diagonal Scale, *Fig.* 4, and which divide each Part of the ſaid Segment into leſs and proportional Particles, as is illuſtrated by *Fig.* 6. For the Lines *bk*, and *cl*, and *dm*, &c. being all parallel one to the other, hence (by *Theorem* 9, *Chap.* 1, *of the Young Gentleman's Geometry*) *ab*: *bk* :: *ac* : *cl* :: *ad* : *dm* :: *ae* : *en* :: *af* : *fo*, &c. But *ab* = $\frac{1}{10}$ *ax*, *ac* = $\frac{2}{10}$ *ax*, *ad* = $\frac{3}{10}$ *ax*, &c. therefore *bk* = $\frac{1}{10}$ *nx*, *cl* = $\frac{2}{10}$ *nx*, *dm* = $\frac{3}{10}$ *nx*, &c. And conſequently, the Line *nx* is divided by the Diagonal *ax*, into ten leſſer and proportional Parts, *viz.* *bk*, *cl*, &c.

portion. For Inſtance: If you would denote the Extent of 245 equal Parts, (*viz.* Inches or Feet, *&c.*) one Foot of your Compaſſes being placed in the middle or prickt Line over-againſt 200, open the other Foot to that Point of the Sub-divided Segment KL, wherein meet the two Lines mark'd, one with 40 on the Side of the Scale, the other with 5 on the Top of the Scale. That Extent of your Compaſſes will denote 245 equal Parts, (*viz.* Inches or Feet, *&c.*) according to the ſaid Scale.

17. Plain *and* ſpherical *Trigonometry, what.*

Further, It is to be here obſerv'd, that *Trigonometry* is two-fold, *Plain* and *Spherical*. *Plain Trigonometry* is ſo call'd, becauſe it teaches how to reſolve any (*) plain (*i. e.* right-lin'd) Triangle: *Spherical Trigonometry* is ſo call'd, becauſe it teaches how to reſolve any *ſpherical* Triangle, *i. e.* any Triangle whoſe Sides are made by the Concurrence of Lines on a ſpherical Surface, and are no other than the Arches of Circles. It will be ſufficient to the (†) Deſign of this Treatiſe, to ſpeak only of plain *Trigonometry*.

(*) Hence, by a *plain* Triangle is to be underſtood throughout this Treatiſe a *right-lin'd* Triangle.

(†) See the Preface.

Laſtly,

Laſtly, The Solution of plain (as alſo of ſpherical) Triangles is two-fold, either by *Calculation*, *i. e.* Working by Numbers, to which are ſubſervient the Tables of Sines and Tangents; or by *Protraction*, *i. e.* drawing Lines and Angles, to which are ſubſervient the Scales of Chords, Sines, *&c.* of each in its Order; and firſt of the Solution of plain Triangles by Calculation. 18. Calculation *and* Protraction, *what.*

CHAP. II.

Of the Solution of plain Triangles by Calculation.

OF the ſix Trigonometrical Parts of a plain Triangle, *viz.* its three Sides, and three Angles, any three being given, excepting the three Angles alone, the reſt may be found either by Calculation or Protraction. From the three Angles given by themſelves, can be inferr'd or found out, only the *Proportion* of the Sides, not the *Sides* themſelves, or their reſpective determinate Lengths. For Inſtance: Suppoſing the three Angles of the Triangle BCD, *Fig.* 7, to be given, *viz.* B 37 ½ Degrees, D 38 ½ Degrees, and conſe- 1. *In order to reſolve a plain Triangle, there muſt be always given one Side, and why.* *Fig.* 7, & 8.

consequently, C 114½ Degrees; from hence it may be known, that the Side CD=½BD, and the Side CB=½BD. But from hence can never be discover'd the determinate Length of BD, (or any other Side,) and consequently not of the Rest CB and CD. Namely, because ½=½=⅓=⅓+⅓=⅔+⅓, *&c.* hence the Proportion of CD to BD will be the same, whether BD be 2 Inches (or Feet, *&c.*) and CD one; or BD 4 Inches, and CD 2; or BD 6 Inches, and CD 3; or BD 20 or 200 Inches, and CD 10 or 100, *&c.* That is in short, it can never be discover'd by the three Angles alone, whether the Triangle sought be BCD, *Fig.* 7, or BCD, *Fig.* 8, (or any other equiangular Triangle,) forasmuch as both these Triangles have the same Angles, and consequently their Sides alike proportional one to the other; though the Sides of the one are much longer than the Sides of the other. Wherefore, to *determine the Length* of the Sides sought, there *must always be given one Side*, or (which is the same) its determinate Length; and hereby will be determin'd the Length of each Side sought. Thus, the Side BD, (*Fig.* 7,) being given, whose Length according to the diagonal Scale, (*Fig.* 4.) is 324 (equal Parts, suppose) Inches; hence, it being found, that CD=⅔BD, and CB=

BD,

BD, thereby it is known, that CD=162 Inches, and CB=216 Inches. But, if the Side BD (*Fig.* 8,) be given, whoſe Length according to the ſame Scale is 135 (equal Parts, ſuppoſe) Inches; then in the ſame, *Fig.* 8, CD=67½ Inches, and CB=45 Inches.

2. *The Solution of plain Triangles reducible to three general Caſes.*

It evidently appearing from what has been ſaid, that, in order to reſolve a plain Triangle, of the three Parts given, one muſt be always a Side; it hence follows, that all the Caſes, that can be propos'd, are reducible *in general* to theſe *Three*, viz.

1. One Side and two Angles being given, to find the other two Sides, and the (*) third Angle: Or,

2. Two Sides and one Angle being given, to find the third Side, and the two other Angles: Or,

3. and laſtly, All the three Sides being given, to find the three Angles.

3. *All the particular Caſes are contain'd in the two following Tables.*

Theſe three general Caſes are diſtinguiſh'd into a great Variety of *particular* Caſes, (according to the particular Sides

(*) Two Angles being given, the third may be known (without *Trigonometry*) by *Corol.* 1, *Theorem* 4, *of the Young Gentleman's Geometry.* And by the ſame Corollary, one Angle being given in *Caſe* 2, and one of the ſought being found, the other ſought is known. And ſo in *Caſe* 3, two of the Angles ſought being found, the third ſought is known.

and

and Angles given or fought,) which are all contain'd in the two following Tables. Whereof, the former contains all the particular Cases, that relate to the Solution of plain *rectangular* Triangles; and the latter contains all the particular Cases, that belong to the Solution of plain *obliquangular* Triangles.

4. *The literal Symbols of the several Parts of a plain Triangle Fig.* 9, & 10.

In Reference to which Tables it is to be observ'd, that for Brevity's sake it is usual to denote the several Parts of a plain Triangle by *Letters*, instead of Words. In order whereto a rectangular Triangle is wont to be denoted by A, B, and C, so placed, as (*Fig.* 9, *viz.*) that A always denotes the right Angle, BA the Basis, CA the Cathetus or Perpendicular, which falling upon the Basis makes the right Angle; BC the Hypotenuse; B the Angle between the Basis and Hypothenuse, call'd in short, *the Angle at the Basis*; C the Angle between the Cathetus and Hypotenuse, call'd in short, *the Angle at the Cathetus*. And an obliquangular Triangle is wont to be denoted by B, C, and D, so placed, as (*Fig.* 10, *viz.*) that by a Cathetus CA, let fall from C, it may be divided into two Triangles, the Cathetus CA being a common Side to both Triangles. And then, as BD is esteem'd the Basis of the whole Triangle BCD, so BA and DA are the particular

particular Baſes, and BC and DC the particular Hypotenuſes of the two Triangles BCA and DCA. Hence, B and D denote the two Angles at the Baſis BD of the whole Triangle BCD; whereof B is always the Angle oppoſite to the Side CD, and D the Angle oppoſite to the Side CB, as C is always the Angle oppoſite to the Baſis BD.

3. Other Symbols uſed in the following Tables.

It is likewiſe to be obſerv'd, that in the following Tables, the Letter ſ denotes a Sine, t a Tangent, and the Greek ς a Coſine. Thus, *Tab. I, Caſe* 1, tB denotes the Tangent of the Angle B. And *Caſe* 3, ſC denotes the Sine of the Angle C. And *Tab. II, Caſe* 16, ςB and ςD denote the Coſines of the Angles B and D. Alſo note, that Z do's (in this Treatiſe, as in *Algebra*) denote the Sum of two Things; X their Difference. *Laſtly*, This Character ∠ denotes an Angle, ∠∠ two Angles; *l* a Leg or Side, *ll* two Legs or Sides. And conſequently, Z*ll* denotes the Sum of two Legs of a Triangle; Z∠∠ the Sum of two Angles, *&c.*

6. Some other Obſervations in Reference to the two following Tables.

It is alſo to be obſerv'd, that a right Angle being always 90 Degrees, hence it is put down, neither among the Parts *given*, nor the Parts *ſought*, in the ſeveral Caſes of *Tab. I.* As for the other two oblique Angles of a plain rectangular Triangle, they are both put down among the

the Parts *given* in each of the first six Cases in *Tab. I*, (not (†) because it is necessary both the said Angles should be actually given, but) because one of them being given, the other is in effect given also, its Quantity being known (by *Corol.* 1, Theorem 4, of the *Young Gentleman's Geometry*,) to be the Complement of the given oblique Angle. On this and the like Accounts, some Tables are drawn up in a much more contracted or shorter Form, than the two which follow. But I judg'd it more agreeably to my Design, to have *all* the particular Cases *distinctly* set down: So, that the young Student may, both more readily find the Case propos'd to him; and also, having found it, may more readily perform the Calculation, having Nothing else to do, but to Work according to the Proportion, which answers to the Case propos'd.

(†) Hence it is said in the first six Cases of *Table I*, thus, *viz.* B *or* C, denoting, that if either be given, it is sufficient.

TAB.

TABLE I.

Shewing the Solution of plain *rectangular* Triangles.

Case.	Parts given.	Sought.	Proportions.
1	B or C . BA	CA	R : tB :: BA : CA.
2	B or C . CA	BA	R : tC :: CA : BA.
3	B or C . BA	BC	sC : R :: BA : BC.
4	B or C . CA	BC	sB : R :: CA : BC.
5	B or C . BC	BA	R : sC :: BC : BA.
6	B or C . BC	CA	R : sB :: BC : CA.
7	BA . CA	B	BA : CA :: R : tB.
8	BA . CA	C	CA : BA :: R : tC.
9	BC . CA	B	BC : R :: CA : sB.
10	BC . BA	C	BC : R :: BA : sC.
11	BA . CA	BC	CA : R :: BA : tC; & then sC : BA :: R : BC.
12	BC . BA	CA	BC : R :: BA : sC; & then tC : BA :: R : CA
13	BC . CA	BA	BC : R :: CA : sB; & then tB : CA :: R : BA.

C TAB.

TABLE II.

Shewing the Solution of plain *obliquangular* Triangles.

Case	Given	Sought	Proportions.
1	BC. BD. D.	C	BC : BD :: sD : (‖) sC.
2	BC. BD. C.	D	BD : BC :: sC : sD.
3	BC. CD. D.	B	BC : CD :: sD : sB.
4	BC. CD. B.	D	CD : BC :: sB : sD.
5	BD. CD. C.	B	BD : CD :: sC : sB.
6	BD. CD. B.	C	CD : BD :: sB : sC.
7	B.C.D. BC	BD	sD : BC :: sC : BD.
8	B.C.D. BC	CD	sD : BC :: sB : CD.
9	B.C.D. BD	BC	sC : BD :: sD : BC.
10	B.C.D. BD	CD	sC : BD :: sB : CD.
11	B.C.D. CD	BC	sB : CD :: sD : BC.
12	B.C.D. CD	BD	sB : CD :: sC : BD.

(‖) It is to be noted, that in this Case it may be doubtful, whether the Angle sought C be Acute or Obtuse. The Reason of which Doubtfulness arises from what is said, §. 10, *Chap.* 1. This happens when two Sides being given, whereof one is the greatest Side, together with the Angle opposite to the lesser of the given Sides, the Angle opposite to the greater of the given Sides is demanded. And in this Case, the Doubtfulness can't be better taken away, than by Protracting the Triangle; as is shewn, *Chap.* 3.

Case

Case	Given	Sought	Propositions.
13	BC . BD . B (†)	C, or D	$\frac{BC+BD}{2} : \frac{BC-BD}{2} :: \frac{D+C}{2} : \frac{C-D}{2}$ (*) Then $\frac{D+C}{2} + \frac{C-D}{2} = C$, and $\frac{D+C}{2} - \frac{C-D}{2} = D$.
14	BC . CD . C (†)	B or D	$\frac{BC+CD}{2} : \frac{BC-CD}{2} :: \frac{B+D}{2} : \frac{B-D}{2}$. Then $\frac{B+D}{2} + \frac{B-D}{2} = B$, and $\frac{B+D}{2} - \frac{B-D}{2} = B$.
15	BD . CD . D (†)	B: or C	$\frac{BD+CD}{2} : \frac{BD-CD}{2} :: \frac{C+B}{2} : \frac{C-B}{2}$. Then $\frac{C+B}{2} + \frac{C-B}{2} = C$, and $\frac{C+B}{2} - \frac{C-B}{2} = B$.
16	BC . CD . BD	B or D or C	DB : DG :: DF : DE. Then DB − DE = 2BA. And DB − BA = DA. Then CB : R :: BA : ϭB, and CD : R :: DA : ϭD. And Complement of B + D to 180 = C.

(*) It is to be obſerv'd, that, in this and the like Caſes, the Sum of the two Angles ſought, C and D, is found by ſubſtracting the Quantity of B, the Angle given, out of 180, according to *Theorem* 4, *of the Young Gentleman's Geometry*.

(††) It is alſo to be obſerv'd in Reference to Caſe 13th, 14th, and 15th, that having found the three Angles B, C, and D, the third Side not given in either Caſe may be found by ſome of the twelve fore-going Caſes.

7. *The Use of the fore-going Tables illustrated, as to artificial Sines, &c.*

The Use of the fore-going Tables sha be illustrated by one or two Example And first, supposing of the rectangul Triangle ABC, (*Fig.* 9,) I have the Sid BA given, *viz.* 8 (‖) Foot long, and th Angle B, *viz.* 36 Degrees, 52 Minutes and it is requir'd to find the Side CA. consult the fore-going Tables, and fin that this is *Case* 1, of *Tab.* I, and cons quently, that R : tB :: BA : CA, th Side requir'd. Hereupon I turn to m Table of artificial Sines, *&c.* whose Ra I find to be 10.000000, and the Tan gent of 36 Degrees, 52 Minutes, to b 9.875010. Then in my Table of Log rithms, I find the Logarithm of 8 to b 0.903090. Having thus found the Num bers, that are to constitute the three fir Terms of the Golden Rule in my Calcu lation, I then (according to the Metho of Working the Golden Rule by Log rithms) add the 2d and 3d Terms, and out of the Sum substract the 1st Term, and the Residue is the 4th

Tangent of B 36 Degr. 52 Min. }	9.8750
BA 8 f. long.	0.9030
Sum	10.7781
The Ray	10.0000
Residue	0.7781

(‖) This Extent of BA in *Fig.* 9, answers to the di gonal Scale, *Fig.* 4, and so of BC.

Ten

Term of my Proportion, and ſo the Number ſought, *viz.* 0.778100. I look for this Number in my Table of Logarithms, and (though I find not exactly the ſame Number, yet) I find one very near it, *viz.* 0.778151, the Logarithm of the Number 6. Wherefore, the Side required CA is 6 Foot long very nearly, *viz.* wanting only $\frac{55}{1000}$ of a Foot.

8. *The Uſe of the Tables illuſtrated, as to natural Sines, &c.*

The Reaſon, why the fore-going Example is wrought by artificial rather than natural Numbers, is, becauſe (as has been (*) afore obſerv'd) Tables of artificial Sines, *&c.* are more eaſy to work by, and ſo are generally made Uſe of, and therefore uſually adjoin'd to Books of *Trigonometry*. Whereas few Books have Tables of natural Sines, *&c.* Howſoever I ſhall here ſhew (for the Satisfaction of the more Curious) how natural Sines, *&c.* may be found by artificial Sines, and alſo, how the fore-going Example is to be wrought by natural Sines or Tangents, *&c.* when they are found.

If then you would have the natural Sine or Tangent of any Angle, and want a Table of them, but have a Table of Artificial Ones; then look out the artificial Sine or Tangent in the Table, and

(*) See the ſecond Note on §. 12, *Chap.* 1.

not much regarding the Characteriſtic thereof, ſee what Number anſwers to th other Part thereof in the Table of Log rithms, and that will ſhew you the nat ral Sine or Tangent deſir'd. For i ſtance; Not regarding the Characteriſtic of the artificial Tangent (above-found of B, *viz.* 9.875010, I look in the Tab of Logarithms for the other Part, *vi* 875010; and the neareſt to it, there be found, is the Logarithm 875061, whic anſwers to the Number 750, which therefore may be taken as the natur Tangent ſought of the Angle B, the D ference being only .....

Having thus found the natural Ta gent of B, if thereby you would find t Side CA requir'd; then you muſt pla (as afore) the Terms according to t Direction anſwering to *Caſe* 1, *Tab.* and muſt work the Golden Rule, (as natural Numbers, *i. e.*) by multiplyi the 2d and 3d Terms together, and d viding the Product by the 1ſt Ter The Quotient will be the Number Soug or CA, *viz.*

Ray. (†) Nat. Tang. BA. CA.
1000 : 750 : : 8 : 6.

(†) The Ray always conſiſts of an Unit, and ſo ma Cyphers, as there are Figures in the (natural or artifici Sine or Tangent.

For 7500 × 8 = 60000, and 10000) 60000 (6.
And thus the Length of the Side CA ſought'd, hath been found to be 6 Foot by both Sort of Operations.

9. Another Example.

Again, of the fore-mention'd Triangle ABC, ſuppoſe CA and B to be given, and BC to be requir'd. I find this to be *Caſe* 4, of *Tab.* I, and conſequently, the Rule for finding BC to be this, *viz.* sB : R :: CA : BC. Wherefore,

As artifi. Sine of B 36 Deg. 52 Min.	9.778119
To the Ray	10.000000
So CA 6 Foot long	0.778151
To BC the Side requir'd (10 f.)	1.000032.

For 10.000000 + 0.778151 = 10.778151; and 10.778151 — 9.778119 = 1.000032, which is (but ——, *i. e.* is nothing different from 1000000) the Logarithm of the Number 10, which, therefore is the Meaſure of the Side requir'd BC. And thus having given the Side BA (= 8 f.) of the Triangle ABC, the Side CA has been found equal to 6 Feet, and BC to ten Feet; agreeable to what is obſerv'd in the Uſe of *Theorem* 12, *of the Young Gentleman's Geometry*; whence may be obſerv'd the Truth of trigonometrical, as well as geometrical Principles.

I pro-

10. *Of the four Theorems, on which plain Trigonometry is founded.*

I proceed now to ſpeak of the *fundamental Theorems* of *Trigonometry*, ſo call'd, becauſe on them is founded the Solution of plain Triangles, in all the Variety of Caſes contain'd in the two fore-going Tables. And theſe Theorems are four, on the firſt of which is founded, *Tab.* I. or the Solution of plain rectangular Triangles; and on the other three is founded, *Tab.* II. or the Solution of plain obliquangular Triangles.

THEOREM I.

Whereon is founded the Solution of plain rectangular Triangles.

Fig. 11. In every plain rectangular Triangle ABC, if BC be the Ray, then BA is the Sine of the Angle C, and CA the Sine of the Angle B. If BA be the Ray, then BC is the Secant, and CA the Tangent of the Angle B. But, if CA be the Ray, then BC is the Secant, and BA the Tangent of the Angle C; as is illuſtrated *Fig.* 11.

Whence it follows,

1. R : sB :: BC : CA.
2. R : sC :: BC : BA.
3. R : tB :: BA : CA.
4. R : tC :: CA : BA.

And

And theſe four Proportions are the ame with thoſe appertaining to *Caſe* 6, , 1, and 2, of *Tab.* I, and from theſe e deduced all the other Proportions, elonging to the ſeveral other Caſes of *ab.* I, as may be ſeen by comparing em together. And therefore it evident- appears, how on this fundamental heorem 1, is founded *Tab.* I. Proceed e to Theorem 2.

THEOREM II.

Whereon is founded the Solution of plain obliquangular *Triangles, when, there being given either all the Angles with one Side, or two Angles with the Side oppoſite to one of the ſaid Angles, the Reſt are ſought.*

In every plain Triangle, the Sides are oportional to the Sines of the oppoſite ngles; that is, sB : DC :: sC : BD :: D : BC. For $\frac{BC}{2} : \frac{CD}{2} :: sD : sB$, and $\frac{BC}{2} : \frac{BD}{2} :: sD : sC$, and $\frac{CD}{2} : \frac{BD}{2} ::$ B : sC; as is illuſtrated, *Fig.* 12. *Fig.* 12.

THE-

THEOREM III.

Whereon is founded the Solution of plain obliquangular *Triangles, when, there being given two Sides with the Angle contain'd between them, the Rest are ſought.*

$\frac{Zll}{2} : \frac{Xll}{2} :: t.\frac{ZLLop}{2} : t.\frac{XLLop}{2}$; *that is, in Words,* as Half the Sum of the (Legs or) Sides given, is to Half the Difference of the Sides given; ſo is the Tangent of Half the Sum of the oppoſite Angles ſought, to the Tangent of Half the Difference of the oppoſite Angles ſought.

Fig. 13. That is, in *Fig.* 13, GE : BE :: GH : FH.

Demonſtration. In the Triangle BCD *Fig.* 13, let the Legs or Sides given be BD and BC. Draw BG=BC; and ſ there will be $GE=\frac{Zll}{2}$, and $BE=\frac{Xll}{2}$ and the (‖) extern Angle GBC=BCD+ BDC, the two intern and oppoſite An

(‖) By *Theorem* 4, *Chap.* 1, *of the Young Gentleman's Geometry.*

gle

gles of the Triangle BCD. Then draw CG, and after that draw EH, and alſo BF parallel to CD; whence will ariſe the Angle (*) GBF=BDC, and (†) FBC =BCD. Then biſect CG by BH, and there will ariſe the Angle (‖) GBH= HBC. But GBH+HBC=GBC=GBF+ FBC=BDC+BCD, *i. e.* Z∠*Lopp*, And therefore, (becauſe GBH=HBC, and GBH+HBC=GBC,) GBH=$\frac{Z\angle Lopp}{2}$; and ſo GH the Tangent of $\frac{Z\angle Lopp}{2}$. Then make the Angle HBK=FBH. Now, becauſe (as afore has been ſhewn) the Angle GBH=HBC, therefore GBH—FBH =HBC—HBK, *i. e.* GBF=KBC. Wherefore FBC (BCD) —KBC (GBF or BDC) =FBK, or X∠*Lopp*. Wherefore FBH= $\frac{X\angle Lopp}{2}$; and ſo FH is the Tangent of $\frac{X\angle Lopp}{2}$. But now GE : BE : : GH : IH, that is, $\frac{Zll}{2}$: $\frac{Xll}{2}$: : t, $\frac{Z\angle Lop}{2}$: t, $\frac{X\angle Lop.}{2}$

(*) By *Theorem* 3, *Chap.* 1, *of the Young Gentleman's Geometry.*

(†) By *Corol.* 2, *Theorem* 3, *Chap.* 1, *of the Young Gentleman's Geometry.*

(‖) By *Theorem* 5, *Chap.* 1. *of the Young Gentleman's Geometry.*

THE-

THEOREM IV.

On which is founded the Solution of plain obliquangular *Triangles, when, the three Sides being given, the Angles are ſought.*

Fig. 14. DB : DG : : DF : DE, *that is in Words*: The greateſt Side DB of any plain Triangle, is to the Sum DG of the other two Sides, as is the Difference DF of thoſe Sides, to that Segment DE of the greateſt Side, which being taken away, the Cathetus CA will biſect the remaining Segment EB of the greateſt Side.

Demonſtration. By *Prop.* 36, 3 *Euclid*, DG×DF=DT*q*=DB×DE. Wherefore, by *Prop.* 15, 6 *Euclid*, DB : DG : : DF : DE.

From the three laſt Theorems is deduced, *Tab.* II, or the Solution of plain *obliquangular* Triangles, in all the Variety of Caſes that can be propos'd. Nay, the ſaid Theorems may ſerve for the Reſolving of all plain Triangles whatever, *Rectangular* as well as Obliquangular. But foraſmuch as the firſt Theorem do's ſuffice to ſhew the Solution of plain rectangular Triangles, and that after a more eaſy Manner in ſome Caſes, hence

the three laſt Theorems are wont to be reſtrain'd to the Solution only of plain obliquangular Triangles. And conſequently, *Tab.* II, is accommodated only to obliquangular Triangles. Of which Table the twelve firſt Caſes are founded on the 2d Theorem, the three next Caſes, (*viz.* 13th, 14th, and 15th,) on the 3d Theorem; and Caſe the 16th or laſt on the 4th and laſt Theorem. And thus much for the Solution of plain Triangles by Calculation.

CHAP. III.

Of the Solution of plain Triangles by Protraction.

1. HERE are two Things to be ſhewn; 1*ſt*, How to *protract* or deſcribe any plain Triangle requir'd. 2*dly*, How to *reſolve* the ſaid Triangle, when it is protracted; *i. e.* how to find the Meaſure of the unknown Angles or Sides in the ſaid Triangle, when protracted. *Two Things to be here taught.*

2. Protraction is either of ſome Side, or of ſome Angle. A given Side is protracted by drawing a right Line BC, *Fig.* 15, taken off from the (ſimple or diagonal) *Firſt, how to protract any given Side. Fig.* 15.

nal) Scale of equal Parts, at ſuch Length, as to contain the ſame Number of equal Parts according to the Scale, as is the Number of Inches or Feet, *&c.* in the Side given, *viz.* 254.

3. *Secondly, how to* protract *any given* Angle.

A given Angle B=48 Degrees, is protracted, by (taking a Chord of 60 Degrees for your Ray, *i. e.* by) opening your Compaſſes to the Extent of 60 Degrees on your Scale of Chords, and then one Foot being faſtened on one End B of the given Side BC, drawing with the other Foot the Arch CA; and on the ſaid Arch ſetting off from A to E, the Extent of 48 Degrees taken from your Scale of Chords; and after that drawing the right Line BD.

4. *A plain Triangle* how *to be deſcrib'd, when the* three Sides *are given, and no Angle.*

After this manner is any given Side and Angle to be protracted, and ſo any plain Triangle to be deſcrib'd, when either one Side and two Angles, or two Sides and one Angle are given. But, if the three Parts given be the three Sides; then the Triangle is to be deſcrib'd, as is ſhewn *Chap.* 2, *Problem* 2, *Caſe* 1*ſt*, *or* 2*d*, *or* 3*d*, *of the Young Gentleman's Geometry.*

5. *How to find the Meaſure of a* Side not given.

The Triangle BCD, (*Fig.* 15,) being protracted, the Meaſure of any Side not given, is found, by applying the Extent thereof with your Compaſſes to your (ſimple or diagonal) Scale of equal Parts, and ſo finding how many ſuch equal Parts

rts are contain'd in the ſaid Side. hich equal Parts are to be eſteem'd In- es, or Feet, or Yards, accordingly as e Meaſure of the given Side is denoted Inches, or Feet, or Yards, *&c.* Thus, ppoſing BC to be 254 Feet long, CD ll be 198 Feet, and BD 204 Feet.

6. *How to find the Meaſure of an* Angle not given.

In like Manner, the ſaid Triangle be- g protracted, the Meaſure of any An- e C or D not given, is found, by open- g your Compaſs to the Extent of 60 egrees on your Scale of Chords, and en fixing one Foot on the Point of the ngle ſought, and making an Arch from e Leg to the other of the ſaid Angle. he Extent of the Arch apply'd to your ale of Chords, will ſhew the Degrees the reſpective Angle. Thus, *Fig.* 15, e Extent of the Arch FG, ſhews C to an Angle of 52 Degrees; and the tent of the Arch EG, ſhews D to be Angle of 80 Degrees.

7. *The Solution of plain Triangles by Protraction*, on what founded.

It remains to be obſerv'd, that the So- tion of plain Triangles by Protraction, founded on *Theorem* 9, *Chap.* 1, *of the oung Gentleman's Geometry.* Namely, ecauſe the protracted Triangle is equi- ngular to the real Triangle, (*i. e.* the iangular Ground, or the like,) there- re, the Sides of that have the ſame roportion one to the other, as have the

Sides

Sides of this; albeit thoſe are in a m
ner infinitely leſs than theſe.

8. *The Uſe of plain* Trigonometry *appears in taking* Heights *and* Diſtances, *&c.*

And thus having ſhewn, how to ſolve any plain Triangle, either by C culation or Protraction; I proceed n to illuſtrate the great Uſefulneſs of pl *Trigonometry*, in taking *Heights* and *ſtances*, as alſo in *Surveying* and *Fort cation*.

CHAP. IV.

Of Taking the Altitude *or* Hei *of an Object*, viz. *Tower*, T &c.

1. *The* Height *of an Object, what.*

IT is to be remember'd, that (accord to *Definition* 42, *Chap.* 1, *of the Y Gentleman's Geometry*,) the Altitude Height of an Object, *viz.* Tower, Tr and the (*) like, is meaſur'd by a P pendicular from its Top to its Botto Thus, the Height of the Tower, 16, and of the Tree, *Fig.* 17, is den

(*) For as to the Height of the Sun, or any o Celeſtial Light, that is computed by its Diſtance a the Horizon on a vertical Circle: Of which in A nomy.

l by the Perpendicular CA; and ſo n all other ſuch Caſes through this Trea-ſe.

3. *The Symbols made Uſe of in taking Heights.*

For the Line CA of Altitude, and the ine BA of Diſtance, (*viz.* between the bſerver B, and the Foot of the Object ,) and the Line BC of Viſion do always onſtitute the three Sides of a rectangular riangle ABC. Wherein the Height of e Object is the Cathetus or Perpendi-ular, and therefore, (according to the ceiv'd Method of denoting the ſeveral arts of a rectangular Triangle in *Trigo-metry*,) is aptly denoted by CA. In ke manner, the Diſtance of the Ob-rver from the Object, making always e Baſis, is aptly denoted by BA; and e Line of Viſion, making always the ypotenuſe, is aptly denoted by BC. nd ſo as to the ſeveral Angles, A de-otes the right Angle between the Line Altitude and the Baſis, B the Angle etween the Baſis and Hypotenuſe, which ay be call'd, *the Angle of Altitude*, be-uſe the Altitude of the Object is al-ays either (†) its Sine or Tangent; and C will denote the remaining Angle

(†) Namely, if BA be taken for the Ray, then CA is e Tangent of B; but if BC be taken for the Ray, en CA is the Sine of B.

between the Lines of Vision and Alt
tude.

3. *How to take the Angle of Altitude.* The best Instrument for taking th
Angle of Altitude is the Quadrant, theno
commonly distinguish'd by the Name o
the Quadrant of Altitude; by which it
done thus. Hold your Quadrant up
right, so as the String may have Liber
to slip to either Edge, and move th
Quadrant so held, 'till through the Sigh
you see the Top of the Object, whos
Height you would take. Then the De
grees, intercepted between the Beginnin
of the Quadrant and the String, are th
Degrees of the Angle of Altitude; name
ly, of the Altitude of the Object abov
your Eye. Wherefore, if you stand up
right, when you take the Angle of A
titude, to the Altitude found by the sai
Angle of Altitude, there must be alway
added the Height of your Eye above th
Ground, to know the Altitude requir
of the Object.

4. *An accessible and inaccessible Height, what.* If one may come to measure from th
Place B of Observation, to the Foot
of the Object, (*i. e.* to the very Botto
of the Perpendicular CA,) then it
call'd an *accessible* Height. But, if an
Thing hinders from coming to A th
Foot of the Object, then it is call'd a
inaccessible Height.

PRO

PROBLEM I.

To find an accessible *Height.*

Measure your Line BA of Distance, and find the Measure of the Angle B by your Quadrant. These two being known, CA is found by *Tab.* 1, *Case* 1, *viz.* R : tB : : BA : CA. For Instance: suppose BA=347 Feet, and B=35 Degrees. Wherefore, 5. *First, by* Calculation. *Fig.* 16.

As the Ray	10,000000	
To the Logarithm of BA 347 Feet	2.540329	Add
To the artificial Tangent of B 35 Degr.	9.845227	
To the Logarithm of the Side sought CA 243 Feet very neat.	12.385556	
	10.000000	Subst.
	2.385556	Residue.

If you would find the Height sought CA by Protraction, work thus. Draw (according to *Chap.* 3, §. 2.) the Line of Distance BA=347 (equal Parts or) Feet. Then, (by *Chap.* 3, §. 3.) make the Angle B=350, and the Angle A= 90 Degrees, or a right Angle. Then draw the Sides BC and CA, 'till they meet in C. Apply CA to your diagonal Scale, and it will be equal to 243 (equal Parts thereof, or) Feet. 6. *Secondly, by* Protraction. *Fig.* 16.

7. *Some other Methods of taking accessible Heights.*

To the Method of taking Heights b Protraction, may be referr'd the Metho of taking Heights by a *Staff*, or a *Bo of Water*, or the like. Forasmuch as b Help of the Staff or Bowl of Wate (*&c.*) there is as it were protracted Triangle, equiangular to the Triang made by the three Lines of Vision, Distance, and of the Altitude sough Namely, in *Fig.* 17, or 18, the Tr angle *bac*, is equiangular to the Triang BAC, (as shall be shewn by and by.) a therefore, the Measure of the Sides and *ca* being known, as also of B thereby is found (according to the Ru of Proportion) CA the Altitude sough For, (according to *Theorem* 9, *Chap.* *of the Young Gentleman's Geometry*,) *ba* : *ca* :: BA : CA.

8. *Accessible Heights may be taken by a Staff two Ways.*

Now the Way to find the Altitude an Object by a Staff, is two-fold, eith by the Staff and its Shade, when t Sun shines; or else by the Staff alon at any Time, or whether the Sun shi or Not.

9. *To take an accessible Height by a Staff and its Shadow.* *Fig.* 17.

To find the Altitude of an Object b a Staff and its Shade, work thus. At the End of the Shadow cast by the T of the Object, set up your Staff *ca* p pendicularly, (as *Fig.* 17.) and ma the End *b* of the Shadow made by yo Staff. For, as *ba* : *ca* :: BA : CA, i

s the Length of the Staff's Shadow, is o the Height of the Staff; ſo is the ength of the Object's Shadow, to the eight of the Object. For Inſtance: uppoſe your Staff *ca* to be three Foot ong, or juſt a Yard, and its Shadow *ba* o be nine Foot long, and the Shadow A of the Object to be 60 Foot long; nen the Height of the Object will be wenty Foot. For 9 : 3 : : 60 : 20. This Method depends (as is afore ſaid) on *Theorem* 9. *Chap.* 1, *of my Geometry*, or) n the Triangles *bac*, and BAC being utually equiangular. And that they e ſo, is thus ſhewn, *viz.* the Staff *ca* eing erected perpendicularly, therefore, he Angle *a* is a right Angle, and ſo e- ual to A. Alſo, becauſe the (‖) Ray f the Sun, (or, which comes to the ame, the Extremity of the ſhaded Air) *c*, is always parallel to the other Ray C, therefore, (by *Theorem* 3. *Chap.* 1, *f my Geometry*,) the Angle *b*=B, and onſequently, the Angle *c*=C; and ſo

(‖) Although the Rays of the Sun, as proceeding from e Sun as their Center, are in Strictneſs, not parallel, yet y Reaſon of their immenſe Length or Diſtance from e Sun, wherein they concur, their Obliquity is not diſ- rnible in ſuch Caſes, but they may be eſteem'd as par- lel; eſpecially as to ſuch ſmall Segments or Portions of em as are herein concern'd.

the Triangle *abc*, equiangular to the Tri
angle ABC.

10. *To take an accessible Height by a* Staff a-lone. *Fig.* 18.

If the Sun do's not shine, then you may nevertheless find the Height of the Object by your Staff alone, thus. Having set up (as afore) your Staff *ca* perpendicularly, go backward 'till you can see the top C of your Object, in a Line with the top *c* of your Staff, as is represented, *Fig.* 18, by the Line *b*C or BC. Then (*) *ba* : *ca* :: BA (or *b*A) : CA; that is, as the Distance *ba* of the Place *b* where you saw the Top C of the Object and the Top *c* of your Staff in one Line *bc* (or Bc) from the Place *a* of your Staff is to the Height *ca* of the Staff; so is the Distance of the former Place (*b* or) B from the Foot A of the Object, to the Height CA of the Object.

11. *To take an accessible Height by a* Dish *or* Bowl of Water. *Fig.* 19.

If you would for Curiosity Sake know how to find the Height of an Object by a Bowl or Dish of Water, it is done thus. Go backward from the Bowl, 'till standing upright, you can see the Top C of the Object in the Water. Then *ba* : *ca* : BA : CA, *i. e.* as is the Distance *ba* of

(*) For here also *bca* and BCA are equiangular Triangles. For CA and *ca* being both Perpendiculars, are therefore Parallel. And consequently, (by *Theor.* 4. *Chap.* 7. *of my Geometry*,) the Angle *c*=C, as well as *a*=A and the Angle B or *b* is common to both Triangles.

you

our Station *a* from (†) the Bowl *b*, to our own Height *ca*. as far as to your Eyes; so is the Distance BA of the Foot of the Object, to the Height CA of the Object. For, according to that principal Theorem of Catoptricks, the Angle of Reflexion P*bc*, is equal to the Angle of Incidence PBC. And therefore, the Angle *abc* (=1 Right—P*bc*=1 Right—PBC)=ABC. And the Angle A=*a*, because both Right; and therefore, the remaining Angles C and *c* are equal; and so the Triangles ABC and *abc* equiangular.

Lastly, There is one Way more to take an accessible Height, which I shall here mention by Reason of its Easiness and Expediteness, when it may be us'd. It is thus. Go towards or from your object CA, if you have Room, 'till the string of your Quadrant falls just on the 45th Degree. Then the Height CA of your Object, will be equal to BA the Distance between your Place B, and the Foot A of the Object. For the Angles A and C will be equal, (*viz.* each 45 Degrees,) and therefore, the Triangle

20. *To take an accessible Height by* Quadrant only. *Fig.* 20.

(†) If you would be exact, your Distance, and the Distance of the Object must be both taken from the very Point of the Water, whereon you see the Top of the Object.

ABC will be an Iſoſceles, *i. e.* BA=C
by *Theorem* 3, *Corol.* 2, *of the Young Gentleman's Geometry.*

PROBLEM II.

To find an inacceſſible *Height.*

13. By Calculation. *Fig.* 21.

Take two Stations B and *b* lying in direct Line with the Foot A of the Object. The Diſtance B*b* between the two Stations, with the two Lines of Viſion BC and *b*C, make an obliquangular Triangle BC*b*. All whoſe Angles are known, by taking the two Angles of Altitude CBA and C*b*A, in their reſpective Triangles BCA and *b*CA. For 180—Angle CBA=CB*b*; and 180—CB*b*=*b*BC*b*. The three Angles of the Triangle *b*CB being thus known, and the Diſtance B*b* between the two Stations being meaſur'd, the Side BC is found by *Caſe* 1 *Tab.* II, (*viz.* sB : CD :: sD : BC. For that is in Reſpect of the Notation of the ſeveral Sides of the Triangle BC*b*, *Fig.* 21,) sBC*b* : B*b* :: sC*b*B : BC. But now BC is alſo the Hypotenuſe of the rectangular Triangle ABC; and the Meaſure of BC being known, as alſo of the Angle CBA, (*viz.* CBA=180—CB hence may be found CA the Height

e Object, by *Case* 6, *Tab.* I, *viz.* R :
B :: BC : CA.

For Inſtance : I would know the eight of the Church-Steeple, *Fig.* 21, etween A the Bottom of which, and *b* ne Place where I am, runs a River, ſo at it is an *inacceſſible* Height. Where- ore, by my Quadrant, I take the Angle *b*A of Altitude at my Station *b*, which find to be 27 ½ Degrees. Then meaſu- ing in a ſtraight Line from *b* to B my econd Station, (which I find to be 92.7 oot,) I take the Angle CBA of Altitude gain at my ſecond Station B, and find there to be 52 ½ Degrees. Hence it ollows, that the Angle CB*b* = 127 ½ De- rees, (for CB*b* = 180 — CBA ;) and al- o, that the Angle BC*b* = 25 Degrees, for BC*b* = 180 — : CB*b* + C*b*B : = 180 — 155 ;) Having thus in the Triangle BC*b* the three Angles, and one Side B*b* iven, thereby is found the Side BC, by *Case* 11, *Tab.* II, *Viz.*

As

As the Sine of BC*b* 25 D. 30 M. 9.63398
To the Log. of B*b* 92.7 Foot 1.96708
So the Sine of C*b*B 27 D. 30 M. 9.66440

To the Log. of BC somewhat above 99.4 f. *viz.* 100 f. very nearly { 11.63148 / 9.63398 / 1.99750 }

Having thus found the Measure of BC and having afore found the Angle CBA thereby is found CA the Height sought by *Case 6. Tab.* 1. *viz.*

As the Ray 10.00000
To the Log. of BC 100 2.00000
So the Sine of CBA 52 D. 30 M. 9.89946

To the Log. of CA 79.3 and somewhat more, *viz.* 80.3 very nearly { 11.89946 / 10.00000 / 1.89946 }

14. *To take an inaccessible Height by* Protraction. *Fig.* 21.

If you would work by Protraction, you must proceed thus. Having (as afore) at the two Stations B and *b*, found the Angles of Altitude, *viz.* C*b*B and CBA and measur'd the Length of B*b*, then draw a right Line B*b* of as many Parts of the Scale, as the Measure of B*b* is found to be of Feet (or Yards, *&c.*) At the two Points B and *b* make two Angles CBA and C*b*A equal to the Angles

Altitude taken by you at B and *b*. rom C the Point, where CB and C*b* inrſect or croſs one the other being rawn, let fall the Perpendicular CA to e Line of Diſtance B*b* extended, if need e, 'till it meets with CA. The Perpenicular CA apply'd to the Scale of equal arts will ſhew the Number of Parts, *i. e.* Feet or Yards, *&c.*

15. *To take an inacceſſible Height by a Staff and its Shadow.* *Fig.* 22, & 23.

An inacceſſible Height may alſo be ound by the Help of a Staff, and its hade, thus. Set up the Staff *ca* perpenicularly, and mark the End *b* of its Shaow *Fig.* 23; and at the very ſame time ark the End B of the Shade of the inacſſible Object CA *Fig.* 22. Then three four Hours after mark again the End *d* the Staff's Shadow, and alſo the End D the Shadow of the inacceſſible Object. hen meaſure the Diſtance between the vo mark'd Ends *b* and *d* of the Shade of e Staff, and alſo between the two ark'd Ends B and D of the Shade of the acceſſible Object. As *bd* to *ca*, ſo BD CA, *i. e.* As the Diſtance between the vo different Ends of the Staff's Shadow, the Height of the Staff; ſo will the iſtance between the two different Ends the Object's Shadow, be to the Height the Object. For (by *Theor.* 9. *Chap.* 1. *my Geom.*) *da* : *ca* :: DA : CA. Wherefore

fore *da*—(†) *ba* : *ca* :: DA—(†) BA
CA, *i. e. bd* : *ca* :: BD : CA.

16. *To take an inacceſſible Height, when there is not Room to take two Stations on the Ground.* *Fig.* 24.

If it ſhould happen, that you have n
Room to take two Stations, as *Fig.* 2
then an inacceſſible Height may be foun
thus. Firſt take the Arch CBA of Al
tude on the Ground B; and then go
to the Top of the Houſe (or Tree) whe
you are, and there take the Angle CD
Draw an indeterminate Line BA, a
thereon at B erect the Perpendicular BD
of an equal Length with the Height
the Houſe DB, (which is ſuppos'd to
known,) and thereon at D make an Ang
CDB equal to the Angle taken on the T
of the Houſe, and at B make an Ang
CBA equal to the Angle of Altitude t
ken on the Ground. Then extend B
and DC 'till they meet in C. A Perpe
dicular CA, let fall from C upon the i
determinate Line BA, being apply'd
the Scale of equal Parts, will ſhew t
Height of the Object CA.

(†) For as the Lines *da* and DA are proportional
the fore-mention'd Theorem, ſo will their Parts *ba*
BA; becauſe, the Shade of the Staff will be propor
ably lengthened or ſhortened, as is the Shade of
Object.

PR

PROBLEM III.

To measure the Height of an Object, which ſtands on the Top of another Object.

I would know the Height of the Spire *d* above the Tower A*a*. Find the whole Height CA (by *Prob.* 1. if A be acceſſible, *Fig.* 24, if inacceſſible, as *Fig.* 25, then by *Probl.* 2.) Then find the Height A*d* (or *ca*) of the Tower alone (as afore by *Probl.* 1. or 2.) Then CA—A*d* (or *ca*) =C*d* the Height of the Spire. *Fig.* 25, & 26.

PROBLEM IV.

To find the Height of an Hill or Mountain.

This may be done in ſome Manner by *Probl.* 2. But becauſe the Tops of Hills or Mountains, if of any conſiderable Height, do appear higher than really they are, by reaſon of the Refraction of the Ray of Viſion; therefore there is another more exact Method to find the Height of Hills and Mountains, by the Help of an Inſtrument called *the Brazen* T, from *Fig.* 27.

from its reſembling the Letter of th
Name, as may be ſeen *Fig.* 27. For t
Inſtrument being plac'd between tw
Staves (or Spikes, divided into a certa
Number of Feet, and thoſe ſubdivid
into Inches) thereby may be obſerv
(at leaſt by the Help of ſlipping Mark
to be mov'd upwards or downward
what Number of Feet and Inches t
Hill riſes or the Aſcent increaſes at all t
ſeveral Poſitions of the ſaid Staves, fro
the Bottom to the Top of the Hill
Mountain. And conſequently, by a
ding together the ſeveral Feet and Inch
which at the ſeveral Poſitions of t
two Staves, have been obſerv'd to
contain'd, between the horizontal or l
vel Lines, thence will ariſe the Sum
all the Feet and Inches from the Botto
to the Top of the Hill, and ſo the Heig
ſought. Thus the Height CA of t
Hill or Mountain *Fig.* 27. is equal
the Sum of $ab+cd+ef+gh$. And th
Method is called *Cultellation.*

CHA

CHAP. V.

Of Taking the Diſtance *between two* Objects.

1. *Two different general Caſes in taking Diſtances.*

IN taking Diſtances (as well as Heights) there are in general two different Caſes. For either we may come to (*) *One*, or *Neither* of the Two Objects, whoſe Diſtance one from the other we would take.

2. *The* Angle of Diſtance, *what.*

The Angle which is oppoſite to the Line of Diſtance (between the two Objects) may be call'd, the *Angle of Diſtance*, and is here denoted by the Letter C: Foraſmuch as the Triangles ariſing in taking Diſtances are generally obliquangular Triangles.

3. *How to take the* Angle of Diſtance.

The moſt commodious Inſtrument for taking Angles of Diſtance is the *Theodolite* or *Circumferentor*, i. e. a Braſs Circle divided into 360 Degrees, and fitted with

(*) As for the third Caſe, *viz.* wherein *both* the Objects, whoſe Diſtance is requir'd, may be come at, that do not ſtand in need of *Trigonometry* to find the Diſtance, becauſe it may be meaſur'd (without any more ado) by a Chain or other long Meaſure.

four

four Sights. For the Sights being mov'd, as that you can through two them (lying in a ſtrait Line AB) ſee o A, of the Objects; and through the oth two (on the Line DE) ſee the other, E, the Objects, the Angle ACE will be Angle of Diſtance; whoſe Meaſure w be the Degrees intercepted between C and CE; and are always equal to t Degrees between CD and CB, becau the Angles ACE and DCB are ver cal Angles, and conſequently equ Whence BCD as well as ACE may be ſteem'd the Angle of Diſtance.

See *Fig.* 40, of *Geometry.*

PROBLEM I.

To take the Diſtance between t Objects, when One *of the O jects is* acceſſible.

4. *By* Calculation. *Fig.* 28.

I would know the Diſtance of the O ject B (*Fig.* 28.) from the Object one, B, of which Objects is acceſſible, b the other, C, is not. On one Side of B chooſe a Station D, at a (†) conſidera Diſtance from B. The Line BC of

(†) *Note,* That the greater the Diſtance BD is tween the acceſſible Object B, and the Station D, much more exact will the Operation be.

ſta

nce between the two Objects B and C, the Lines BD and CD of Diſtance be-en each Object and my Station, make obliquangular Triangle BCD. Now the gles CBD and BDC being found by the eodolite, and conſequently the Angle B being known, (*viz.* DCB=180— D+BDC) and the Side BD being ſur'd, hereby may be found BC the tance requir'd between the two Ob-s B and C, by *Tab.* II, *Caſe* 9, *viz.* BD : : ſD : BC.

5. *An Example.*

or Inſtance: Suppoſe BD, the Diſtance my Station D from the acceſſible Ob-B, to be one hundred and four Yards; the Angle BDC=86 Degr. and the gle DBC=63 Degr. and conſequently Angle BCD=31 Degr. It follows, rding to the forementiond *Caſe* 9, . II, that

he Sine of C 31 Deg.	9.711839
he Logarithm of BD 104 Yards	2.017033
he Sine of D 86 Deg.	9.999404
	12.016437
he Log. of BC 202 very nearly	9.711839
	2.304598

6. *By Protraction. Fig.* 28.

you would take the Diſtance by Pro-ion, you muſt work thus. Draw containing 104 equal Parts upon your dia-

diagonal Scale, equal to the Number Yards found upon measuring BD. A then at one End B describe the Angl equal to its Measure of 63 Degrees fo by the Theodolite; and likewise at other End D describe the Angle D= Deg. as found by the Theodolite. T draw the Lines BC and DC, 'till they m in C, the Line BC apply'd to your agonal Scale, will shew the Distance the two Objects B and C, *viz.* about Yards.

7. *By a Chain or Pole only.* *Fig.* 29.

If you have no Theodolite, or s like Instrument by you, the Distance may be taken only by a Chain or P (or the like) thus. Measure with Chain the Distance BD between the ceſſible Object B and your Station which your Station choose so in this C as that it may be (as near as you gues's) at an equal Distance with B fr the inaccessible Object C. Then in Lines of Vision DC and BC mark out two Points *d* and *b*, and measure with Chain the several Lines DB, B*b*, *bd*, and *d*B. Then protract a Trapezi BD*db* of the same Sides; the two Si B*b* and D*d* of which being produc'd, meet in C, which will denote the Pl of your inaccessible Object. And con quently the Line BC, being apply'd the Scale of equal Parts, will thereon s

e Diſtance ſought. For by meaſuring d protracting the Sides of the Trapezi- n BD*db*, you do in effect find and pro- & the Angles B and D of the obliquan- lar Triangle BCD, as well as the Side D of the ſaid Triangle. And therefore e protracted Triangle BCD is equian- lar to the Triangle made by the Lines Viſion at B and D, and the Line BD; d conſequently the Sides of the protra- d Triangle are proportional to the Sides the real Triangle, repreſented by the otracted one.

By this laſt Method may the Breadth a River be taken. For if you ſuppoſe D to be one Bank of the River, and C denote a Point of the Bank on the o- r Side of the River; then the Diſtance the Banks or Breadth of the River will equal to BC. Where it is to be noted, t in this Method the Trapezium BD*bd* y be taken or meaſur'd out, on either e of BD, *viz.* either towards the inac- ible Object C, or on the other Side of , as you have Room or moſt Conve- ncy.

8. *To take the* Breadth of a River. *Fig.* 29.

9. *To take* the Breadth of a River, *or any ſuch like Diſtance, by a* Staff *and a* Square-rule.

The ſame may be performed alſo after Manner. Set up a Staff perpendicu- y on the Bank where you are, and reon place a ſquare Rule with its An- downwards on the Head of the Staff. ve the ſquare Rule, 'till the Line BA

 Fig. 30.

of Vision from B a Point in the oppo Bank of the River, falls in directly w the upper Edge of one Side of the squ Rule; and then from the Point C (wh the upper Edges of the two Sides of square Rule meet) draw, along the up Edge of the other Side, a String, or Ro or Chain, 'till it comes in a ſtraight L to the Ground at *b*. As (†) *b*A : CA CA : BA, *i. e.* As the Diſtance *b*A tween the Point *b*, where the String c to the Ground, and the Foot A of y Staff, is to CA the Height of your Sta ſo is the Height CA of your Staff to Diſtance BA of A (the Foot of your St or) the Bank where your Staff is, fro the other Bank. And after the ſame M ner may any other Diſtance be tak where only one Object is inacceſſible.

10. *To take the Diſtance of an* Object of Height, *as of a Tower*, &c. *Fig.* 21, or 22, & 23.

If the inacceſſible Object be an Ob of Height, *viz.* a Tower, or Tree, or like, then having found (by the Prob for taking an inacceſſible Height) Height CA of the ſaid Object, and Angle B of Altitude, thereby may found the Diſtance ſought BA by *Ta Caſe* 2; *viz.* R : *t*C : : CA : BA. For Angle B being known, thereby is the gle C known; *viz.* C=90—B.

(†) Namely, by *Corol.* 1, *Theorem* 11, *of the Young tleman's Geometry.*

P R

PROBLEM II.

To take the Diſtance between two Objects, when both *are* inacceſſible.

II. *By* Calculation. *Fig.* 31.

Take your firſt Station D as you pleaſe; l on the Side of the former chooſe a ond Station *d*, ſo as D*d* may be paral- to BC as nearly as you can. Then with ur Theodolite at D one of your Stati- , take the Angles BDC and CD*d*, ich together make the Angle BD*d*. en at your other Station *d* take the gles B*d*C and C*d*F, (the Mark F being up in a right Line with your two Sta- ns D and *d*) which together make the gle B*d*F. Now in the Triangle BD*d* Angles BD*d*+DB*d*=B*d*F. Where- e the Meaſure of the Angle B*d*F being nd by the Theodolite, DB*d*=B*d*F BD*d*. And therefore the ſtationary Di- nce D*d* being meaſur'd, sDB*d* : D*d* : : D*d* : B*d* by *Caſe* 12, *Tab.* II; *viz.* sB :) : : sC : BD. In like manner in the iangle C*d*D, the Angles CD*d*+DC*d* C*d*F. Wherefore the Meaſure of C*d*F ng found by the Theodolite, the An- DC*d*=C*d*F—CD*d*. And therefore *Caſe* 12, *Tab.* II, sDC*d* : D*d* : sCD*d* :

Cd. Wherefore the Angle C*d*B of t
Triangle B*Cd* being found afore, and t
Angle CB*d*=CD*d* found alſo afore,
follows by *Caſe* 12, *Tab*. II, that sCB
Cd : : sC*d*B : BC the Diſtance ſought.

12. *By* Protraction. *Fig*. 31. To take the like Diſtance by Protrac
on, you muſt work thus. Take (as afo
at your two Stations, D and *d*, the ſe
ral Angles BDC and C*dd*, and C*d*B a
B*d*D. Then having meaſur'd your ſt
onary Diſtance D*d*, and protracted it,
one End D thereof protract its reſpect
Angles BDC and CD*d*; and on the ot
End *d* its reſpective Angles C*d*B a
B*d*D. Draw out the ſeveral Sides of t
Angles meet reſpectively, two BD a
B*d* in B, the other two, C*d* and CD,
B. The Diſtance BC apply'd to yo
Scale of equal Parts, will give you t
Diſtance ſought.

PROBLEM III.

To take the Draught of a City, Port, or the like.

Fig. 32, & 33. This is done after the ſame Manner
is taken the Diſtance between two
jects, when both are inacceſſible; it
ing no other than a Repetition of the
Operation, according to the Variety

remarkable Objects which you would ineate or deſcribe in the Draught. For tance: Let A, B, C, D, E and F de-te ſo many remarkable Buildings (*viz.* urches, or Pillars, or other publick E-ices) in a City; or elſe let them de-te ſo many Ships lying in a Port or rbour. I take two Stations M and N ſo conſiderable a Diſtance, as that the e MN of the ſaid Diſtance may be g enough to ſerve conveniently for the is of all the Triangles to be taken. en at M placing the fixt Index of my eodolite in a right Line with MN, I the moveable Index, as Occaſion re-res; and ſo take in order the ſeveral gles MNA, MNB, MNC, *&c.* and ſo he laſt MNF. Then removing to my er Station N, and placing (as afore) fixt Index of my Inſtrument in a Line h MN, I take the ſeveral Angles MA, NMB, NMC, *&c.*

Having done this, I (†) protract the e MN of Diſtance between my two tions M and N, taking from my Scale equal Parts the Line *mn* anſwerable to

(†) Having the Baſis MN, and the two Angles at the in each Triangle, the other Sides in each Triangle be found by *Calculation*; but this Method being ous, therefore Protraction is generally made Uſe of his Caſe.

MN. And then at the End *m* of the L
mn, I protract the Angles *mna*=MN
mnb=MNB, *mnc*=MNC, &c. Li
wise at the other End *n* of the Line
I protract the Angles *nma*=NMA,
=NMB, *nmc*=NMC, &c. The L
of the several Angles being produced,
shew in the several Points of Interse
on *a*, *b*, *c*, &c. the respective Places
the several Objects A, B, C (&c.); a
consequently their respective Distan
both one from the other, and also fr
each of my Stations M and N; *viz.*
Extents *ab*, or *ac*, &c. or *bc*, or
&c. being apply'd to my Scale of eq
Parts, give the Measure of the said
stances.

PROBLEM IV.

To find the Horizontal Line Length of an Hill or Mountain.

Fig. 34. On the Top of the Hill, at C, set a Staff or Mark of an equal Height w your Eye (above the Ground) or w the Staff to which the Quadrant is f ned at B the Bottom of the Hill. T move the Quadrant 'till you see the T of the (Staff or) Mark at C, where

yo

You'll take the Angle CBA. Then measure up the Side BC of the Hill. Being come up to C, measure the Level *Cc* on the Top of the Hill. Then measure the other Side *bc* of the Hill, and take the Angle *cba*, as you did afore the Angle CBA. Having done this, you may find the Length of the Hill, either by Calculation or Protraction.

If you work by Calculation, then in the Triangle ABC, having the Angle (CBA or) B, and the Hypotenuse BC, the Basis BA is found by *Case* 5, *Tab.* I, *viz.* R : sC : : BC : CA. In like manner, in the Triangle *abc* having the Angle *b*, and the Hypotenuse *bc*, the Basis *ba* is found by *Case* 5, *Tab.* II.

If you work by Protraction, then having protracted the Hypotenuse BC, and on one End B thereof protracted also the Angle (CBA or) B; from the End C of the Hypotenuse let fall a Perpendicular CA upon the other Leg of the Angle B. The Extent BA will be the Basis of the Triangle ABC. In like manner will be found *ba*, the Basis of the other Triangle *abc*.

Having thus found (either by Calculation or Protraction) the Measure of the two Bases BA and *ba*, these two added together, and also to the Length of the Level-Line *Cc* on the Top of the Hill, will give the total Measure of the horizontal

zontal Line or Length of the whole Hill. Namely, Supposing BA=160 Yards or Chains, *&c.* and *ba*=182, and the Level-Line Cc=122, the total Measure of the Length of the Hill will be 464. And thus much for taking of Distances. Proceed we next to Surveying.

CHAP. VI.

Of Surveying *or* Measuring *of* Land.

1. Surveying, *why esteem'd as belonging to* Trigonometry.

THE *Surveying* or *Measuring* of *Land* is esteem'd as belonging to Trigonometry, and so usually taught in Treatises of Trigonometry, because the Way formerly us'd to survey, was by taking the Angles and Sides of the several Triangles, into which any Spot or Tract of Ground may be divided; and then protracting the said Angles and Sides on Paper; and after that finding the Area or Content of the several Triangles so protracted.

2. *An Example of the* old *Method of Surveying.*

This Method is illustrated *Fig.* 35, wherein the several Lines AB, BC, CD, *&c.* represent the several Sides or Hedges of a Ground; and the small Circle T represents the Place of the Theodolite,

te. Which being there plac'd, by turning the moveable Index thereof as Occasion requires, the ſeveral Angles ATB, BTC, CTD, *&c.* are taken. And then by meaſuring with a Chain (or the like) from T reſpectively to A, B, C (*&c.*) hereby is found the Length of the ſeveral Sides TA, TB, TC, TD, *&c.* of the ſeveral Triangles ATB, BTC, CTD, *&c.* And ſo by the Help of the Angles and Sides thus found, the other unknown Angles and Sides of the ſeveral Triangles may be found, (either by Calculation or Protraction;) and ſo the ſeveral Triangles may be protracted, and conſequently the whole Ground, made up of the ſaid Triangles. The Ground being thus protracted, its Area or Content is found (as has been obſerv'd in the Uſe of *Theor.* 7, *Chap.* 1, *of the Young Gentleman's Geometry*) by letting fall a Perpendicular from any Angle in each Triangle to its oppoſite Side, and multiplying the ſaid oppoſite Side into half the Perpendicular. Thus the Content of the Triangle ATB may be found, by letting fall a Perpendicular from the Angle ABT to the Side AT, and multiplying the Side AT into half the ſaid Perpendicular, (or, which comes to the ſame, by multiplying half the Side AT into the whole Perpendicular.) And in like manner may be fonnd

the

the Contents of the other ſeveral Trian-gles BTC, CTD *(&c.)* and conſequent-ly of the whole Ground ABCDEF.

3. *A better Method of Surveying.* But now foraſmuch as the Finding o the Angles and Sides in each Triangle into which any Ground may be divided is only in order to protract the Triangles and ſo to find the Contents of the ſaid Triangles when protracted; hence there is found out another and much more ex-pedite and exact Way for finding the ſaid Contents, (without being at the Trouble firſt to find the Angles and Sides of each Triangle, and then to protract them when found, *viz.*) by finding the Mea-ſure of the Perpendicular and its Baſis *in the Field* or *Ground it ſelf.* I ſhall there-fore ſay no more of the old Method o Surveying, (which properly belongs to Trigonometry) but ſhall forthwith pro-ceed to ſhew the other Method, as being much to be preferr'd before the old Me-thod, for the Reaſons already mention'd. And this Method I ſhall illuſtrate, by ſhewing how to ſurvey thereby the ſame Ground ABCDEF.

4. *An Example.* *Fig.* 36. And this Method is illuſtrated *Fig.* 36. For being come into the ſaid Ground after viewing it, I perceive it may be con-veniently diſtinguiſh'd into two Trapezi-um's ABCD and ADEF. Wherefore ha-

vin-

ing ſet up (*) Marks at the ſeveral An-
les or Corners of the Ground ABCDEF,
he ſtrait Line AD (ſuppos'd to be drawn
etween the Angles A and D) will divide
he Ground into the two Trapezium's a-
oremention'd. Then I (†) meaſure with
ny Chain from A to C for the Diagonal
C, whereby the Trapezium ABCD is di-
inguiſh'd into two Triangles ABC and
DC. And (either (‖) as I go along mea-
uring the Diagonal AC, or elſe after I
ave meaſur'd it) I find the Perpendicu-
rs PB and PD by the Inſtrument for that
urpoſe; which from its Uſe and Shape

(*) Theſe need be no other than Sticks, with ſome
hite Thing (as linnen Rags or Paper) faſtened on their
ops.

(†) In order to meaſure aright the Diagonal AC, (or
ny other ſtraight Line or Side of your Ground,) with
he Chain, you muſt take great Care, that you keep in
ſtraight Line (as you go meaſuring) from A to C. In
rder whereto, he that carries the End of the Chain,
uſt take Care that he that carries the Beginning of it, is
ways (at leaſt that Hand of his which carries the Be-
inning of the Chain) directly between him and the
ark C they are meaſuring to, every Time the Chain
laid down, And in order to prevent any Miſtake as
the *Number* of Chains, it is adviſable for him that
oes firſt at the Head of every Chain's Length to thruſt
faſten a Stick into the Ground, to be taken up by
im that carries the End of the Chain. And ſo at
ſt the Number of Sticks will ſhew the Number of
hole Chains.

(‖) It ſeems moſt adviſable to meaſure the whole Dia-
onal firſt, becauſe, hereby there is the leſs Danger of
iſtaking the Length of the Diagonal.

may

may be call'd the (*) *Perpendicular-T*
In like manner I measure from A to E f
the Diagonal AE, which distinguishes t
Trapezium ADEF into two Triangl
ADE and AEF. And then by my Pe
pendicular-Top, I find (one after the
ther) the two Perpendiculars PD and P
and measure them with my Chain. An
thus, the outer Sides or Hedges AB, B
CD, DE, EF, FA, being all of the
straight or right Lines, the Work is do
in the Field; all that remains to be don
being only to multiply half the Perpend
culars into their respective Diagonals,
order to know the Area or Content
each Triangle or Trapezium; and so
the whole Ground: Which may be do
at Home, the Numbers of the Measu

(*) For this Instrument is best made in the Shape
a common Top, which Boys use to whip. It is b
made of a dried Piece of Box or Pear-tree, that w
bear three Inches, or three Inches and an half Diame
This being turn'd flat on the upper Round, (with
Neck to fit to the Head of a Staff,) find the Center
the said upper Round or circular Surface, and there
draw two or three concentrical Circles, (as you see, *F*
38,) divide the said Circles exactly into four eq
Parts, (as in *Fig.* 38,) and then, with a Whipsaw v
thin, saw by the Marks *a*, *b*, *c*, *d*, the two Lines
and *cd* at right Angles pretty Deep. Thus, your Inst
ment is made; only in order to use it, it must be
on the Head of a Staff, as has been above intimated.
which End it is to be made with an Hole at the Bot
to slip on upon the Head of a Staff.

f the ſeveral Diagonals and Perpendicu-
rs being ſet down in Writing, as ſoon
s taken.

And here it muſt be ſhewn, how the
lumbers, denoting the Number of whole
hains or of ſingle Links, are to be mul-
plied in order to find the Content. The
hain then here ſuppoſ'd to be made uſe
f by the Surveyor is that, which from
s Contriver goes by the Name of *Gun-
er's Chain*, and which is beſt contrived
or Surveying. It contains four Statute-
oles or Perches, each Perch containing
fteen Feet and an half, or five Yards and
n half: ſo that the whole Chain is ſixty-
x Feet or twenty-two Yards long. The
hole Chain is divided into an hundred
qual Links, ſo that twenty-five Links
re juſt a Pole or Perch. And for the
ore ready counting the Links, it is uſu-
l to diſtinguiſh every Ten with a Braſs
late, (or the like) ſo contriv'd as to
ew, ſome way or other, whether it be
he firſt, or ſecond, or third, *&c.* Length
i. e. the 10th, or 20th, or 30th, *&c.*)
ink from either End; the 50th or Mid-
le Link having a more remarkable Plate
han the reſt.

5. *Of* Gunter's Chain, *and the Way of* Working by Numbers, *to find the Content of a Ground by the ſaid Chain.*

In meaſuring any Line or Extent by
his Chain, regard need be had only to
hole Chains or whole Links; which are
o be writ down with a ſeparating Note,
(whe-

(whether Full-point or Comma, or any o
ther Mark) between the Chains a
Links, (as is uſual to do between Intege
and Decimals,) *viz.* 6.35 denotes
Chains and 35 Links. If the Links
under 10, a Cypher muſt be prefix'd, *vi*
7.09 denotes 7 Chains and 9 Links.

The Numbers denoting the Chains a
Links being ſet down, as has been direc
ed; to find the Content (of an exa
Square or other rectangular Parallelogra
you multiply (*) the Length by t
Breadth; but it ſeldom hap'ning th
Grounds are of ſuch a Shape, hence it
generally requiſite to diſtinguiſh yo
Ground into Trapeziums, as has been
fore ſhewn, and then to find the Co
tent) you multiply the Diagonal by (†
half the Perpendicular, (or (‖) the lik
and from the Product cut off five Figur
(accounting Cyphers for ſuch,) reckon
from the right Hand, with a Daſh

(*) Agreeably to what has been taught in *Defin.* 4
Chap. 1, *of the Young Gentleman's Geometry.*

(†) Agreeably to what has been taught in the *Uſe*
Theorem 7, *Chap.* 1, *of the Young Gentleman's Geometry.*

(‖) Namely, the Product will be the ſame, whet
you multiply the whole Diagonal (or Baſis) by Half t
Perpendicular, or the whole Perpendicular by Half t
Diagonal, or the whole Diagonal by the whole Perp
dicular; and take Half the ſaid Product for the Prod
ſought.

you

ır Pen: So that the Figures to the left nd denote the Acres.

If the five Figures cut off to the right nd were not all Cyphers, multiply m by 4; and from the Product cut again five Figures towards the right nd, and the reſt will denote Roods or arters of an Acre.

Laſtly, if among the five Figures of the Product, cut off towards the right d, there be any Figures beſides Cyrs, multiply all the five Figures by From the Product cut off again five ures to the right Hand; and thoſe of left Hand will denote ſquare Perches oles. I ſhall now illuſtrate the foreg Rules by ſome Examples.

uppoſe then in *Fig.* 36, the Diagonal o be juſt 7 Chains long, and the Perdicular PF to be 2 Chains and 88 s, and the other ſhort Perpendicular o be 60 Links. I (†) add the two endiculars together, and the Sum of makes 3 Chains and 48 Links.

† For this is the moſt expedite Way, hereby have'd the Trouble of two diſtinct Multiplications, PF by 7ch, and then of ½ pD.

Then the Half of the said Sum of my Perpendiculars, viz. 174, I multiply by the whole of my Diagonal (‖) 700. I cut off the five Figures to the right Hand, and the remaining 1 denotes one Acre.

Then I multiply the five Figures c by 4, and the Product, (viz. 21800 87200, not amounting to above fi gures, I perceive the Trapezium A do's not contain a full Rood (or Q of an Acre) above the whole Acre f afore.

Therefore I multiply again the f Product 87200 by 40 ; and cutting off again the five Figures to the right Hand in this third Product, the two remaining Figures 34 denote so many square Perches or Poles. Which wants almost six square Perc a Rood, or Quarter of an Acre, thi taining 40 square Perches. For tute-Acre contains 160 square S Perches.

(‖) I choose my Diagonal for the Multiplier, it being 700, I need multiply only by the F and to the Product thereof add the two Cypher

Upon the Whole therefore I find, that
e Trapezium ABEF contains one Acre,
d a Quarter of an Acre over, wanting
ſquare Perches. I proceed then to find
e Content of the other Trapezium
BCD; whoſe Diameter AC ſuppoſe
be 5 Chains and 15 Links; the Perpen-
cular PB to be 1 Chain and 62 Links;
d the other Perpendicular PD to be 1
ain and 89 Links. Where-

e I multiply the Diagonal	515
5 by Half the Sum of the	175
o Perpendiculars, *viz.* 175;	2575
d the Product not ariſing to	3605
ove five Figures, I perceive	515
t the ſaid Trapezium does	90125
t contain a Whole Acre.	

Then I multiply the Product found by
and from the ſecond Pro-

t cutting off five Figures to	90125
right Hand, there remains	4
Figure 4, which denotes the	360500

ntents of the Trapezium to
3 Roods or three Quarters of
Acre, and ſomewhat more.
I proceed therefore to find what more
is, which I do by multiplying the 5 Fi-

es cut off by 40. From	60500
is third Product I cut off as	40
in the five Figures to the	24|20000
ht Hand, and the remain-	
24 denote ſo many ſquare	

Perches; which is a little above Ha
Quarter of an Acre. So that upon
Whole I find the Trapezium ABCD
contain 3 Roods and a little (viz. 4 squ
Perches) above half a Rood.

In the last Place then, by adding
Contents of the two Trapeziums AB
and ADEF toge-
gether, I find the
whole Ground
ABCDEF to con-
tain one Acre,
three Roods, and
fifty-eight square
Perches: Which
square Perches a-

Acres	Roods	Sq. Per.
1 .	00 .	34
0 .	03 .	24
1 .	3 .	58, that
2 .	0 .	18

mounting to a Rood more, and eight
square Perches over, therefore the C
tent of the said Ground is in all, (w
expres'd after the more proper Man
two Acres, and eighteen odd square P
ches.

After the same Manner may the C
tent of the Ground ABCDEFGHIKL,
presented *Fig.* 37, be found. Na
ly, first, set out in it as large a Tra
zium ABGL as you can; and find
Content of the said Trapezium, by m
tiplying Half the Diagonal, AG into
Whole Sum of the two Perpendiculars
and PL. Then find the Content of
Triangle BFG by multiplying its B

into Half the Perpendicular PF. Then the Content of the Nook CDE by inguishing it into as many Trian- (*) as you see requisite, *viz.* CD*c*, , and *d*cE; and finding the Content ach Triangle, as you did of the Tri- e BFG. And in like manner is the tent of the other two Nooks GHI and to be found. And then at last, the ral Contents of the several Parts of said Ground being added all together, give the Content of the Whole und. And after this Method may a- Piece of Land be survey'd, the several s that can happen in measuring the or Hedges of any Ground being re- ented in *Fig.* 37.

6. *Of* Laying *or* Setting out *a Piece of Land requir'd.*

Hitherto it has been shewn, how to ey or measure a Piece of Land *already* or *laid out* within its respective nds. It may be of use to observe next, how to *set* or *lay out* a Piece and or Ground of such a Measure, all be requir'd, or containing such or a Number of Acres, *&c.*

(*) Namely, by this Method circular Nooks may be d as exactly as by any other; at least so exactly, as at is wanting of Exactness, shall be so little as altogether inconsiderable. But it must be noted, e Triangles such circular Nooks or Windings are into, the nearer you'll come to the exact Measure

7. *A Statute-Acre, what,*

In order hereto, it is to be known in the first Place, that an Acre according to Statute-Law does contain one hundred and sixty square Statute-Poles or Perches, each such Pole or Perch containing (as has been above observ'd) fifteen Feet and an Half, or five Yards and an Half.

Such being the Content of an Acre, it follows, that all rectangular Pieces of Ground, any of whose contiguous Sides of any of the two Lengths set oppositе hereto, will contain just an Acre. Whence it is obvious, how a Ground of a single Acre may be laid out, if there Room to make its contiguous Sides pendicular one to the other, *viz.* by king the two contiguous Sides of so the two Lengths set together a Hence also it is obvious, that Gr which have very different Sides, ma have the very same Content.

Perches			
80	*	2	Square Per
40	*	4	=160=1 A
32	*	5	
20	*	8	
16	*	10	
&c.			

If there be not Conveniency to la the Acre of Ground in a recta Shape, then it may be laid out it i other Shape or Figure, by consid what Proportions are to be take make the Content of the said Figur

al to 160 equal Perches. For In-
ce; if I can lay out the Acre in
Triangular

Basis		Perpend.	
160	&	2	160 × 1
80	&	4	80 × 2
64	&	5	for 64 × 2½ = 160
40	&	8	40 × 4
32	&	10	32 × 5
(&c.)			&c.
Perpend.		Basis	

pe; then
Base and
pendicular
the said
ngle must
e some of
opposite
such like) Proportions one to the o-
r; forasmuch as the Basis multiplied
o Half the Perpendicular (or *vice ver-*
gives the Content of a Triangle.

The Way, how to lay out an Acre of
ound in a triangular Shape, being
own, thereby may be known the Way
ay out an Acre of Ground in any mul-
gular Figure; forasmuch as all Poly-
ns may be constituted by adjoining to-
her several Triangles. And it being
known how to lay out a *single Acre*
Ground in any Shape or Figure;
nce may be known how to lay out
ounds of *more Acres then one* in any
pe, by proportionably encreasing the
ngths of the respective Parts. And thus
ave taken notice of so much of Sur-
ying, as seems requisite to the Design
this Treatise.

CHAP. VII.

Of Fortification.

1. *A Fort, what.*

BY a *Fort* or *Fortreſs* is ſignify'd a Place *fortify'd*, i. e. made ſtrong by the Art of Military Architecture, or Fortification.

2. *A Rampart, what.*

The principal Part of a Fort, is that which is called the *Rampart* or *Rampire*, being a Wall of a good Height and Breadth, compos'd chiefly of Earth, and encompaſſing the Place fortify'd. It is repreſented *Fig.* 39, by DEWXYZ.

3. *A Bulwark or Baſtion, what.*

The *Bulwark* or *Baſtion* of a Rampart is that Part of it, which runs farther than the reſt, ending in an Angle. It is repreſented *Fig.* 39. by IFEGH, &c.

4. *The Face, Gorge, Shoulder, and Flank of a Baſtion, what.*

The ſeveral Parts of a Baſtion are the *Face* or *Front* FE; its *Gorge* (or Neck) IH; its Shoulder F; and its *Flank* (or Wing) FI; which, according to the Modern Method of the *French*, is not built in a ſtrait Line, but rounding, as is repreſented in ſome of the other Baſtions, by the Prick'd-circular Lines about the Flank.

5. *A Curtain, what.*

The Part of the Rampart between two Baſtions is called the *Curtain* KI, which joyns together the two Baſtions IFE and KLDMN.

Mo

Moreover, forasmuch as (according the Rules of Fortification) every Rampart is either a (*) Polygon, or at least a Tetragon; hence the Line DE is called *the Side of the outer Polygon*; BC *the Side of the inner Polygon*; AC *the Semidiameter* of the *inner* Polygon; AE the *Semidiameter* of the *outer* Polygon; pA the *Cathetus* of the *inner* Polygon; PA the *Cathetus* of the *outer* Polygon; Pp the *Distance of the Polygons*.

6. *The* Side of the outer *and* inner Polygon, *&c. what.*

Besides which there are wont to be peculiarly distinguish'd also these several Lines, *viz.* the *Gorge-line* IC; the *Head-line* CE; the (†) *Longest Line of Defence* KE, the *Shortest Line of Defence* FE.

7. *The* Gorge-Line, Head-Line, *&c. what.*

By the forementioned Lines and Parts of the Rampire are constituted several Angles, whereof the Chief are these; *viz.* BAC the *Angle of the Center*; BCA or CBA the *Angle of the Polygon*; FEG the *Angle of the Bestion*, otherwise called the *Diamond Point*, and the *Flanked Angle*; DOE the *outer Flanking Angle*; FkI the

8. *Of the principal* Angles *in Fortification.*

(*) The Reason of this Rule of Fortification depends upon the due Proportion there ought to be between the several Parts of the Rampart. And hence, according to the said Rules, a regular Polygon is to be preferr'd before an Irregular. The Rampart, *Fig.* 39, is a regular Hexagon.

(†) See *Sect.* 30, 31, 32, of this Chapter.

inner

inner Flanking Angle; FCE *the Fro angle*, or Angle forming the Face; F *the Flank-angle*, or Angle forming t Flank; CEF *the Head-line Angle*, &c.

9. *How Fortification* belongs to Trigonometry.

It is also obvious, that by the forem tion'd Lines are made several plain Tri gles, *viz.* ABC, ADE, APD, APE, A ApC; *p*FI, *k*FI, FIC, FCE, *&c.* Hen any three Parts of each of the said Tri gles (except the (‖) three Angles) bei known, the other three unknown may found by Trigonometry, as has been fore shewn. Such as would have par cular Illustrations by Examples, may co sult such Treatises as are professedly wr ten of Fortification. Those young Ge tlemen, for whose Use this Treatise is specially design'd, being such as are n likely to concern themselves in the prac cal Part of Fortification, it seems abu dantly sufficient to my Design, to hav thus shewn in general how Fortificatio depends upon Trigonometry.

10. *The Explication of other Terms of Fortification.*

However being thus led, by (the Su ject of this Treatise) Trigonometry, take notice of the foremention'd Par and Terms of Fortification, because may be of good Use, in order to ma

(‖) The Reason of this Exception is largely given, Ch 2. §. 1, of this Trigonometrical Treatise,

you

ung Gentlemen understand, in a com-
tent measure. Relations given of Sieges,
hall therefore here add the Explication
such other Parts and Terms of Fortifi-
tion, as are of principal Note and Use,
d so most frequently occur in the Ac-
unts of Sieges.

A *Parapet* then is (properly) a Work 11.
is'd on the Rampart, behind which the *A Parapet, what.*
oldiers stand, and where the Cannon is
lac'd for the Defence of the Fort. As in
ig. 51 and 52, *ab* denotes the Breadth
f the Rampart at its Basis or Bottom, so
g denotes the Breadth of the Parapet at
s Bottom, or where it is rais'd on the
ampart.

A *Banquet* is no other than a Foot- 12.
ace at the Bottom of a Parapet, upon *A Banquet, what.*
which the Soldiers stand up to fire into
he Ditch, or upon the Covert-way be-
ond the Ditch. Some Parapets have on-
y one Foot-pace, as is represented by
le, Fig. 51. But it is the more commo-
dious to have two, one for the taller Sol-
diers, and another (somewhat above the
former) for the shorter, as is represented
between *d* and *e*, *Fig.* 52.

According to the old Way of Fortifica- 13.
tion, there is erected between the Ram- *A Fausse-*
parts and the Ditch, a Work called the *bray, what.*
Fausse-braye, or *Half-braye*, being a smal-
ler Sort of a Rampart, having a Parapet,
and

and running round the whole main Ra
part of the Fort. It is represented *F*
51, by *xyz*. But the *French* omit th
Work now-a-days in their Fortificatio
leaving between the Rampart and t
Ditch only a Space about three or fo
Foot wide, to receive the Earth whic
falls from the Parapet. This Spac
they call the *Berme*. See *Fig.* 5
Wherein, as also *Fig.* 51, *klmn* denot
the Ditch.

14. *The Covert-way, what.* On the outer-side of the Ditch is th *Covert-way*, being a Way or Walk (round the out-side of the Ditch) cover'd or defended by the Glacis. It is represented *Fig.* 51 and 52, by *wp*.

15. *The Glacis, what.* The *Glacis* serves as a Parapet to the Covert-way, and loses it self insensibly in the Field. Hence the Largest are esteem'd the Best, because they lose themselves more insensibly than shorter ones. It is denoted *Fig.* 51 and 52, by *prs*.

16. *The Counterscarp, what.* The *Counterscarp* is properly the Slope *nm* of the Ditch on its Out-side, as being opposite to the *Scarp* or Slope *kl* of the Ditch on its Inside. But by this Term now-a-days is generally understood the Covert-way with its Parapet.

17. *Of the additional Works of Fortification.* Besides the Works hitherto spoken of, and which are requisite to all Fortifications, there are also other *additional* Works contrived by Masters of Military Architecture,

re, for to further fortify the ſeveral Parts
Fortification already deſcrib'd. And
ch are *Half-Moons*, *Horn-works*, *Crown-*
rks, *Star-works*, *Ravelins*, *Tenailles*, &c.
f which the four former take their
ames from their Shape. They will all
better apprehended by their reſpective
raughts, than by verbal Deſcriptions.

I begin with the *Tenaille*; of which 18.
ere are three Sorts, repreſented *Fig.* 40, *A Tenaille, what.*
, and 42. They are placed in the
itch, and before the Curtains of the
ampart. They are ſaid to be contriv'd
the *French* inſtead of the *Fauſſe-braye*.
The Tenaille, *Fig.* 41, is diſtinguiſh'd
the Name of the *ſingle* Tenaille; and
at *Fig.* 42, by the Name of the *double*
enaille.

A *Ravelin* is repreſented *Fig.* 43; be- 19.
g a ſmall triangular Work. It is gene- *A Ravelin, what.*
lly rais'd alſo before ſome Curtain, ei-
er of the main Rampart, as *Fig.* 43,
d 46, or of ſome Out-work, as the
avelin, *Fig.* 44. before the Curtain of
e Horn-work, *Fig.* 47, or elſe before
me other Part analagous to a Curtain,
the Ravelin, *Fig.* 45, before the ſingle
enaille, *Fig.* 41.

An *Half-Moon* (*Fig.* 53.) is generally 20.
nfounded with a Ravelin, its Diffe- *An Half-Moon, what.*
nce being principally (if not only)
is, that its Gorge *df* is an Arch in Shape
of

of the Concave Part of an Half-Mo
and it is placed overagainst Z, the Po
of a Baſtion to cover it; whereas a Ra
lin conſiſts of two ſtraight Demigor
as *de ef. Fig.* 46, and is placed overaga
ſome Curtain, as is afore obſerv'd.

21. *An* Horn-work, *what.* An *Horn-work* is repreſented *Fig.* The Fore-part of it conſiſts of a Cur between two Half Baſtions, which run like Horns, whence it takes its Deno nation. It is placed either before a C tain or a Baſtion.

22. *A* Crown-work, *what.* A *Crown-work* is repreſented *Fig.* 48. is ſo called, becauſe the Fore-part of reſembles a Crown. It is uſually m before an Horn-work, to keep the E my at a Diſtance. It is alſo placed imm diately before a Curtain or Baſtion of Rampart.

23. *A* Star-work, *what.* A *Star* is a Work with ſeveral ces, which flank one the other, as 49.

24. *A* Redoubt, *what.* A *Redoubt* is a ſmall ſquare Fort, Guard-houſe, having no Defence but Front. It is uſually deſigned for ma taining the Lines of Circumvallation, Countervallation. In watery Places th are often made of Maſon's Work. It repreſented *Fig.* 50.

25. Out-works, *what.* All the forementionʼd Works, wh lie without the Rampart of the Forts, in general callʼd the *Out-works.*

A *Battery* is a Place rais'd whereon to plant the great Cannon, in order to batter or play upon the Place besieg'd. 26. *A Battery, what.*

Cavaliers are Works rais'd on the Parapet of a Rampart, whereon to plant Cannon in order to scour the Field, and to oppose any Work rais'd by the Besiegers, which commands the Place besieg'd. 27. *Cavalliers, what.*

Fascins are a Sort of Faggots, made to throw into the Ditch, where there is much Water, in order to make the Passage over to the Wall or Rampart of the Place besieg'd, more safe. 28. *Fascins, what.*

Gabions are Baskets filled with Earth, and usually placed upon Batteries, and Parapets, that have suffer'd very much, and also before other Places, to secure them from the Enemies Shot. Lesser Gabions are call'd *Corbeilles.* 29. *Gabions, what.*

A *Gallery* is a made Way or Walk, cover'd with Earth or Turf, and upheld with Planks. They are made in the Ditch already fill'd with Fascins, to the end the Miners may approach safe to the Bastion. 30. *A Gallery, what.*

The *Line of Defence* is that, which is represented by the Discharge of the small Shot towards the Face of a Bastion. It is distinguish'd into the *Longest* and *Shortest*, otherwise called *Fichant* and *Rasant.* 31. *Line of Defence, what.*

The

32. *Line of Defence* Fichant, *what.*

The *Line of Defence Fichant* (or *Longest Line of Defence*) is the Line KE, called, because the Bullets or Shot, which pass along this Line, may fix on the Face of the opposite Bastion.

33. *Line of Defence* Razant, *what.*

The *Line of Defence Rasant* (or the *Shortest Line of Defence*) is the Line KE drawn from the Face of the Bastion, to that Point E, of the adjoining Curtain, where the Face of the said Bastion begins to be discovered. It is so call'd because the Shot, that passes along the said Line, only rases the Face of the said Bastion.

34. *A* Lodgment, *what.*

A *Lodgment* is a Work cast up in a dangerous Post won by Attack, to secure the Besiegers against the Enemies Fire. It is made of any Materials, that are capable of making Resistance.

35. *To* Nail up a Cannon, *what.*

To *nail up a Cannon*, is to drive a Nail (or the like) into its Touch-hole.

36. *To* Sap, *what.*

To *Sap*, is to dig deep into the Earth of the Covert-way and Glacis in the Form of a Trench. The Earth dug out serves for a Security on the Right and Left, and covers above, the Soldiers (or others) in the Trench against the Enemies Fire works, by the help of Hurdles laden with the said Earth.

Militar

Military *Trenches* are of the ſame Kind with common Trenches, only larger, being uſually from ſix to ſeven Foot in Depth, and from eight to ten Foot in Breadth. They are made by the Beſiegers, to approach more ſecurely the Place beſieged; whence they are otherwiſe called the *Lines of Approaches*, or ſometimes *the Approaches*. And they are therefore to be carried on, as not to be in View of the Enemy, or preſently and eaſily diſcoverable by the Enemy or Beſieg'd. 37. Trenches, *what.*

Lines of *Circumvallation* are Trenches made by the Beſiegers about their Camp: well to hinder the Relief of the Beſieged, as to ſtop Deſerters from their own Army. They are guarded with a Parapet. 38. *Lines of* Circumvallation, *what.*

Lines of *Countervallation* are Trenches which the Beſiegers make to ſecure themſelves from the Sallies of the Beſieg'd. They are alſo guarded with a Parapet. 39. *Lines of* Countervallation, *what.*

Lines of *Communication* are Trenches made from one Work to another by the Beſiegers, for the better holding Communication one with the other, and helping the other, in caſe of an Attack by the Enemy, in any of their Works. 40. *Lines of* Communication, *what.*

A *Retrenchment* is a Work made by the Beſieg'd to ſecure themſelves, and ſo to enable them to hold out ſome time longer 41. *A* Retrenchment, *what.*

ger against the Enemy, who is so far ad vanc'd as to be lodg'd on the Rampar It has either a good Parapet, or inste thereof Gabions. And thus I have e plain'd the more usual Terms of Forti cation, and such as more frequently o cur in Relations of Sieges.

CHAP. VIII.

Of the Tables of Logarithms, *an* Artificial Sines *and* Tangents.

1. *The Method of the Tables of* artificial Sines *and* Tangents, *adjoin'd to this Treatise.*

THE Tables of Artificial Sines a Tangents belonging properly Trigonometry, they are therefore h adjoin'd to the End of this Treati They are so contriv'd, by placing t Sines and Cosines by the Side one of t other, and likewise the Tangents and C tangents, that each Page shews the Si and Cosine of two Angles, one where is set down on the Top of the Pag the other (being the Complement of t former to 99) at the Bottom of t Page. Thus the Page (or Pages,) t hath Degree 2 at the Top, hath D gree 87 at the Bottom. And as the Si and Cosines, Tangents and Cotange belong

elonging to the Degrees ſpecified at ne Top, are reſpectively ſhewn by the eſpective Words ſet at the Top of the veral Columns in each Page; ſo the nes and Coſines, *&c.* belonging to the egrees ſpecified at the Bottom, are re-ectively ſhewn by the reſpective Words t at the Bottom of the ſeveral Columns each Page.

The Minutes between each two De-rees are ſet down on the Sides of the ages, thoſe appertaining to the Degrees the Top increaſing downwards; the her appertaining to the Degrees at the ottom increaſing upwards. And on the de of each Minute ſtands its reſpective ne and Coſine, Tangent or Cotangent.

And, although Logarithms are of Uſe, other Parts of Mathematicks as well in Trigonometry, yet becauſe they uſt always be uſed with Artificial Sines nd Tangents; I have therefore judg'd is the moſt proper Place alſo for the able of Logarithms belonging to my athematical Treatiſes: of which I need y no more here, the Uſe of the ſaid Ta-e of Logarithms having been already plain'd, in the Appendix to the *Young entleman's Arithmetick.*

2. *The Table of Logarithms, why adjoin'd alſo to this Treatiſe.*

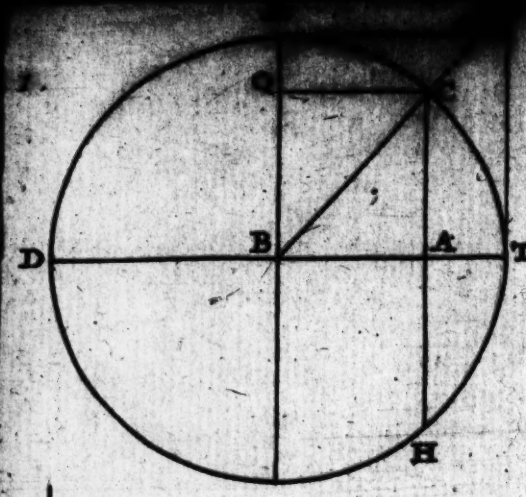


Fig. 2.

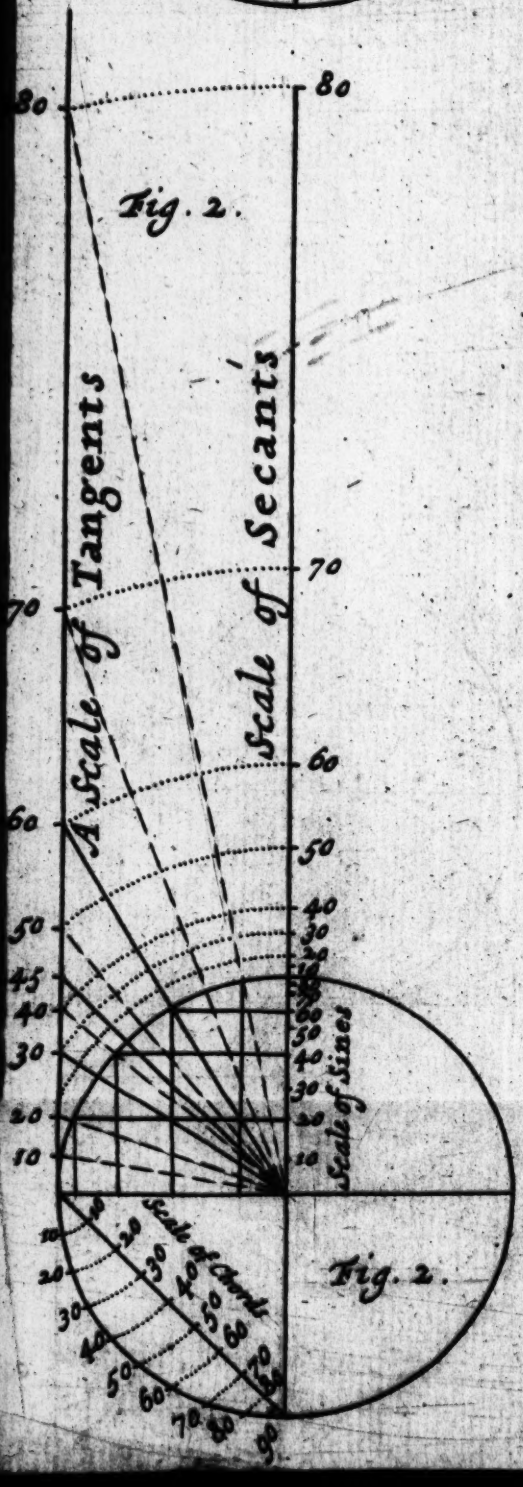


Fig. 3.

A Simple or Plain Scale of Equal Parts

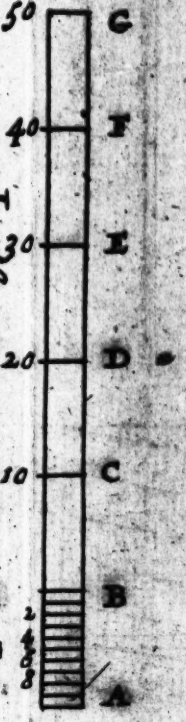


Fig. 4.

A DIAGONAL SCALE

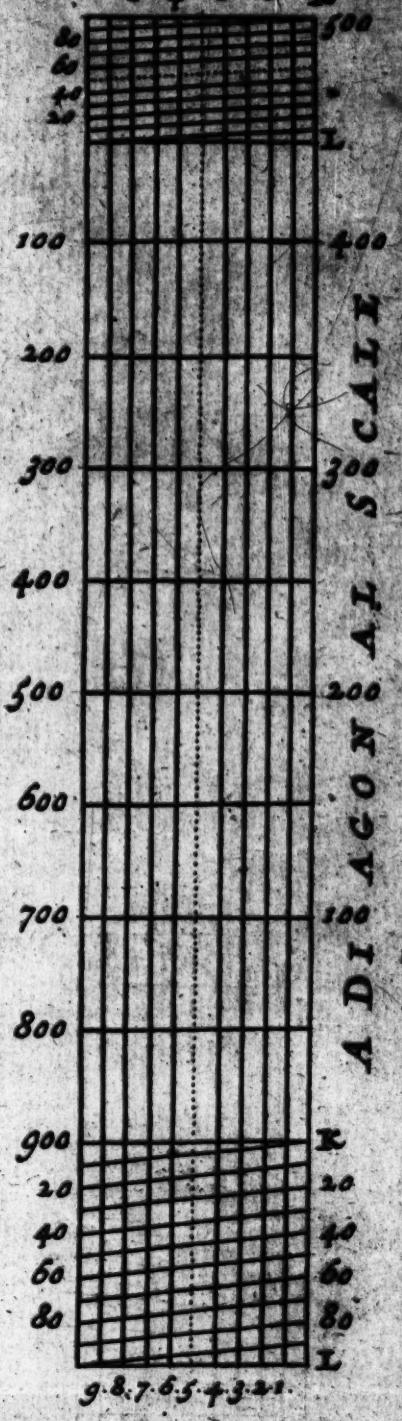
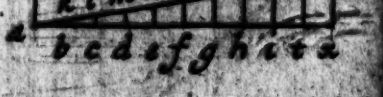


Fig. 5.

Fig. 6.



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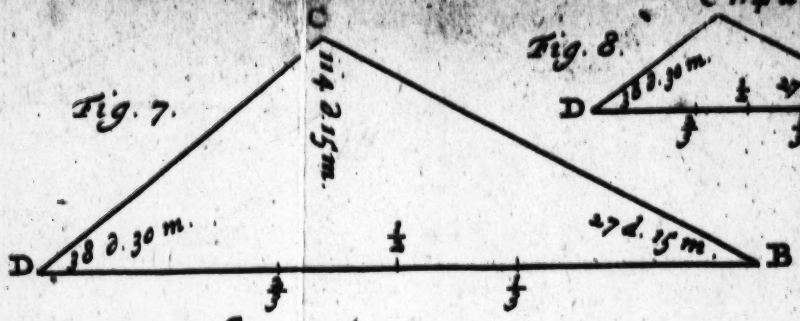

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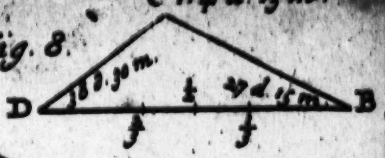


Fig. 9.

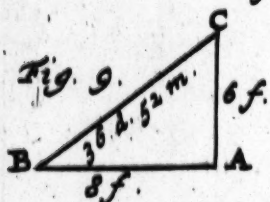
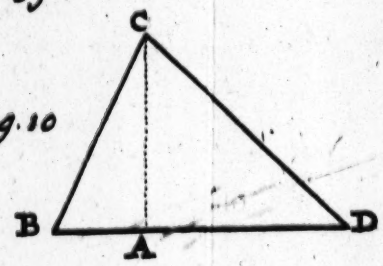


Fig. 10



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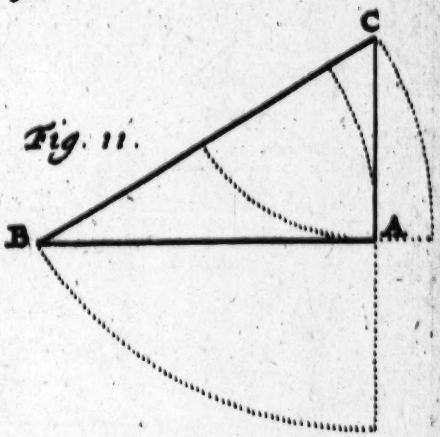

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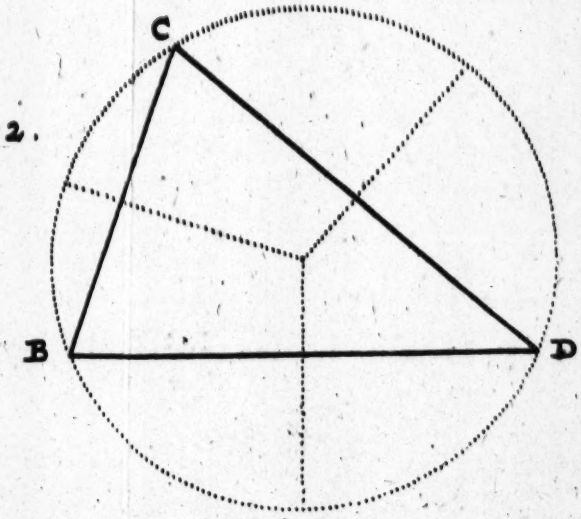
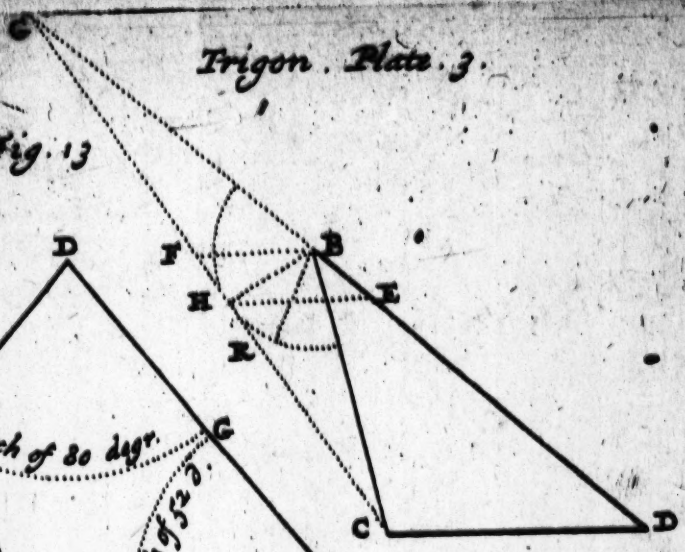


Trigon. Plate 3.

Fig. 13



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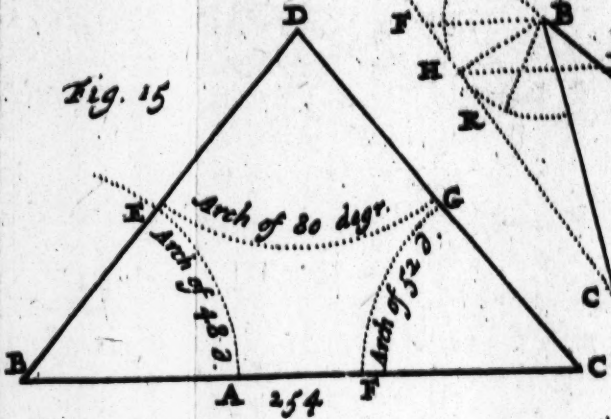

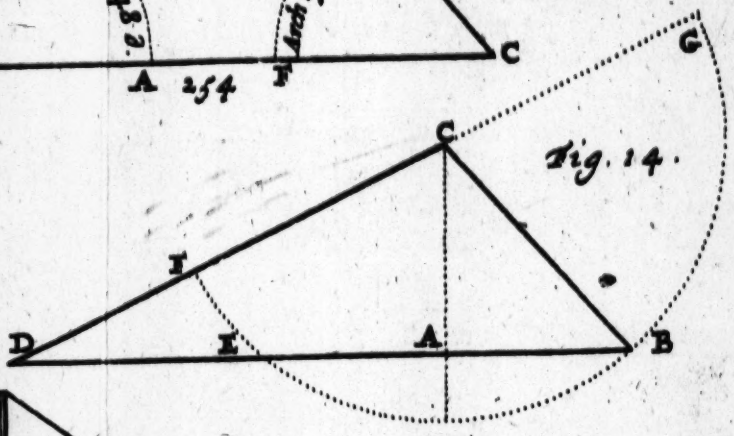

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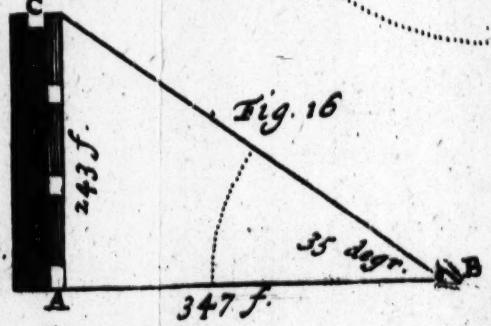


Fig. 16

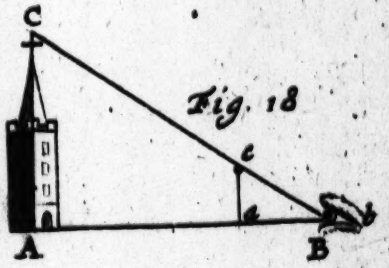
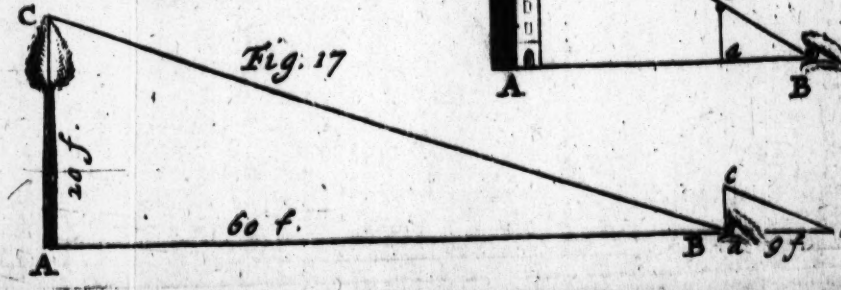


Fig. 18



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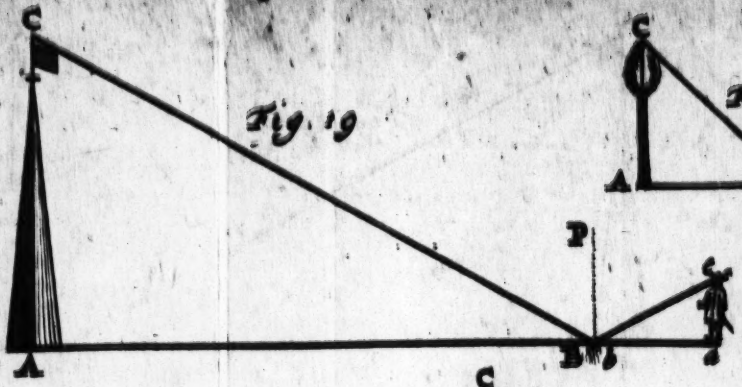


Fig. 19

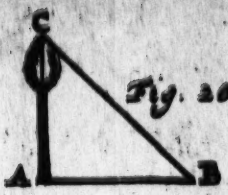


Fig. 20

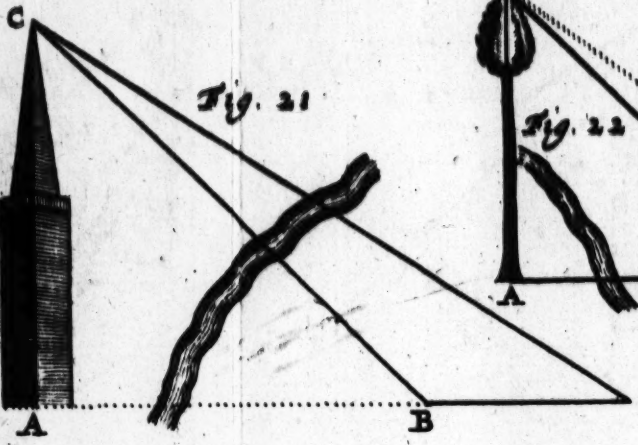


Fig. 21

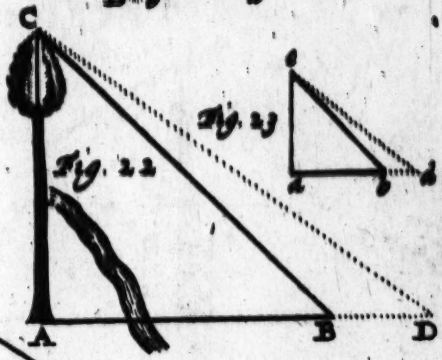


Fig. 22

Fig. 23

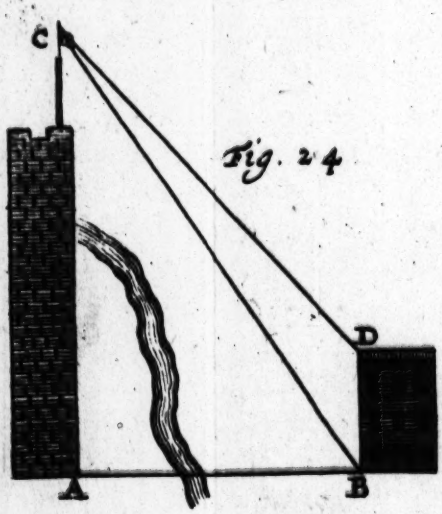
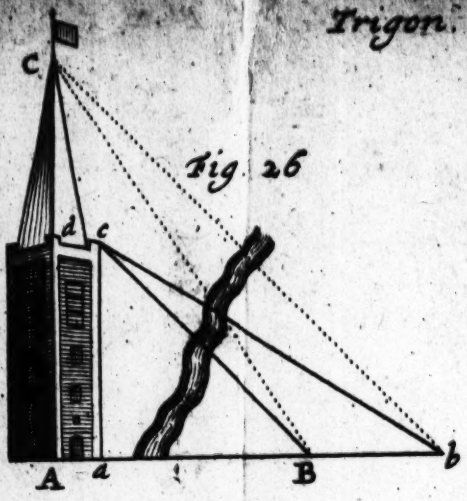


Fig. 24

Fig. 25

Fig. 26



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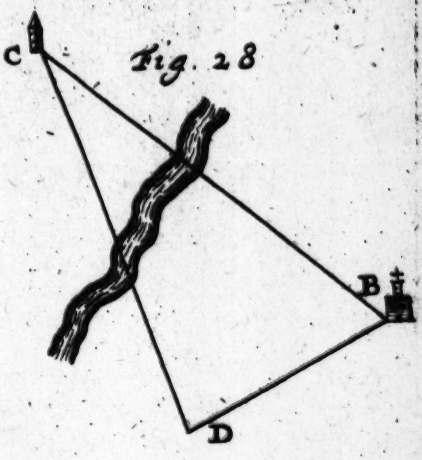
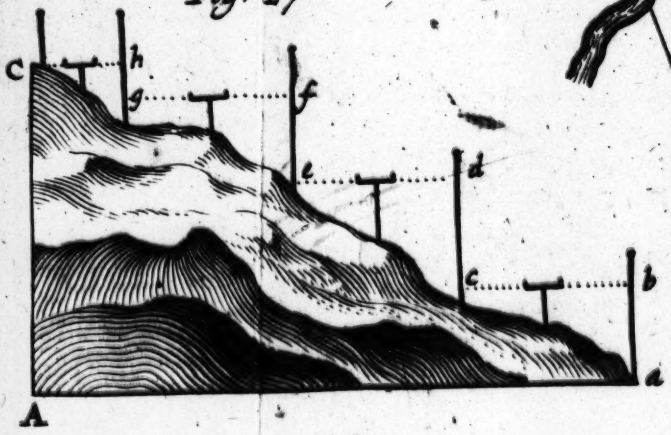


Fig. 27



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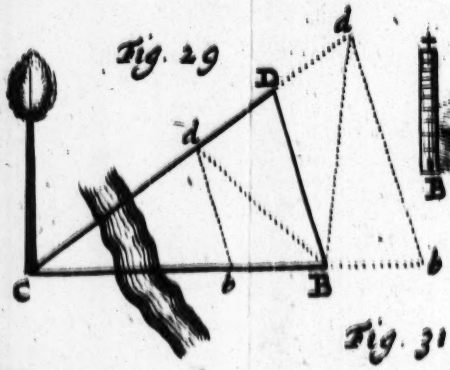

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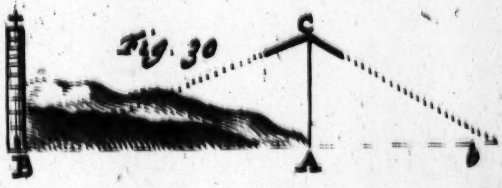

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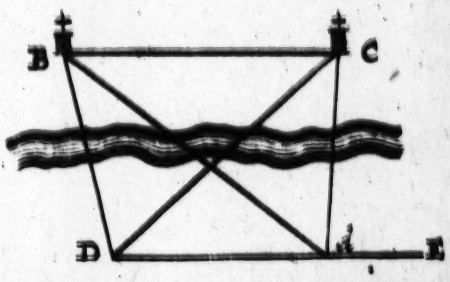
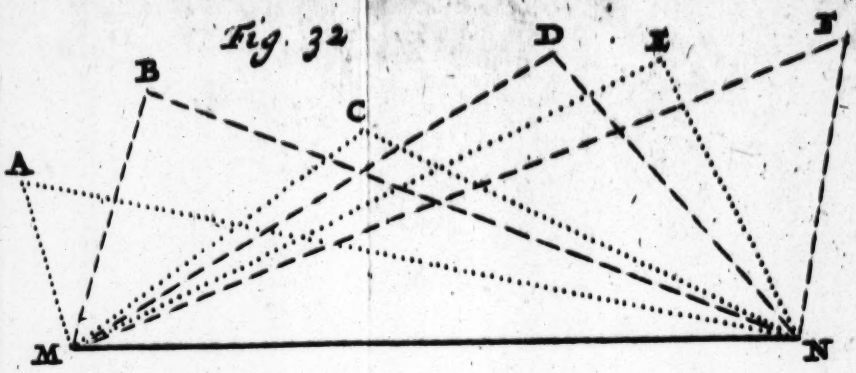


Fig. 32



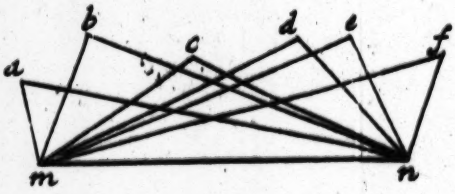


Fig. 33

Fig. 34

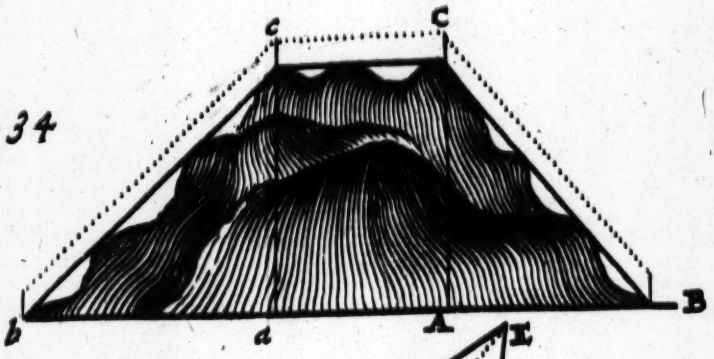
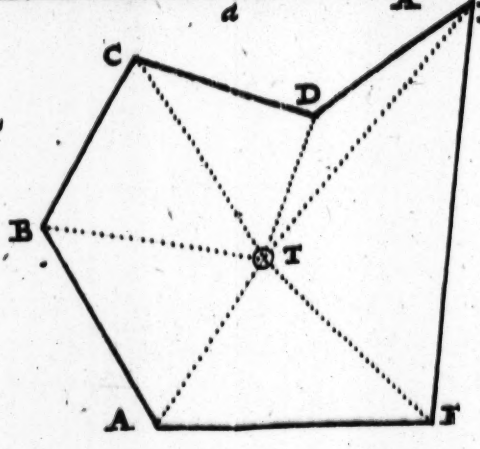


Fig. 35



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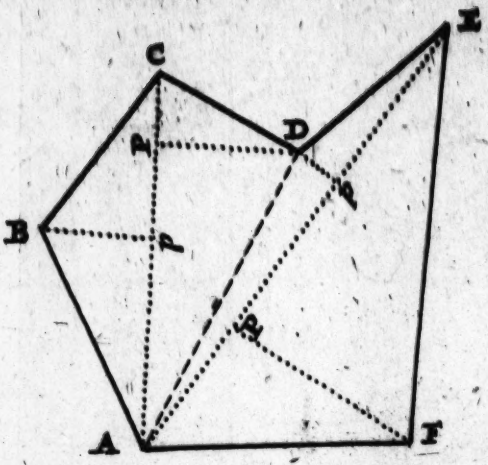

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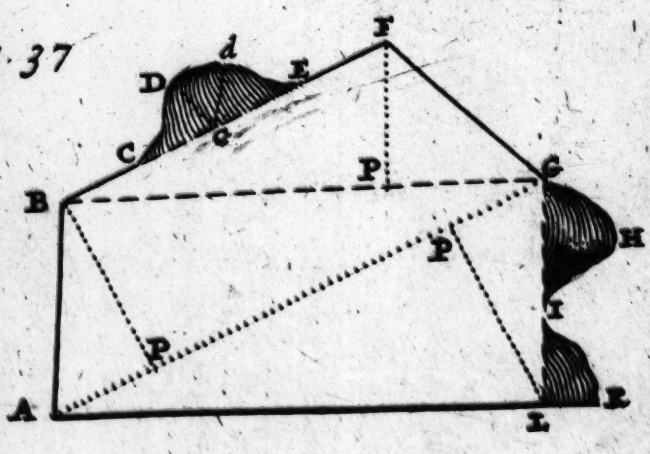

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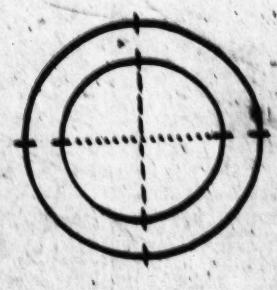

Fig. 48

Fig. 44

Fig. 47

Fig. 49.

Fig. 50

Fig. 46

Fig. 43

Fig. 42

Fig. 53

Fig. 39

Fig. 51.

Fig. 52.

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0.301030	35	1.544068	68	1.832509
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1.431364	60	1.778151	93	1.968483
1.447158	61	1.785330	94	1.973128
1.462398	62	1.792391	95	1.977723
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5447	955595	955543	955592	955639	48
5928	955976	956024	956072	956120	48
6402	956457	956505	956553	956601	48
6888	956936	956984	957032	957080	48
7368	957416	957464	957512	957559	48
7847	957894	957942	957990	958038	48
8325	958373	958421	958468	958516	48
8802	958850	958898	958946	958994	48

N	0	1	2	3	4
910	959041	959089	959137	959185	959232
911	959518	959566	959614	959661	959709
912	959995	960042	960090	960138	960185
913	960471	960518	960566	960613	960661
914	960946	960994	961041	961089	961136
915	961421	961469	961516	961563	961611
916	961895	961943	961990	962038	962085
917	962369	962417	962464	962511	962559
918	962842	962889	962937	962985	963032
919	963315	963363	963410	963457	963504
920	963788	963835	963882	963929	963977
921	964259	964307	964354	964401	964448
922	964731	964778	964825	964872	964919
923	965202	965249	965296	965343	965390
924	965672	965719	965766	965813	965860
925	966142	966189	966239	966283	966330
926	966611	966658	966705	966752	966799
927	967079	967127	967173	967220	967267
928	967548	967595	967642	967688	967735
929	968016	968062	968109	968156	968203
930	968483	968529	968576	968623	968670
931	968949	968996	969043	969089	969136
932	969416	969463	969509	969556	969602
933	969882	969928	969975	970021	970068
934	970347	970393	970439	970486	970532
935	970812	970858	970904	970951	970997
936	971276	971322	971369	971415	971461
937	971739	971786	971832	971879	971925
938	972203	972249	972295	972342	972388
939	972666	972712	972758	972804	972851

	6	7	8	9	D
279	959328	959375	959423	959471	48
757	959804	959852	959899	959947	48
233	960280	960328	960376	960423	48
709	960756	960804	960851	960899	48
184	961231	961279	961326	961374	47
658	961706	961753	961801	961848	47
132	962179	962227	962275	962322	47
606	962653	962701	962748	962795	47
079	963126	963174	963221	963268	47
552	963599	963646	963693	963741	47
024	964071	964118	964165	964212	47
495	964542	964589	964637	964684	47
966	965013	965061	965108	965155	47
437	965484	965531	965578	965624	47
906	965954	966001	966048	966095	47
376	966423	966470	966517	966564	47
845	966892	966939	966986	967033	47
314	967361	967408	967454	967501	47
782	967829	967875	967922	967969	47
249	968296	968343	968389	998436	47
719	968763	968809	968856	968902	47
183	969229	969276	969323	969369	47
649	969695	969741	969789	969835	47
114	970161	970207	970254	970300	47
579	970626	970672	970719	970765	46
044	971090	971137	971183	971229	46
508	971554	971601	971647	971693	49
971	972018	972064	972110	972157	46
434	972481	972527	972573	972619	46
897	972943	972989	973039	973082	46

N	0	1	2	3	4
940	973128	973174	973220	973266	973
941	973589	973636	973682	973728	973
942	974050	974097	974143	974189	974
943	974512	974558	974604	974649	974
944	974972	975018	975064	975109	975
945	975432	975478	975524	975569	975
946	975891	975937	975983	976029	976
947	976349	976396	976442	976488	976
948	976808	976854	976899	976946	976
949	977266	977312	977358	977403	977
950	977724	977769	977815	977861	977
951	978181	978226	978272	978317	978
952	978637	978683	978728	978774	978
953	979093	979138	979184	979229	979
954	979548	979594	979639	979685	979
955	980003	980049	980094	980139	980
956	980458	980503	980549	980594	980
957	980912	980957	981003	981048	981
958	981366	981411	981456	981501	981
959	981819	981864	981909	981954	981
960	982271	982316	982362	982407	982
961	982723	982769	982814	982859	982
962	983175	983220	983265	983310	983
963	983626	983671	983716	983762	983
964	984077	984122	984167	984212	984
965	984527	984572	984617	984662	984
966	984977	985022	985067	985112	985
967	985426	985471	985516	985561	985
968	985875	985920	985965	986009	986
969	986324	986369	986413	986458	986

5	6	7	8	9	D
359	973405	973451	973497	973543	46
320	973866	973913	973959	974005	46
281	974327	974374	974419	974466	46
742	974788	974834	974881	974926	46
202	975248	975294	975339	975386	46
662	975707	975753	975799	975845	46
121	976167	976212	976258	976304	46
579	976625	976671	976717	976763	46
037	977083	977129	977175	977220	46
495	977541	977586	977632	977678	46
952	977998	978042	978089	978135	46
409	978454	978500	978546	978591	46
865	978911	978956	979002	979047	46
321	979366	979412	979457	979503	45
776	979821	979867	979912	979958	46
231	980276	980322	980367	980412	45
685	980730	980776	980821	980867	45
139	981184	981229	981275	981320	45
592	981637	981683	981728	981773	45
045	982090	982135	982181	982226	45
497	982543	982588	982633	982678	45
949	982994	983039	983085	983129	45
401	983446	983490	983536	983581	45
852	983897	983942	983987	984032	45
302	984347	984392	984437	984482	45
752	984797	984842	984887	984932	45
202	985247	985292	985337	985382	45
651	985696	985741	985787	985830	45
099	986144	986189	986234	986279	45
548	986593	986637	986682	986727	45

N	0	1	2	3	
970	986772	986817	986861	986906	986
971	987219	987264	987309	987353	987
972	987666	987711	987756	987800	987
973	988113	988157	988202	988247	988
974	988559	988604	988748	988693	988
975	989005	989049	989094	989138	989
976	989449	989494	989539	989584	989
977	989895	989939	989983	990028	990
978	990339	990383	990428	990472	990
979	990783	990827	990871	990916	990
980	991226	991270	991315	991359	991
981	991669	991713	991758	991802	991
982	992111	992156	992199	992244	992
983	992554	992598	992642	992686	992
984	992995	993039	993083	993127	993
985	993436	993480	993524	993568	993
986	993877	993921	993965	994009	994
987	994117	994361	994405	994449	994
988	994356	994801	994845	994889	994
989	995196	995240	995284	995328	995
990	995635	995679	995723	995764	995
991	996074	996117	996161	996205	996
992	996512	996555	996599	996643	996
993	996949	996993	997037	997080	997
994	997389	997430	997474	997517	997
995	997823	997867	997910	997954	997
996	998259	998303	998347	998390	998
997	998695	998739	998783	998826	998
998	999133	999174	999218	999261	999
999	999565	999609	999652	999696	999

5	6	7	8	9	D
996	987040	987085	987129	987175	45
443	987488	987532	987577	987622	45
889	987934	987979	988024	988068	45
336	988381	988425	988469	988514	45
782	988826	988871	988916	988960	45
227	989272	989316	989361	989405	45
672	989717	989761	989806	989850	44
117	990161	990206	990250	990294	44
561	990605	990649	990694	990738	44
004	991049	991093	991137	991182	44
448	991492	991536	991580	991625	44
890	991935	991979	992023	992067	44
333	992377	992421	992465	992509	44
774	992819	992863	992907	992951	44
216	993259	993304	993348	993392	44
657	993701	993745	993789	993833	44
097	994141	994185	994229	994273	44
537	994581	994625	994669	994713	44
977	995021	995065	995108	995152	44
416	995459	995504	995547	995591	44
854	995898	995942	995986	996029	44
293	996337	996380	996424	996468	44
731	996774	996818	996862	996906	44
168	997212	997255	997299	997343	44
605	997648	997692	997736	997779	44
041	998085	998129	998172	998216	44
477	998521	998564	998608	998652	44
912	998959	998999	999043	999087	44
348	999392	999435	999479	999522	44
783	999826	999869	999913	999957	43

A

TABLE

OF

Artificial Sines

AND

TANGENTS

To Every

EGREE and MINUTE

OF THE

UADRANT.

Common Radius being 10,000,000.

LONDON,

ed in the Year MDCCXIV.

Degree 0.

M	Sine	Co-sine	Tangent	Co-tang
0	0.000000	10.000000	0.000000	Infinit.
1	6.463726	9.999999	6.463726	13.536274
2	6.764756	9.999999	6.764756	13.235244
3	6.940847	9.999999	6.940847	13.059153
4	7.065786	9.999999	7.065786	12.934214
5	7.162696	9.999999	7.162696	12.837304
6	7.241877	9.999999	7.241878	12.758122
7	7.308824	9.999999	7.308825	12.691175
8	7.366816	9.999999	7.366817	12.633183
9	7.417968	9.999999	7.417970	12.582030
10	7.463726	9.999998	7.463727	12.536273
11	7.505118	9.999998	7.505120	12.494880
12	7.542906	9.999997	7.542909	12.457091
13	7.577668	9.999997	7.577672	12.422328
14	7.609853	9.999996	7.609857	12.390143
15	7.639816	9.999996	7.639826	12.360174
16	7.667844	9.999995	7.667849	12.332151
17	7.694173	9.999995	7.694179	12.305821
18	7.718977	9.999994	7.719003	12.281997
19	7.742477	9.999993	7.742484	12.257516
20	7.764754	9.999993	7.764761	12.235239
21	7.785943	9.999992	7.785951	12.214049
22	7.806146	9.999991	7.806146	12.193854
23	7.825451	9.999990	7.825460	12.174540
24	7.843934	9.999989	7.843944	12.156056
25	7.861662	9.999989	7.861674	12.138326
26	7.878695	9.999988	7.878708	12.121292
27	7.895085	9.999987	7.895099	12.104901
28	7.910879	9.999986	7.910894	12.089106
29	7.926119	9.999985	7.926134	12.073866
30	7.940842	9.999983	7.940858	12.059142
	Co-sine	Sine	Co-tang.	Tang

Degree 89.

Degree 9.

Sine	Co-sine.	Tangent	Co-tang.	
940842	7·999983	7·940858	12·059143	60
955082	9·999982	7·955100	12·044900	59
968870	9·999981	7·968889	12·031111	58
982233	9·999980	7·982253	12·017747	57
995198	9·999978	7·995215	12·004781	56
007787	9·999978	8·007810	11·992191	55
020021	9·999976	8·020044	11·979956	54
031919	9·999975	8·031945	11·968055	53
043501	9·999973	8·043527	11·956473	52
054781	9·999972	8·054809	11·945181	51
065776	9·999971	8·065806	11·934191	50
076500	9·999969	8·076531	11·923469	49
086965	9·999968	8·086997	11·913003	48
097183	9·999966	8·097217	11·902783	47
107167	9·999964	8·107203	11·892797	46
116926	9·999963	8·116963	11·883037	45
126471	9·999961	8·126510	11·873490	44
135810	9·999959	8·135851	11·864149	43
144953	9·999958	8·144996	11·855004	42
153907	9·999956	8·153952	11·846048	41
162681	9·999954	8·162727	11·837273	40
171280	9·999952	8·171328	11·828672	39
179713	9·999950	8·179763	11·820237	38
187985	9·999948	8·188036	11·811964	37
196102	9·999946	8·196156	11·803844	36
204076	9·999944	8·204126	11·795874	35
211895	9·999942	8·211953	11·788047	34
219581	9·999940	8·219641	11·780359	33
227134	9·999938	8·227195	11·772805	32
234557	9·999936	8·234621	11·765379	31
241855	9·999934	8·241921	11·758079	30
Sine	Sine	Co-tang.	Tangent	M.

Degree 89.

Degree 1.

M	Sine	Co-sine	Tangent	Co-tang.
0	8.241855	9.999934	8.241921	11.7
1	8.249033	9.999932	8.249102	11.750
2	8.256094	9.999929	8.256165	11.743
3	8.263042	9.999927	8.263115	11.736
4	8.269881	9.999925	8.269956	11.730
5	8.276614	9.999922	8.276691	11.723
6	8.283243	9.999920	8.283323	11.716
7	8.289773	9.999918	8.289856	11.710
8	8.296207	9.999915	8.296292	11.703
9	8.302546	9.999913	8.302634	11.697
10	8.308794	9.999910	8.308884	11.691
11	8.314954	9.999907	8.315046	11.684
12	8.321027	9.999905	8.321122	11.678
13	8.327016	9.999902	8.327114	11.672
14	8.332924	9.999899	8.333025	11.666
15	8.338753	9.999897	8.338856	11.661
16	8.344504	9.999894	8.344610	11.655
17	8.350180	9.999891	8.350289	11.649
18	8.355783	9.999888	8.355895	11.644
19	8.361315	9.999885	8.361430	11.638
20	8.366777	9.999882	8.366895	11.633
21	8.372171	9.999879	8.372292	11.627
22	8.377499	9.999876	8.377622	11.622
23	8.382762	9.999873	8.382889	11.617
24	8.387962	9.999870	8.388092	11.611
25	8.393101	9.999867	8.393234	11.606
26	8.398179	9.999864	8.398315	11.601
27	8.403199	9.999861	8.403338	11.595
28	8.408161	9.999858	8.408304	11.590
29	8.413068	9.999854	8.413213	11.585
30	8.417919	9.999851	8.418068	11.580
	Co-sine	Sine	Co-tang.	Tan

Degree 88.

Degree 1.

Sine	Co-sine	Tangent	Co-tang.	
4179[illegible]	9.999851	8.418068	11.581932	30
4227[illegible]	9.999848	8.422869	11.577131	29
4276[illegible]	9.999844	8.427618	11.572382	28
4325[illegible]	9.999841	8.432315	11.567685	27
4368[illegible]	9.999838	8.436962	11.563038	26
4413[illegible]	9.999834	8.441560	11.558440	25
4459[illegible]	9.999831	8.446110	11.553890	24
4504[illegible]	9.999827	8.450613	11.549387	23
4548[illegible]	9.999824	8.455070	11.544930	22
4593[illegible]	9.999820	8.459481	11.540519	21
4636[illegible]	9.999816	8.463849	11.536151	20
4679[illegible]	9.999812	8.468172	11.531828	19
4722[illegible]	9.999809	8.472454	11.527546	18
4764[illegible]	9.999805	8.476693	11.523307	17
4806[illegible]	9.999801	8.480892	11.519108	16
4848[illegible]	9.999797	8.485050	11.514940	15
4889[illegible]	9.999794	8.489171	11.510830	14
4930[illegible]	9.999790	8.493250	11.506750	13
4970[illegible]	9.999786	8.497295	11.502707	12
5010[illegible]	9.999782	8.501298	11.498702	11
5050[illegible]	9.999778	8.505267	11.494733	10
5089[illegible]	9.999774	8.509200	11.490800	9
5128[illegible]	9.999769	8.513098	11.486902	8
5167[illegible]	9.999765	8.516961	11.483039	7
5205[illegible]	9.999761	8.520790	11.479210	6
5243[illegible]	9.999756	8.524586	11.475414	5
5281[illegible]	9.999753	8.528349	11.471651	4
5318[illegible]	9.999748	8.532080	11.467920	3
5355[illegible]	9.999744	8.535779	11.464221	2
5391[illegible]	9.999740	8.539447	11.460553	1
5428[illegible]	9.999735	8.543084	11.456916	0
[illegible]ine	Sine	Co-tang.	Tangent	

Degree 88.

Degree 2.

M	Sine	Co-sine	Tangent	Co-tang
0	8.542819	9.999735	8.543084	11.4569
1	8.546422	9.999731	8.546691	11.4533
2	8.549995	9.999726	8.550268	11.4497
3	8.553558	9.999722	8.553817	11.4461
4	8.557054	9.999717	8.557336	11.4426
5	8.560540	9.999713	8.560827	11.4391
6	8.563999	9.999708	8.564291	11.4357
7	8.567431	9.999703	8.567727	11.4322
8	8.570836	9.999699	8.571137	11.4288
9	8.574214	9.999694	8.574520	11.4254
10	8.577566	9.999689	8.577877	11.4221
11	8.580892	9.999685	8.581208	11.4187
12	8.584193	9.999680	8.584514	11.4154
13	8.587469	9.999675	8.587795	11.4121
14	8.590721	9.999670	8.591051	11.4088
15	8.593948	9.999665	8.594283	11.4056
16	8.597152	9.999660	8.597492	11.4024
17	8.600332	9.999655	8.600677	11.3992
18	8.603488	9.999650	8.603838	11.3960
19	8.606622	9.999645	8.606978	11.3929
20	8.609734	9.999640	8.610094	11.3898
21	8.612823	9.999635	8.613189	11.3867
22	8.615891	9.999629	8.616262	11.3837
23	8.618937	9.999624	8.619313	11.3807
24	8.621967	9.999619	8.622343	11.3777
25	8.624965	9.999614	8.625352	11.3747
26	8.627948	9.999608	8.628340	11.3717
27	8.630911	9.999603	8.631308	11.3688
28	8.633854	9.999597	8.634456	11.3659
29	8.636776	9.999592	8.637184	11.3630
30	8.639679	9.999586	8.640093	11.3601

Co-sine	Sine	Co-tang	Tan

Degree 87.

Degree 2.

Sine	Co sine	Tangent	Co-tang.	
39679	9.999586	8.640093	11.359907	30
42563	9.999581	8.642982	11.357017	29
45428	9.999575	8.645653	11.354147	28
48274	9.999570	8.648704	11.351296	27
51102	9.999564	8.651538	11.348463	26
53911	9.999558	8.654352	11.345648	25
56702	9.999553	8.657149	11.342851	24
59475	9.999547	8.659928	11.340272	23
62230	9.999541	8.662689	11.337311	22
64968	9.999535	8.665433	11.334567	21
67689	9.999529	8.668160	11.331840	20
70393	9.999523	8.670869	11.329130	19
73080	9.999518	8.673563	11.326437	18
75751	9.999512	8.676239	11.323761	17
78405	9.999506	8.678899	11.321100	16
81043	9.999499	8.681544	11.318456	15
83665	9.999493	8.684172	11.315828	14
86272	9.999487	8.686784	11.313216	13
88892	9.999481	8.689381	11.310619	12
91438	9.999475	8.691963	11.308037	11
93998	9.999469	8.694529	11.305471	10
96543	9.999462	8.697081	11.302919	9
99073	9.999456	8.699617	11.300383	8
01589	9.999450	8.702135	11.297861	7
04090	9.999443	8.704646	11.295354	6
06576	9.999437	8.707139	11.292860	5
09046	9.999431	8.709618	11.290381	4
11507	9.999424	8.712083	11.287917	3
13952	9.999418	8.714534	11.285466	2
16383	9.999411	8.716972	11.283028	1
18800	9.999404	8.719396	11.280604	0
ine	Sine	Co-tang.	Tangent.	M

Degree 87.

Degree 3.

	Sine	Co-sine	Tangent	Co-tang
0	8.718800	9.999404	8.719346	11.280
1	8.721204	9.999398	8.721806	11.278
2	8.723595	9.999391	8.724254	11.275
3	8.725972	9.999384	8.726588	11.273
4	8.728336	9.999378	8.728959	11.271
5	8.730688	9.999371	8.731317	11.268
6	8.733027	9.999364	8.733663	11.266
7	8.735354	9.999357	8.735996	11.264
8	8.737667	9.999350	8.738317	11.261
9	8.739969	9.999343	8.740626	11.259
10	8.742259	9.999336	8.742922	11.257
11	8.744536	9.999329	8.745207	11.254
12	8.746801	9.999322	8.747479	11.252
13	8.745955	9.999315	8.749740	11.250
14	8.751297	9.999308	8.751989	11.248
15	8.753528	9.999301	8.754227	11.245
16	8.755747	9.999294	8.756453	11.243
17	8.757955	9.999286	8.758668	11.241
18	8.760151	9.999279	8.760872	11.239
19	8.762337	9.999272	8.763065	11.237
20	8.764511	9.999265	8.765246	11.235
21	8.766675	9.999257	8.767417	11.232
22	8.768828	9.999250	8.769578	11.230
23	8.770970	9.999242	8.771727	11.228
24	8.773101	9.999235	8.773866	11.226
25	8.775223	9.999227	8.775995	11.224
26	8.777333	9.999220	8.778114	11.222
27	8.779434	9.999212	8.783222	11.220
28	8.781524	9.999204	8.782320	11.218
29	8.783605	9.999197	8.784404	11.216
30	8.785675	9.999119	8.786486	11.214
	Co-sine	Sine	Co-tang.	T

Degree 86.

Degree 3.

Sine	Co-sine	Tangent	Co-tang.	
785675	9 999185	8.786486	11.213514	30
787736	9,999181	8,781554	11,211446	29
789787	9,999174	8,790613	11,209387	28
791828	9,999166	8,792662	11,207338	27
793859	9,999158	8,794701	11,205299	26
795881	9,999150	8,796731	11,203269	25
797894	9,999142	8,798752	11,201248	24
799897	9,999134	8,800763	11,199237	23
801891	9,999126	8,802765	11,197235	22
803876	9,999118	8,804758	11.195242	21
805852	9 999110	8,806742	11,193258	20
807819	9,999102	8,808717	11,191283	19
809777	9,999094	8,812683	11,189317	18
811726	9,999086	8,812641	11,187359	17
813667	9,999077	8,814589	11,185411	16
815598	9,999069	8,816529	11,183471	15
817522	9,999061	8,818461	11,181539	14
819436	9,999052	8,820384	11,179616	13
821342	9,999044	8,822298	11,177702	12
823240	9,999036	8,824205	11,175795	11
825130	9,999027	8,826103	11,173897	10
827011	9,999019	8,827992	11,172008	9
828884	9,999010	8,829874	11,170 26	8
830749	9,999002	8,831748	11,168252	7
832106	9,998993	8.833613	11,166377	6
834456	9,998984	8,835471	11,164529	5
836297	9,998976	8,837321	11,162679	4
838130	9,998967	8,839163	11,160837	3
839956	9 998958	8,840998	11,159002	2
841774	9,998940	8,842825	11,157175	1
843585	9,998941	8,844644	11,155356	0
Sine	Sine	Co-tang.	Tangent.	M

Degree 86.

Degree 4.

M	Sine	Co-sine	Tangent	Co-tang
0	8.843584	9.998941	8.844644	11.1553
1	8.845387	9.998931	8.846455	11.1534
2	8.847183	9.998923	8.848240	11.1517
3	8.848971	9.998914	8.850057	11.1499
4	8.850752	9.998905	8.851846	11.1481
5	8.852525	9.998896	8.853628	11.1463
6	8.854291	9.998887	8.855403	11.1445
7	8.856049	9.998878	8.857171	11.1428
8	8.857801	9.998869	8.858932	11.1410
9	8.859546	9.998860	8.860686	11.1393
10	8.861283	9.998851	8.862433	11.1375
11	8.863014	9.998842	8.864173	11.1358
12	8.864738	9.998832	8.865906	11.1340
13	8.866454	9.998823	8.867632	11.1323
14	8.868165	9.998813	8.869351	11.1306
15	8.869868	9.998804	8.871064	11.1289
16	8.871565	9.998795	8.872750	11.1272
17	8.873255	9.998785	8.874469	11.1255
18	8.874938	9.998776	8.876162	11.1238
19	8.876615	9.998766	8.877849	11.1221
20	8.878285	9.998757	8.879529	11.1204
21	8.879949	9.998747	8.881202	11.1187
22	8.881607	9.998738	8.882896	11.1171
23	8.883258	9.998728	8.884530	11.1154
24	8.884903	9.998718	8.886185	11.1137
25	8.886542	9.998708	8.887833	11.1121
26	8.888174	9.998699	8.889476	11.1104
27	8.889801	9.998689	8.891112	11.1088
28	8.891421	9.998679	8.892742	11.1071
29	8.893035	9.998669	8.894366	11.1055
30	8.894643	9.998659	8.895984	11.1038

Co-sine	Sine	Co tang	Tang

Degree 85.

Degree 4.

Sine	Co-sine.	Tangent	Co-tang.	
94642	9.998659	8.845984	11.104016	30
96247	9.998649	8.897596	11.102404	29
97842	9.998639	8.899209	11.100797	28
99432	9.998629	8.900803	11.099197	27
01017	9.998619	8.902398	11.097602	26
02596	9.998609	8.903987	11.096013	25
04169	9.998599	8.905570	11.094430	24
05736	9.998589	8.907147	11.092853	23
07297	9.998577	8.908719	11.091281	22
08853	9.998568	8.910285	11.089715	21
10404	9.998558	8.911846	11.088154	20
11949	9.998548	8.913401	11.086599	19
13488	9.998537	8.914951	11.085049	18
15022	9.998527	8.916498	11.083505	17
16550	9.998516	8.918034	11.081966	16
18073	9.998506	8.919568	11.080432	15
19591	9.998495	8.921096	11.078904	14
21103	9.998485	8.922619	11.077381	13
22610	9.998474	8.924136	11.075864	12
24112	9.998464	8.925649	11.074351	11
25609	9.998453	8.927156	11.072844	10
27100	9.998442	8.928658	11.071344	9
28587	9.998431	8.930155	11.069845	8
30068	9.998421	8.931647	11.068353	7
31544	9.998410	8.933134	11.066866	6
33015	9.998399	8.934616	11.065384	5
34481	9.998388	8.936093	11.063907	4
35942	9.998377	8.937565	11.062435	3
37398	9.998366	8.939032	11.060968	2
38850	9.998355	8.940494	11.059506	1
40296	9.998344	8.941952	11.058048	0
Co-sine	Sine	Co-tang.	Tangent	

Degree 85.

Degree 5.

M	Sine	Co-sine	Tangent	Co-tang
0	8.940296	9.998344	8.941952	11.05
1	8.941738	9.998333	8.943404	11.056
2	8.943174	9.998322	8.944852	11.055
3	8.944606	9.998311	8.946295	11.053
4	8.946034	9.998300	8.947734	11.052
5	8.957456	9.998289	8.949168	11.050
6	8.958814	9.998277	8.950597	11.049
7	8.950287	9.998266	8.953021	11.047
8	8.951696	9.998255	8.953441	11.046
9	8.953099	9.998243	8.954856	11.044
10	8.954499	9.998232	8.956267	11.043
11	8.955804	9.998220	8.957674	11.041
12	8.957284	9.998209	8.959075	11.040
13	8.958670	9.998197	8.960472	11.038
14	8.960052	9.998186	8.961866	11.037
15	8.961429	9.998174	8.963255	11.035
16	8.962801	9.998163	8.964639	11.034
17	8.964170	9.998151	8.966019	11.033
18	8.965534	9.998139	8.967394	11.032
19	8.966893	9.998128	8.968766	11.031
20	8.968249	9.998106	8.970133	11.029
21	8.969600	9.998104	8.971495	11.028
22	8.970947	9.998092	8.972855	11.027
23	8.972289	9.998080	8.974209	11.025
24	8.973626	9.998068	8.975560	11.024
25	8.974962	9.998056	8.976906	11.023
26	8.976293	9.998044	8.978248	11.021
27	8.977619	9.998032	8.979586	11.020
28	8.978941	9.998020	8.98092	11.019
29	8.980259	9.998008	8.982251	11.017
30	8.981573	9.997996	8.98357	11.016
	Co-sine	Sine	Co-tang.	Tang

Degree 84.

Degree 5.

ine	Co-sine	Tangent	Co-tang.	
1573	9.997998	8.983577	11.016423	30
2883	9.997984	8.984899	11.015101	29
4189	9.997971	8.986217	11.013783	28
5491	9.997959	8.987532	11.012468	27
6789	9.997947	8.988842	11.011158	26
8083	9.997935	8.990149	11.009851	25
9374	9.997922	8.991451	11.008549	24
0660	9.997910	8.992750	11.007250	23
1943	9.997897	8.994045	11.005955	22
3228	9.997885	8.995337	11.004663	21
4497	9.997873	8.996624	11.003376	20
5768	9.997860	8.997908	11.002092	19
7036	9.997847	8.999188	11.000812	18
8299	9.997835	9.000465	10.999534	17
9560	9.997822	9.001738	10.998262	16
0816	9.997809	9.003007	10.996993	15
2069	9.997797	9.004272	10.995728	14
3318	9.997784	9.005534	10.994466	13
4563	9.997771	9.006792	10.993208	12
5805	9.997758	9.008047	10.991953	11
7044	9.997722	9.009298	10.990702	10
8278	9.997732	9.010546	10.989454	9
9510	9.997719	9.011790	10.988210	8
0737	9.997706	9.013031	10.986969	7
1962	9.997693	9.014268	10.985732	6
3182	9.997680	9.015502	10.984498	5
4399	9.997667	9.016732	10.983268	4
5613	9.997654	9.017959	10.982041	3
6824	9.997641	9.019183	10.980817	2
8031	9.997628	9.020403	10.979597	1
9235	9.997614	9.021620	10.978380	0
ne	Sine	Co-tang.	Tangent.	M

Degree 84.

Degree 6.

M	Sine	Co-sine	Tangent	Co-tan
0	9.019225	9.997614	9.021620	10.97
1	9.020435	9.997601	9.022834	10.97
2	9.021632	9.997588	9.024044	10.97
3	9.022825	9.997574	9.025251	10.97
4	9.024016	9.997561	9.026455	10.97
5	9.025203	9.997548	9.027655	10.97
6	9.026386	9.997534	9.028852	10.97
7	9.027567	9.997520	9.030046	10.96
8	9.028744	9.997507	9.031237	10.96
9	9.029918	9.997493	9.032425	10.96
10	9.031089	9.997480	9.033609	10.96
11	9.032257	9.997466	9.034791	10.96
12	9.033421	9.997452	9.035969	10.96
13	9.034582	9.997439	9.037144	10.96
14	9.035741	9.997425	9.038316	10.96
15	9.036896	9.997411	9.039485	10.96
16	9.038048	9.997397	9.040651	10.95
17	9.039197	9.997383	9.041813	10.95
18	9.040342	9.997369	9.042973	10.95
19	9.041485	9.997355	9.044130	10.95
20	9.042625	9.997341	9.045284	10.95
21	9.043762	9.997327	9.046434	10.95
22	9.044895	9.997313	9.047582	10.95
23	9.046026	9.997299	9.048727	10.95
24	9.047154	9.997285	9.049869	10.95
25	9.048279	9.997271	9.051008	10.95
26	9.049400	9.997256	9.052144	10.95
27	9.050519	9.997242	9.053277	10.95
28	9.051635	9.997228	9.054408	10.95
29	9.052749	9.997214	9.055535	10.94
30	9.053859	9.997119	9.056660	10.94
	Co-sine	Sine	Co-tang.	T

Degree 83.

Degree 6.

Sine	Co-sine		Tangent	Co-tang.	
538859	9.997199		9.056640	10.943340	30
554966	9.997185		9.057781	10.942219	29
560071	9.997170		9.058900	10.941100	28
565172	9.997156		9.060016	10.939984	27
570271	9.997141		9.061130	10.938870	26
575367	9.997127		9.062240	10.937760	25
580460	9.997112		9.063348	10.936652	24
585551	9.997098		9.064453	10.935547	23
590638	9.997083		9.065556	10.934445	22
595723	9.997068		9.066655	10.933345	21
600806	9.997053		9.067752	10.932248	20
605885	9.997039		9.068847	10.931153	19
610962	9.997024		9.069938	10.930062	18
616036	9.997009		9.071027	10.928973	17
621107	9.996994		9.072113	10.927887	16
626176	9.996979		9.073197	10.926803	15
631242	9.996964		9.074278	10.925722	14
636306	9.996949		9.075356	10.924644	13
641366	9.996934		9.076432	10.923568	12
646424	9.996919		9.077505	10.922495	11
651480	9.996904		9.078576	10.921424	10
656533	9.996886		9.079644	10.920356	9
661583	9.996874		9.080710	10.919290	8
666631	9.996858		9.081773	10.918227	7
671676	9.996843		9.082833	10.917167	6
676719	9.996823		9.083891	10.916109	5
681759	9.996812		9.084947	10.915053	4
686797	9.996797		9.085999	10.914000	3
691832	9.996782		9.087050	10.912950	2
696864	9.996766		9.088098	10.911902	1
701894	9.996751		9.089144	10.910856	0
e 1 Sine		Co-tang.	Tangent.	M	

Degree 83.

Degree 8.

M	Sine	Co-sine	Tangent	Co tang.
0	9.143555	9.995753	9.147803	10.852197
1	9.144453	9.995735	9.148718	10.851282
2	9.145349	9.995717	9.149632	10.850368
3	9.146243	9.995699	9.150544	10.849456
4	9.147136	9.995681	9.151454	10.848546
5	9.148026	9.995664	9.152363	10.847637
6	9.148915	9.995646	9.153269	10.846731
7	9.149801	9.995628	9.154174	10.845826
8	9.150686	9.995610	9.155077	10.844923
9	9.151569	9.995592	9.155978	10.844022
10	9.152451	9.995573	9.156877	10.843123
11	9.153330	9.995555	9.157775	10.842225
12	9.154208	9.995537	9.158671	10.841329
13	9.155082	9.995519	9.159565	10.840435
14	9.155957	9.995501	9.160457	10.839543
15	9.156830	9.995482	9.161347	10.838653
16	9.157700	9.995464	9.162236	10.837764
17	9.158569	9.995446	9.163123	10.836877
18	9.159436	9.995427	9.164008	10.835992
19	9.160301	9.995409	9.164892	10.835108
20	9.161164	9.995390	9.165773	10.834227
21	9.162025	9.995372	9.166654	10.833346
22	9.162885	9.995353	9.167532	10.832468
23	9.163743	9.995334	9.168409	10.831591
24	9.164600	9.995316	9.169284	10.830716
25	9.165454	9.995297	9.170157	10.829843
26	9.166307	9.995278	9.171029	10.828971
27	9.167158	9.995260	9.171899	10.828101
28	9.168008	9.995241	9.172767	10.827233
29	9.168856	9.995222	9.173634	10.826366
30	9.169702	9.995203	9.174499	10.825501
M	Co-sine	Sine	Co-tang.	Tang

Degree 81.

Degree 8.

Sine	Co-sine	Tangent	Co-tang.	
1697029.995203		9.174495	10.825504	30
1705469.995184		9.175362	10.824638	29
1713899.995165		9.176224	10.823776	28
1722309.995146		9.177082	10.822918	27
1730709.995127		9.177942	10.822058	26
1739089.995108		9.178799	10.821202	25
1747449.995089		9.179655	10.820345	24
1755789.995070		9.180508	10.819492	23
1764119.995051		9.181360	10.818640	22
1772429.995032		9.182211	10.817789	21
1780729.995012		9.183060	10.816940	20
1789009.994993		9.183907	10.816093	19
1797269.994974		9.184752	10.815248	18
1805519.994955		9.185597	10.814403	17
1813749.994935		9.186439	10.813560	16
1821969.994916		9.187280	10.812720	15
1830169.994896		9.188120	10.811880	14
1838349.994876		9.188957	10.811042	13
1846519.994857		9.189794	10.810206	12
1854669.994838		9.190629	10.809371	11
1862809.994818		9.191462	10.808538	10
1870929.994798		9.192294	10.807706	9
1879039.994779		9.193124	10.806876	8
1887129.994759		9.193953	10.806047	7
1895199.994739		9.194780	10.805220	6
1903259.994719		9.195606	10.804394	5
1911309.994699		9.196429	10.803569	4
1919339.994680		9.197253	10.802747	3
1927349.994660		9.198074	10.801926	2
1935349.994640		9.198894	10.801106	1
1943329.994620		9.199712	10.800287	0
Sine	Sine	Co-tang.	Tangent	

Degree 81.

Degree 9.

M	Sine	Co-sine	Tangent	Co-tang.
0	9.194332	9.994620	9.199712	10.800288
1	9.195129	9.994600	9.200529	10.799471
2	9.195925	9.994580	9.201345	10.798655
3	9.196718	9.994560	9.202159	10.797841
4	9.197511	9.994540	9.202971	10.797029
5	9.198302	9.994519	9.203782	10.796218
6	9.199091	9.994499	9.204592	10.795408
7	9.199879	9.994479	9.205400	10.794600
8	9.200666	9.994459	9.206207	10.793793
9	9.201451	9.994438	9.207013	10.792987
10	9.202234	9.994418	9.207817	10.792183
11	9.203017	9.994398	9.208619	10.791381
12	9.203797	9.994377	9.209420	10.790580
13	9.204577	9.994357	9.210220	10.789780
14	9.205354	9.994336	9.211018	10.788982
15	9.206131	9.994316	9.211815	10.788185
16	9.206906	9.994295	9.212611	10.787389
17	9.207679	9.994274	9.213405	10.786595
18	9.208452	9.994254	9.214198	10.785802
19	9.209222	9.994233	9.214989	10.785011
20	9.209992	9.994212	9.215780	10.784220
21	9.210760	9.994191	9.216568	10.783432
22	9.211526	9.994171	9.217356	10.782644
23	9.212291	9.994150	9.218142	10.781858
24	9.213055	9.994129	9.218929	10.781071
25	9.213818	9.994108	9.219710	10.780290
26	9.214579	9.994087	9.220491	10.779509
27	9.215338	9.994066	9.221272	10.778728
28	9.216097	9.994044	9.222052	10.777948
29	9.216854	9.994024	9.222830	10.777170
30	9.217609	9.994003	9.223607	10.776393

Co-sine	Sine	Co-tang.	Tang.

Degree 80.

Degree 9.

Sine	Co-sine	Tangent	Co-tang.	
217609	9.994003	9.223607	10.776393	30
218363	9.993982	9.224382	10.775618	29
219116	9.993960	9.225156	10.774844	28
219868	9.993939	9.225929	10.774071	27
220618	9.993918	9.226704	10.773300	26
221367	9 993897	9.227471	10.772529	25
222115	9.993875	9.228240	10.771760	24
222861	9.993854	9.229007	10.770993	23
223606	9.993832	9.229774	10.770226	22
224349	9.993811	9.230539	10.769461	21
225092	9.993789	9.231302	10.768698	20
225833	9.993768	9.232065	10.767935	19
226573	9.993746	9.232826	10.767174	18
227311	9.993725	9.233586	10.766414	17
228048	9.993703	9.234345	10.765655	16
228784	9.993681	9.235103	10.764897	15
229518	9.993660	9.235859	10.764141	14
230252	9.993638	9.236614	10.763386	13
230984	9.993616	9.237368	10.762632	12
231715	9 993594	9.238120	10.761880	11
232444	9.993572	9.238872	10.761128	10
233172	9 993550	9.239622	10.760378	9
233899	9.993528	9.240371	10.759629	8
234625	9.993506	9.241118	10 758882	7
235349	9.993484	9.241865	10.758135	6
236073	9.993462	9.242610	10.757390	5
236795	9.993440	9.243354	10.756646	4
237515	9.993418	9.244097	10.755903	3
238235	9.993396	9.244839	10.755161	2
238952	9.993374	9.245579	10 754421	1
239670	9.993351	9.246319	10.753681	0
Sine	Sine	Co-tang.	Tangent.	M

Degree 80.

Degree 10.

′	Sine	Co-sine	Tangent	Co-tang
0	9.239670	9.993351	9.246319	10.7536
1	9.240386	9.993329	9.247057	10.7529
2	9.241101	9.993307	9.247794	10.7522
3	9.241814	9.993284	9.248530	10.7514
4	9.242526	9.993262	9.249264	10.7507
5	9.243237	9.993240	9.249998	10.7500
6	9.243947	9.993117	9.250730	10.7492
7	9.244656	9.993195	9.251461	10.7485
8	9.245363	9.993172	9.252191	10.7478
9	9.246070	9.993149	9.252920	10.7471
10	9.246775	9.993127	9.253648	10.7464
11	9.247478	9.993104	9.254374	10.7457
12	9.248181	9.993011	9.255200	10.7449
13	9.248883	9.993059	9.255824	10.7442
14	9.249583	9.993036	9.256547	10.7435
15	9.250282	9.993013	9.257269	10.7428
16	9.250980	9.992990	9.257990	10.7421
17	9.251677	9.992967	9.258710	10.7414
18	9.252373	9.992944	9.259429	10.7407
19	9.253067	9.992921	9.260146	10.7399
20	9.253761	9.992898	9.260863	10.7392
21	9.254453	9.992875	9.261578	10.7385
22	9.255144	9.992852	9.262292	10.7378
23	9.255834	9.992829	9.263005	10.7371
24	9.256523	9.992806	9.263717	10.7364
25	9.257211	9.992783	9.264426	10.7357
26	9.257897	9.992759	9.265138	10.7350
27	9.258583	9.992736	9.265847	10.7343
28	9.259268	9.992713	9.266555	10.7336
29	9.259951	9.992690	9.267261	10.7329
30	9.260633	9.992666	9.267967	10.7322
′	Co-sine	Sine	Co-tang.	Tang

Degree 79.

Degree 10.

Sine	Co-sine		Tangent	Co-tang.	
60635	9.992666		9.267967	10.732033	3(
61314	9.992643		9.268671	10.731329	29
61994	9.992616		9.269375	10.730625	28
62673	9.992596		9.270778	10.729923	27
63351	9.992572		9.271479	10.729221	2(
64027	9.992549		9.271479	10.728521	2
64703	9.992525		9.272178	10.727822	24
65378	9.992501		9.272876	10.727124	2
66051	9.992478		9.273573	10.726427	2
66723	9.992454		9.274269	10.725731	21
67395	9.992430		9.274964	10 725036	2(
68065	9.992406		9.275658	10.724342	1
68734	9.992382		9.276351	10 723649	18
69402	9.992362		9.277043	10.722957	17
70069	9.992335		9.277734	10.722267	16
70735	9.992311		9.278424	10.721576	1
71400	9.992287		9.279113	10.720887	14
72063	9.992263		9.279801	10.720199	13
72726	9.992239		9.280488	10.719512	12
73388	9.992214		9.281174	10.718826	11
74049	9.992190		9.281858	10.718142	1(
74708	9.992166		9.282542	10.717458	9
75367	9.992142		9.283225	10.716775	8
76025	9.992118		9.283907	10.716093	7
76681	9.992093		9.284588	10.715412	6
77337	9.992069		9.285268	10.714732	5
77991	9.992045		9.285946	10.714053	4
78685	9.992020		9.286624	10.713376	3
79297	9.991996		9.287301	10.712699	2
79948	9.991971		9.287977	10.712023	1
80599	9.991947		9.288652	10.711348	0
fine	Sine		Co-tang	Tangent	M

Degree 79.

Degree 11.

M	Sine	Co sine	Tangent	Co-tang.
0	9.280599	9.991947	9.288652	10.711348
1	9.281229	9.991922	9.289326	10.710674
2	9.281897	9.991897	9.289999	10.710001
3	9.282544	9.991873	9.290671	10.709329
4	9.283190	9.991848	9.291342	10.708658
5	9.283836	9.991823	9.292013	10.707987
6	9.284480	9.991799	9.292682	10.707318
7	9.285124	9.991774	9.293350	10.706650
8	9.285766	9.991749	9.294017	10.705983
9	9.286408	9.991724	9.294684	10.705316
10	9.287048	9.991699	9.295349	10.704651
11	9.287688	9.991674	9.296013	10.703987
12	9.288326	9.991649	9.296677	10.703323
13	9.288964	9.991624	9.297339	10.702661
14	9.289600	9.991599	9.298001	10.702000
15	9.290236	9.991574	9.298662	10.701338
16	9.290870	9.991549	9.299322	10.700678
17	9.291504	9.991524	9.299980	10.700020
18	9.292137	9.991498	9.300638	10.699362
19	9.292768	9.991473	9.301295	10.698705
20	9.293398	9.991448	9.301951	10.698049
21	9.294029	9.991422	9.302607	10.697393
22	9.294658	9.991397	9.303261	10.696739
23	9.295286	9.991372	9.303914	10.696086
24	9.295913	9.991346	9.304567	10.695433
25	9.296539	9.991321	9.305218	10.694782
26	9.297164	9.991295	9.305867	10.694133
27	9.297788	9.991270	9.306519	10.693481
28	9.298412	9.991244	9.307168	10.692832
29	9.299034	9.991218	9.307816	10.692184
30	9.299655	9.991193	9.308463	10.691537

Co-sine	Sine	Co-tang.	Tang

Degree 78.

Degree 11.

Sine	Co-sine.		Tangent	Co-tang.	
99655	9.9911937		9.3084631	10.6915371	30
00276	9,9911867		9,3090109	10,6909891	29
00895	9,9911141		9,3097541	10,6902461	28
01514	9,9911115		9,3103991	10,6896011	27
02132	9,9911090		9,3110421	10,6889581	26
02749	9,9910641		9,3116851	10,6883151	25
03364	9,9911038		9,3123271	10,6876731	24
03979	9,9911012		9,3129681	10,6870321	23
04593	9,9909986		9,3136081	10,6863921	22
05207	9,9909960		9,3142471	10,6857531	21
05819	9,9909934		9,3148851	10,6851151	20
06430	9,9909908		9,3155231	10,6844771	19
07041	9,9909882		9,3161591	10,6838411	18
07650	9,9909855		9,3167951	10,6832051	17
08259	9,9909829		9,3174301	10,6825701	16
08867	9,9909803		9,3180641	10,6819361	15
09474	9,9909777		9,3186971	10,6813031	14
10080	9,9909750		9,3193301	10,6806701	13
10685	9,9909724		9,3199611	10,6800391	12
11289	9,9909697		9,3205921	10,6794081	11
11893	9,9909671		9,3212221	10,6787781	10
12495	9,9909645		9,3218511	10,6781491	9
13097	9,9909618		9,3224791	10,6775211	8
13698	9,9909591		9,3231061	10,6768941	7
14297	9,9909565		9,3237331	10,6762671	6
14897	9,9909538		9,3243581	10,6756421	5
15495	9,9909512		9,3249831	10,6750171	4
16092	9,9909485		9,3256071	10,6743931	3
16689	9,9909458		9,3262311	10,6737691	2
17284	9,9909431		9,3268531	10,6731471	1
17879	9,9909404		9,3274751	10,6725251	0
ine	Sine		Co-tang.	Tangent	

Degree 78.

Degree 12.

M	Sine	Co-sine	Tangent	Co-tang.
0	9.317879	9.990404	9.327475	10.672525
1	9.318473	9.990377	9.328096	10.671904
2	9.319066	9.990350	9.328715	10.671285
3	9.319658	9.990324	9.329334	10.670666
4	9.320250	9.990297	9.329953	10.670047
5	9.320840	9.990270	9.330570	10.669430
6	9.321430	9.990242	9.331187	10.668813
7	9.322019	9.990215	9.331803	10.668197
8	9.322607	9.990188	9.332418	10.667582
9	9.323194	9.990161	9.333033	10.666967
10	9.323780	9.990134	9.333646	10.666354
11	9.324366	9.990107	9.334259	10.665741
12	9.324950	9.990079	9.334871	10.665129
13	9.325534	9.990052	9.335482	10.664518
14	9.326117	9.990025	9.336093	10.663907
15	9.326699	9.989997	9.336702	10.663298
16	9.327281	9.989970	9.337311	10.662689
17	9.327862	9.989942	9.337919	10.662081
18	9.328441	9.989915	9.338527	10.661473
19	9.329020	9.989887	9.339133	10.660867
20	9.329599	9.989860	9.339739	10.660261
21	9.330176	9.989832	9.340344	10.659656
22	9.330753	9.989804	9.340948	10.659052
23	9.331328	9.989777	9.341552	10.658448
24	9.331903	9.989749	9.342155	10.657845
25	9.332478	9.989721	9.342757	10.657243
26	9.333051	9.989693	9.343358	10.656642
27	9.333624	9.989665	9.343958	10.656042
28	9.334196	9.989637	9.344554	10.655446
29	9.334767	9.989609	9.345157	10.654843
30	9.335337	9.989581	9.345755	10.654245
	Co-sine	Sine	Co-tang.	Tang.

Degree 77.

Degree 12.

Sine	Co-sine	Tangent	Co-tang.	
35337	9.989581	9.345755	10.654245	30
35906	9.989553	9.346353	10.653647	29
36475	9.989525	9.346949	10.653051	28
37043	9.989597	9.347545	10.652455	27
37610	9.989469	9.348141	10.651859	26
38176	9.989441	9.348735	10.651265	25
38742	9.989413	9.349329	10.650671	24
39306	9.989384	9.349922	10.650078	23
39870	9.989356	9.350514	10.649486	22
40434	9.989328	9.351106	10.648894	21
40996	9.989299	9.351697	10.648303	20
41558	9.989271	9.352287	10.647713	19
42115	9.989248	9.352876	10.647124	18
42679	9.989214	9.353465	10.646535	17
43239	9.989185	9.354053	10.645947	16
43797	9.989157	9.354640	10.645360	15
44355	9.989128	9.355227	10.644773	14
44912	9.989100	9.355812	10.644187	13
45469	9.989071	9.356398	10.643602	12
46024	9.989042	9.356982	10.643018	11
46579	9.989014	9.357566	10.642434	10
47134	9.988985	9.358149	10.641851	9
47687	9.988956	9.358731	10.641269	8
48240	9.988927	9.359313	10.640687	7
48792	9.988898	9.359893	10.640107	6
49343	9.988869	9.360474	10.639526	5
49893	9.988840	9.361053	10.638947	4
50443	9.988811	9.361632	10.638369	3
50992	9.988782	9.362210	10.637790	2
51540	9.988754	9.362787	10.637213	1
52088	9.988724	9.363364	10.636636	0
Sine	Sine	Co-tang.	Tangent	M

Degree 77.

Degree 13.

M	Sine	Co-sine		Tangent	Co-tang.
0	9.352088	9.988724		9.363364	10.636
1	9.352635	9.988695		9.363940	10.636
2	9.353181	9.988666		9.364515	10.635
3	9.353726	9.988636		9.365090	10.634
4	9.354271	9.988607		9.365664	10.634
5	9.354185	9.988578		9.366237	10.633
6	9.355358	9.988548		9.366810	10.633
7	9.355901	9.988519		9.367382	10.632
8	9.356443	9.988489		9.367953	10.632
9	9.356984	9.988460		9.368524	10.631
10	9.357524	9.988430		9.369094	10.630
11	9.358064	9.988401		9.369663	10.630
12	9.358603	9.988371		9.370232	10.629
13	9.359141	9.988342		9.370799	10.629
14	9.359679	9.988312		9.371367	10.628
15	9.360215	9.988282		9.371933	10.628
16	9.360752	9.988252		9.372499	10.627
17	9.361287	9.988223		9.373064	10.626
18	9.361822	9.988193		9.373629	10.626
19	9.362356	9.988163		9.374193	10.625
20	9.362889	9.988133		9.374756	10.625
21	9.363422	9.988103		9.375319	10.624
22	9.363954	9.988073		9.375881	10.624
23	9.364485	9.988043		9.376442	10.623
24	9.365016	9.988013		9.377003	10.622
25	9.365546	9.987983		9.377563	10.622
26	9.366075	9.987953		9.378122	10.621
27	9.366604	9.987922		9.378681	10.621
28	9.367132	9.987892		9.379239	10.620
29	9.367659	9.987862		9.379797	10.620
30	9.368185	9.987832		9.380354	10.619
	Co-sine	Sine		Co-tang.	Tang

Degree 76.

Degree 13.

Sine	Co-sine	Tangent	Co-tang.	
68185	9.987832	9.380354	10.619646	30
68711	9.987801	9.380910	10.619090	29
69236	9.987771	9.381466	10.618534	28
69761	9.987740	9.382021	10.617980	27
70285	9.987710	9.382575	10.617425	26
70808	9.987679	9.383129	10.616871	25
71330	9.987649	9.383682	10.616318	24
71852	9.987618	9.384234	10.615766	23
72373	9.987588	9.384786	10.615214	22
72894	9.987557	9.385337	10.614663	21
73414	9.987526	9.385888	10.614112	20
73933	9.987496	9.386438	10.613562	19
74452	9.987465	9.386987	10.613013	18
74970	9.987434	9.387536	10.612464	17
75487	9.987403	9.388084	10.611916	16
76003	9.987372	9.388631	10.611369	15
76519	9.987341	9.389178	10.610822	14
77035	9.987310	9.389724	10.610276	13
77549	9.987279	9.390270	10.609730	12
78063	9.987248	9.390815	10.609185	11
78577	9.987217	9.391360	10.608640	10
79089	9.987186	9.391907	10.608097	9
79601	9.987155	9.392467	10.607553	8
80113	9.987124	9.392989	10.607011	7
80624	9.987092	9.393531	10.606469	6
81134	9.987061	9.394074	10.605927	5
81643	9.987030	9.394614	10.605386	4
82152	9.986998	9.395154	10.604846	3
82661	9.986967	9.395694	10.604306	2
83168	9.986938	9.396233	10.603767	1
83675	9.986904	9.396771	10.603229	0
	Sine	Co-tang.	Tangent.	M

Degree 76.

Degree 14.

M	Sine	Co-fine	Tangent	Co-tang.
0	9.383675	9.986904	9.396771	10.603229
1	9.384181	9.986873	9.397309	10.602691
2	9.384687	9.986841	9.397846	10.602154
3	9.385192	9.986809	9.398383	10.601617
4	9.385697	9.986778	9.398919	10.601081
5	9.386201	9.986746	9.399455	10.600545
6	9.386704	9.986714	9.399990	10.600010
7	9.387207	9.986683	9.400524	10.599476
8	9.387709	9.986651	9.401058	10.598942
9	9.388210	9.986619	9.401591	10.598409
10	9.388711	9.986587	9.402124	10.597876
11	9.389211	9.986555	9.402656	10.597344
12	9.389711	9.986523	9.403187	10.596813
13	9.390210	9.986491	9.403718	10.596282
14	9.390708	9.986459	9.404249	10.595751
15	9.391206	9.986427	9.404778	10.595222
16	9.391703	9.986395	9.405306	10.594694
17	9.392199	9.986363	9.405836	10.594164
18	9.392695	9.986331	9.406364	10.593636
19	9.393190	9.986299	9.406892	10.593108
20	9.393685	9.986266	9.407419	10.592581
21	9.394179	9.986234	9.407945	10.592055
22	9.394673	9.986201	9.408471	10.591529
23	9.395166	9.986169	9.408996	10.591004
24	9.395654	9.986137	9.409521	10.590479
25	9.396150	9.986104	9.410045	10.589955
26	9.396641	9.986072	9.410569	10.589431
27	9.397131	9.986039	9.411092	10.588908
28	9.397621	9.986007	9.411615	10.588385
29	9.398111	9.985974	9.412137	10.587863
30	9.398600	9.985942	9.412658	10.587342

Co-fine	Sine	Co-tang.	T

Degree 75.

Degree 14.

ine	Co-sine	Tangent	Co-tang.	
98600	9.985942	9.412658	10.587342	30
99087	9.985909	9.413179	10.586821	29
99575	9.985876	9.413699	10.586301	28
00062	9.985843	9.414219	10.585781	27
00549	9.985811	9.414738	10.585262	26
01035	9.985778	9.415257	10.584742	25
01520	9.985745	9.415775	10.584225	24
02005	9.985712	9.416293	10.583707	23
02489	9.985679	9.416810	10.583190	22
02972	9.985646	9.417326	10.582674	21
03455	9.985613	9.417842	10.582157	20
03938	9.985580	9.418357	10.581642	19
04420	9.985547	9.418873	10.581127	18
04901	9.985513	9.419387	10.580613	17
05382	9.985480	9.419901	10.580099	16
05862	9.985447	9.420415	10.579585	15
06341	9.985414	9.420927	10.579072	14
06820	9.985380	9.421440	10.578560	13
07299	9.985347	9.421951	10.578048	12
07776	9.985314	9.422463	10.577537	11
08254	9.985280	9.422973	10.577026	10
08731	9.985247	9.423484	10.576516	9
09207	9.985213	9.423993	10.576007	8
09682	9.985180	9.424503	10.575497	7
10157	9.985146	9.425011	10.574989	6
10632	9.985112	9.425518	10.574480	5
11106	9.985079	9.426027	10.573973	4
11579	9.985045	9.426534	10.573466	3
12052	9.985011	9.427041	10.572959	2
12524	9.984977	9.427547	10.572453	1
12996	9.984943	9.428052	10.571947	0
e	Sine	Co-tang	Tangent	M

Degree 75.

Degree 15.

M	Sine	Co-sine	Tangent	Co-tang
0	9.412996	9.984944	9.428052	10.571
1	9.413467	9.984910	9.428557	10.571
2	9.413938	9.984876	9.429062	10.570
3	9.414408	9.984842	9.429566	10.570
4	9.414878	9.984808	9.420070	10.579
5	9.415347	9.984774	9.420573	10.179
6	9.415815	9.98474(	9.431075	10.568
7	9.416283	9.984706	9.431577	10.568
8	9.416850	9.98467:	9.432079	10.567
9	9.417217	9.984637	9.432580	10.567
10	9.417684	9.984603	9.433080	10.566
11	9.418149	9.984569	9.433580	10.566
12	9.418615	9.984535	9.434080	10.565
13	9.419079	9.984500	9.434579	10.565
14	9.419544	9.984466	9.435078	10.565
15	9.420007	9.984431	9.435576	10.564
16	9.420470	9.984397	9.436073	10.563
17	9.420933	9.984363	9.436570	10.563
18	9.421395	9.984328	9.437067	10.562
19	9.421856	9.984293	9.437563	10.562
20	9.422317	9.984259	9.438059	10.561
21	9.422778	9.984224	9.439554	10.560
22	9.423238	9.984189	9.439548	10.560
23	9.423697	9.984155	9.439543	10.560
24	9.424156	9.984120	9.440036	10.559
25	9.424615	9.984085	9.440529	10.559
26	9.425072	9.984050	9.441022	10.558
27	9.425530	9.984015	9.441514	10.558
28	9.425987	9.983980	9.441006	10.558
29	9.426443	9.983945	9.442497	10.557
30	9.426899	9.983910	9.442988	10.557
	Co-sine	Sine	Co-tang.	T

Degree 74.

Degree 15.

Sine	Co-sine.	Tangent	Co-tang.	
26899	9.983910	9.442988	10.557011	30
27354	9.983875	9.443479	10.556521	29
27809	9.983840	9.443968	10.556031	28
28264	9.983805	9.444458	10.555542	27
28717	9.983770	9.444947	10.555053	26
29170	9.983735	9.445435	10.554565	25
29623	9.983699	9.445923	10.554077	24
30075	9.983664	9.446411	10.553589	23
30527	9.983629	9.446898	10.553102	22
30978	9.983593	9.447384	10.352616	21
31429	9.983558	9.447870	10.552129	20
31879	9.983523	9 448356	10.551644	19
32328	9.983487	9.448841	10.551159	18
32778	9.983452	9 449326	10.550674	17
33206	9.983416	9.449870	10.550181	16
33674	9.983380	9.450294	10.559706	15
34122	9.983345	9.450777	10.549223	14
34569	9.983309	9.451260	10.548740	13
35016	9.983273	9.451743	10.548257	12
35462	9.983238	9 452225	10.547775	11
35918	9.983202	9 452706	10.547294	10
36353	9.983166	9.453187	10.546813	9
36798	9.983130	9.453668	10.546332	8
37242	9.983094	9.454148	10.545852	7
37686	9.983058	9.454629	10.545372	6
38129	9.983022	9.455107	10.544893	5
38572	9.982986	9.455586	10.544414	4
39014	9.982950	9.456064	10.542936	3
39456	9.982914	9.456542	10.543458	2
39897	9.982878	9.457019	10.542980	1
40338	9.982842	9.457496	10.542503	0
Sine	Sine	Co-tang.	Tangent	M

Degree 74.

Degree 16.

M	Sine	Co-sine	Tangent	Co-tang.
0	9.440338	9.982842	9.457496	10.542
1	9.440778	9.982805	9.457973	10.542
2	9.441218	9.982769	9.458449	10.541
3	9.441658	9.982732	9.458925	10.541
4	9.442096	9.982696	9.459400	10.540
5	9.442535	9.982660	9.459875	10.540
6	9.442973	9.982623	9.460349	10.539
7	9.443416	9.982587	9.460829	10.539
8	9.443848	9.982550	9.461297	10.538
9	9.444284	9.982514	9.461770	10.538
10	9.444720	9.982477	9.462242	10.537
11	9.445155	9.982441	9.462714	10.537
12	9.445590	9.982404	9.463186	10.536
13	9.446025	9.982367	9.463658	10.536
14	9.446459	9.982330	9.464129	10.535
15	9.446893	9.982294	9.464599	10.535
16	9.447326	9.982257	9.465069	10.534
17	9.447759	9.982220	9.465539	10.534
18	9.448191	9.982183	9.466008	10.533
19	9.448623	9.982146	9.466476	10.533
20	9.449054	9.982109	9.466945	10.533
21	9.449485	9.982072	9.467413	10.532
22	9.449915	9.982035	9.467880	10.532
23	9.440345	9.981998	9.468347	10.532
24	9.440775	9.981961	9.468814	10.532
25	9.441203	9.981923	9.469280	10.532
26	9.451632	9.981886	9.469746	10.530
27	9.452060	9.981849	9.470211	10.529
28	9.452488	9.981812	9.470676	10.529
29	9.452915	9.981774	9.471141	10.528
30	9.453342	9.981737	9.471605	10.528
M	Co-sine	Sine	Co-tang.	Tan

Degree 73.

Degree 16.

Sine	Co-sine	Tangent	Co-tang.	
5334	9.981737	9.471605	10.528395	30
537689	9.981695	9.471968	10.528032	29
541948	9.981652	9.472322	10.527678	28
546199	9.981624	9.472995	10.527005	27
550444	9.981587	9.473457	10.526543	26
554683	9.981549	9.473919	10.526081	25
558916	9.981512	9.474381	10.525619	24
563143	9.981474	9.474842	10.525158	23
567364	9.981436	9.475303	10.524697	22
571582	9.981398	9.475763	10.524237	21
575794	9.981361	9.476223	10.523777	20
580006	9.981323	9.476683	10.523317	19
584214	9.981285	9.477142	10.522858	18
588416	9.981247	9.477601	10.522399	17
592613	9.981209	9.478059	10.521941	16
596804	9.981171	9.478517	10.521483	15
601081	9.981133	9.478975	10.521025	14
605257	9.981095	9.479432	10.520568	13
609426	9.981057	9.479889	10.520111	12
613590	9.981019	9.480345	10.519655	11
617748	9.980980	9.480801	10.519199	10
621901	9.980942	9.481257	10.518743	9
626048	9.980904	9.481712	10.518288	8
630189	9.980866	9.482167	10.517833	7
634325	9.980827	9.482621	10.517379	6
638456	9.980789	9.483075	10.516925	5
642579	9.980750	9.483528	10.516471	4
646698	9.980712	9.483982	10.516018	3
650811	9.980672	9.484434	10.515565	2
654918	9.980635	9.484887	10.515113	1
659019	9.980596	9.485339	10.514661	0
Sine	Sine	Co-tang.	Tangent	

Degree 73.

Degree 17.

M	Sine	Co-sine	Tangent	Co-tang
0	9,465935	9,980596	9,485339	10,514
1	9,466348	9,980558	9,485791	10,514
2	9,466762	9,980519	9,486242	10,513
3	9,467173	9,980480	9,486693	10,513
4	9,467585	9,980441	9,487143	10,512
5	9,467996	9,980401	9,487591	10,512
6	9,468407	9,980364	9,488043	10,511
7	9,468817	9,980325	9,488493	10,511
8	9,469227	9,980286	9,488941	10,510
9	9,469637	9,980247	9,489390	10,510
10	9,469446	9,980208	9,489838	10,510
11	9,470455	9,980169	9,490286	10,509
12	9,471863	9,980130	9,490733	10,509
13	9,471071	9,980091	9,491180	10,508
14	9,471678	9,980052	9,491627	10,508
15	9,472086	9,980012	9,492073	10,507
16	9,472492	9,979973	9,492519	10,507
17	9,472898	9,979934	9,492964	10,507
18	9,473304	9,979894	9,493410	10,506
19	9,473710	9,979855	9,493854	10,506
20	9,474115	9,979816	9,494299	10,505
21	9,474519	9,979776	9,494743	10,505
22	9,474923	9,979737	9,495186	10,504
23	9,475327	9,979697	9,495630	10,504
24	9,475730	9,979658	9,496073	10,503
25	9,476133	9,979618	9,496515	10,503
26	9,476536	9,979578	9,496957	10,503
27	9,476938	9,979539	9,497399	10,502
28	9,477340	9,979499	9,497840	10,502
29	9,477741	9,979455	9,498282	10,501
30	9,478142	9,9794 9	9,498722	10,501
	Co-sine	Sine	Co-tang.	Tan

Degree 72.

Degree 17.

Sine	Co-sine		Tangent	Co-tang.	
78142	9.979419		9.498722	10.501278	30
78542	9.979380		9.499163	10.500837	29
78942	9.979340		9.499602	10.500398	28
79342	9.979300		9.500042	10.499958	27
79741	9.979260		9.500481	10.499519	26
80140	9.979220		9.500920	10.499080	25
80538	9.979180		9.501359	10.498641	24
80936	9.979140		9.501797	10.498203	23
81334	9.979099		9.502234	10.497765	22
81731	9.979059		9.502672	10.497328	21
82128	9.979019		9.503109	10.496891	20
82525	9.978980		9.503546	10.496454	19
82921	9.978939		9.503982	10.496018	18
83316	9.978898		9.504418	10.495582	17
83711	9.978858		9.504854	10.495146	16
84106	9.978817		9.505289	10.494711	15
84501	9.978777		9.505724	10.494276	14
84895	9.978736		9.506158	10.493841	13
85289	9.978696		9.506593	10.493407	12
85682	9.978655		9.507026	10.492973	11
86075	9.978615		9.507459	10.492540	10
86467	9.978574		9.507892	10.492107	9
86859	9.978533		9.508326	10.491674	8
87251	9.978493		9.508759	10.491241	7
87642	9.978452		9.509191	10.490809	6
88033	9.978411		9.509622	10.490377	5
88424	9.978370		9.510044	10.489946	4
88814	9.978329		9.510486	10.489515	3
89204	9.978288		9.510916	10.489084	2
89593	9.978247		9.511346	10.488654	1
89982	9.978206		9.511776	10.488225	0
′	Sine		Co-tang.	Tangent.	M

Degree 72.

Degree 18.

M	Sine	Co-sine	Tangent	Co-tang.
0	9.489982	9.978206	9.511776	10.48
1	9.490371	9.978165	9.512206	10.48
2	9.490759	9.978124	9.512635	10.48
3	9.491147	9.978083	9.513064	10.48
4	9.491534	9.978042	9.513493	10.48
5	9.491922	9.978000	9.513921	10.48
6	9.492308	9.977959	9.514349	10.48
7	9.492695	9.977918	9.514777	10.48
8	9.493080	9.977877	9.515204	10.48
9	9.493466	9.977835	9.515631	10.48
10	9.493851	9.977794	9.516057	10.48
11	9.494236	9.977752	9.516484	10.48
12	9.494620	9.977711	9.516910	10.48
13	9.495005	9.977669	9.517335	10.48
14	9.495388	9.977628	9.517761	10.48
15	9.495771	9.977586	9.518185	10.48
16	9.496154	9.977544	9.518610	10.48
17	9.496537	9.977503	9.519034	10.48
18	9.496919	9.977461	9.519458	10.48
19	9.497301	9.977419	9.519882	10.48
20	9.497682	9.977377	9.520305	10.48
21	9.498063	9.977335	9.520728	10.47
22	9.498444	9.977293	9.521151	10.47
23	9.498824	9.977251	9.521573	10.47
24	9.499204	9.977209	9.521995	10.47
25	9.499584	9.977167	9.522417	10.47
26	9.499963	9.977125	9.522838	10.47
27	9.500342	9.977083	9.523259	10.47
28	9.500720	9.977041	9.523679	10.47
29	9.501099	9.976999	9.524099	10.47
30	9.501476	9.976956	9.524520	10.47
	Co-sine	Sine	Co-tang.	Tang.

Degree 71.

ine	Co-sine		Tangent	Co-tang.	
1476	9.977956		9.524520	10.475480	30
1854	9.976914		9.524936	10.475060	29
2291	9.976872		9.525359	10.474641	28
2807	9.976830		9.525778	10.474222	27
2984	9.976787		9.526197	10.473803	26
3360	9.976745		9.526615	10.473385	25
3735	9.976702		9.527033	10.472967	24
4110	9.976660		9.527451	10.472549	23
4485	9.976617		9.527868	10.472132	22
4840	9.976574		9.528285	10.471715	21
5234	9.976532		9.528702	10.471298	20
5608	9.976489		9.529118	10.470881	19
5981	9.976446		9.529535	10.470465	18
6354	9.976404		9.529950	10.470049	17
6727	9.976361		9.530366	10.469634	16
7099	9.976318		9.530781	10.469219	15
7471	9.976275		9.531196	10.468804	14
7843	9.976232		9.531611	10.468389	13
8214	9.976185		9.532025	10.467975	12
8585	9.976146		9.532439	10.467561	11
8955	9.976103		9.532852	10.467147	10
9326	9.976060		9.533266	10.466734	9
9696	9.976027		9.533679	10.466321	8
0065	9.975970		9.534092	10.465908	7
0434	9.975930		9.534504	10.465496	6
0803	9.975887		9.534916	10.465084	5
1171	9.975844		9.535328	10.464672	4
1549	9.975800		9.535739	10.464261	3
1907	9.975757		9.536150	10.463849	2
2275	9.975713		9.536561	10.463439	1
2642	9.975670		9.536972	10.463028	0
ine	Sine		Co-tang	Tangent	M

Degree 19.

M	Sine	Co-sine	Tangent	Co-tang
0	9.512644	9.975670	9.536672	10.463
1	9.513009	9.975626	9.537382	10.462
2	9.513375	9.975583	9.537792	10.462
3	9.513741	9.975539	9.538202	10.461
4	9.514107	9.975496	9.538610	10.461
5	9.514472	9.975452	9.539020	10.460
6	9.514837	9.975408	9.539429	10.460
7	9.515202	9.975364	9.539837	10.460
8	9.515566	9.975321	9.540245	10.459
9	9.515930	9.975277	9.540653	10.459
10	9.516294	9.975233	9.541061	10.458
11	9.516657	9.975189	9.541468	10.458
12	9.517020	9.975145	9.541875	10.458
13	9.517382	9.975101	9.542281	10.457
14	9.517745	9.975057	9.542688	10.457
15	9.518107	9.975013	9.543094	10.456
16	9.518468	9.974969	9.543499	10.456
17	9.518829	9.974925	9.543903	10.456
18	9.519190	9.974880	9.544310	10.456
19	9.519551	9.974836	9.544715	10.455
20	9.519911	9.974792	9.545119	10.455
21	9.520271	9.974747	9.545524	10.454
22	9.520631	9.974703	9.545927	10.454
23	9.520990	9.974659	9.546331	10.454
24	9.521349	9.974614	9.546735	10.453
25	9.521707	9.974570	9.547138	10.453
26	9.522065	9.974525	9.547540	10.452
27	9.522423	9.974480	9.547943	10.452
28	9.522781	9.974436	9.548345	10.451
29	9.523138	9.974391	9.548747	10.451
30	9.523495	9.974346	9.549149	10.450
	Co-sine	Sine	Co-tang.	Tang

Degree 70.

Degree 19.

Sine	Co-sine.	Tangent	Co-tang.	
24349	9.974346	9.549145	10.450851	30
24385	9.974302	9.549551	10.450450	29
24208	9.974257	9.549951	10.450049	28
24564	9.974212	9.550352	10.449648	27
24920	9.974167	9.550752	10.449248	26
25275	9.974122	9.551152	10.448848	25
25630	9.974077	9.551552	10.448448	24
25984	9.974032	9.551952	10.448048	23
26339	9.973987	9.552351	10.447649	22
26693	9.973942	9.552750	10.447250	21
27046	9.973897	9.553149	10.446951	20
27400	9.973852	9.553548	10.446452	19
27753	9.973807	9.553946	10.446054	18
28105	9.973761	9.554344	10.445656	17
28458	9.973716	9.554741	10.445259	16
28810	9.973671	9.555139	10.444861	15
29161	9.973625	9.555536	10.444464	14
29513	9.973580	9.555932	10.444068	13
29864	9.973535	9.556329	10.443671	12
30214	9.973489	9.556725	10.443275	11
30565	9.973443	9.557121	10.442879	10
30915	9.973398	9.557517	10.442483	9
31265	9.973352	9.557912	10.442088	8
31614	9.973307	9.558308	10.441693	7
31963	9.973261	9.558702	10.441298	6
32312	9.973215	9.559097	10.440903	5
32661	9.973169	9.559491	10.440509	4
33009	9.973123	9.559885	10.440115	3
33357	9.973078	9.560279	10.439721	2
33704	9.973032	9.560673	10.439327	1
34052	9.972986	9.561066	10.438934	0
ine	Sine	Co-tang.	Tangent	

Degree 70.

Degree 20.

M	Sine	Co-sine	Tangent	Co-tang.
0	9.534052	9.972986	9.561066	10.438
1	9.534395	9.972940	9.561459	10.438
2	9.534746	9.972894	9.561851	10.438
3	9.535091	9.972848	9.562244	10.437
4	9.535437	9.972801	9.562636	10.437
5	9.535781	9.972755	9.563028	10.436
6	9.536129	9.972709	9.563419	10.436
7	9.536474	9.972663	9.563811	10.436
8	9.536818	9.972617	9.564202	10.435
9	9.537163	9.972570	9.564592	10.435
10	9.537507	9.972524	9.564983	10.435
11	9.537851	9.972477	9.565373	10.434
12	9.538194	9.972431	9.565763	10.434
13	9.538537	9.972384	9.566153	10.433
14	9.538880	9.972338	9.566542	10.433
15	9.539222	9.972291	9.566932	10.432
16	9.539565	9.972245	9.567320	10.432
17	9.539907	9.972198	9.567709	10.432
18	9.540249	9.972151	9.568097	10.431
19	9.540590	9.972105	9.568486	10.431
20	9.540931	9.972058	9.568873	10.431
21	9.541272	9.972011	9.569261	10.430
22	9.541612	9.971964	9.569647	10.430
23	9.541953	9.971917	9.570035	10.429
24	9.542292	9.971870	9.570422	10.429
25	9.542632	9.971823	9.570809	10.429
26	9.542971	9.971776	9.571195	10.428
27	9.543310	9.971729	9.571581	10.428
28	9.543649	9.971682	9.571967	10.428
29	9.543987	9.971635	9.572352	10.427
30	9.544325	9.971588	9.572738	10.427

Co-sine	Sine	Co-tang.	Tan

Degree 69.

Degree 20.

ine	Co-sine	Tangent	Co-tang.	
4215	9.971588	9.572738	10.427262	30
4663	9.971540	9.573133	10.426877	29
5000	9.971493	9.573507	10.426492	28
5338	9.971445	9.573892	10.426108	27
5674	9.971398	9.574276	10.425724	26
6011	9.971351	9.574660	10.425340	25
6347	9.971303	9.575044	10.424956	24
6683	9.971256	9.575427	10.424573	23
7019	9.971208	9.575811	10.424189	22
7354	9.971161	9.576193	10.423807	21
7689	9.971113	9.576576	10.423424	20
8024	9.971065	9.576958	10.423041	19
8358	9.971018	9.577341	10.422659	18
8693	9.970970	9.577723	10.422277	17
9026	9.970922	9.578104	10.421896	16
9360	9.970874	9.578486	10.421514	15
9693	9.970826	9.578867	10.421133	14
0026	9.970779	9.579248	10.420752	13
0359	9.970731	9.579628	10.420371	12
0692	9.970683	9.580009	10.419991	11
1024	9.970634	9.580389	10.419611	10
1355	9.970586	9.580769	10.419231	9
1687	9.970538	9.581149	10.418851	8
2018	9.970490	9.581528	10.418472	7
2349	9.970442	9.581907	10.418092	6
2680	9.970394	9.582286	10.417713	5
3010	9.970345	9.582665	10.417335	4
3340	9.970247	9.583043	10.416956	3
3670	9.970249	9.583422	10.416578	2
4000	9.970200	9.583800	10.416200	1
4325	9.970152	9.584177	10.415823	0
Co-sine	Sine	Co-tang.	Tangent	M.

Degree 69.

Degree 21.

M	Sine	Co-sine	Tangent	Co-t
0	9,554329	9,970152	9,584177	10,415
1	9,554658	9,970103	9,584555	10,415
2	9,554987	9,970055	9,584932	10,415
3	9,555315	9,970006	9,585308	10,414
4	9,555643	9,969957	9,585686	10,414
5	9,555971	9,969909	9,586062	10,413
6	9,556299	9,969860	9,586439	10,413
7	9,556626	9,969811	9,586815	10,413
8	9,556953	9,969762	9,587190	10,412
9	9,557279	9,969713	9,587566	10,412
10	9,557606	9,969665	9,587941	10,412
11	9,557932	9,969616	9,588316	10,411
12	9,558258	9,969567	9,588691	10,411
13	9,558583	9,969518	9,589066	10,410
14	9,558909	9,969469	9,589440	10,410
15	9,559234	9,969419	9,589814	10,410
16	9,559558	9,969370	9,590188	10,409
17	9,559883	9,969321	9,590561	10,409
18	9,560207	9,969272	9,590935	10,409
19	9,560531	9,969223	9,591308	10,408
20	9,560855	9,969173	9,591681	10,408
21	9,561178	9,969124	9,592054	10,407
22	9,561501	9,969075	9,592426	10,407
23	9,561824	9,969025	9,592798	10,407
24	9,562146	9,968976	9,593170	10,406
25	9,562468	9,998926	9,593542	10,406
26	9,562790	9,968877	9,593914	10,406
27	9,563112	9,968827	9,594285	10,405
28	9,563433	9,968777	9,594656	10,405
29	9,563754	9,968728	9,595027	10,405
30	9,564076	9,968678	9,595397	10,404
	Co-sine	Sine	Co-tang.	Tange

Degree 68.

Degree 21.

ine	Co-sine	Tangent	Co-tang.	
4075	9.968678	9.595397	10.404602	30
4396	9.968628	9.595768	10.404232	29
4716	9.968578	9.596138	10.403862	28
5036	9.968528	9.596508	10.403492	27
5356	9.968478	9.596878	10.403122	26
5675	9.968428	9.597247	10.402753	25
5995	9.968378	9.597616	10.402384	24
6314	9.968328	9.597985	10.402015	23
6632	9.968278	9.598354	10.401646	22
6951	9.968228	9.598722	10.401277	21
7269	9.968178	9.599091	10.400909	20
7587	9.968128	9.599459	10.400541	19
7904	9.968078	9.599827	10.400173	18
8222	9.968027	9.500194	10.399806	17
8539	9.967977	9.500562	10.399438	16
8855	9.967927	9.500929	10.399071	15
9172	9.967876	9.601296	10.398704	14
9488	9.967826	9.601662	10.398337	13
9804	9.967775	9.602029	10.397971	12
0120	9.967725	9.602395	10.397605	11
0435	9.967674	9.602761	10.397239	10
0751	9.967623	9.603127	10.396873	9
1065	9.967573	9.603493	10.396507	8
1380	9.967522	9.603858	10.396142	7
1695	9.967471	9.604223	10.395777	6
2009	9.967420	9.604588	10.395412	5
2322	9.967370	9.604953	10.395047	4
2636	9.967319	9.605317	10.394683	3
2949	9.967268	9.605681	10.394318	2
3263	9.967217	9.606046	10.393954	1
3575	9.967166	9.606409	10.393590	0
ine	Sine	Co-tang.	Tangent.	M

Degree 68.

Degree 22.

M	Sine	Co-sine	Tangent	Co-
0	9.573575	9.967166	9.606409	10.33
1	9.573888	9.967115	9.606773	10.33
2	9.574200	9.967064	9.607136	10.33
3	9.574512	9.967012	9.607500	10.33
4	9.574824	9.966961	9.607863	10.33
5	9.575135	9.966910	9.608225	10.33
6	9.575447	9.966859	9.608588	10.33
7	9.575758	9.966807	9.608950	10.33
8	9.576068	9 966756	9.609312	10.33
9	9.576379	9.966705	9.609674	10.33
10	9.576689	9.966653	9.610036	10.33
11	9.576999	9.966602	9.610397	10.33
12	9.577309	9.966550	9.610758	10.33
13	9.577618	9.966499	9.611119	10.33
14	9.577927	9.966447	9.611480	10.33
15	9.578236	9.966395	9.611840	10.33
16	9.578545	9.966344	9.612201	10.33
17	9.578853	9.966292	9.612561	10.33
18	9.579161	9.966240	9.612921	10.33
19	9.579469	9.966188	9.613281	10.33
20	9.579777	9.966136	9.613641	10.33
21	9.580084	9.966084	9.614000	10.33
22	9.580392	9.966032	9.614359	10.33
23	9.580698	9.965980	9.614718	10.33
24	9.581005	9.965928	9.615077	10.33
25	9.581311	9.965876	9.615435	10.33
26	9.581618	9.965824	9.615793	10.33
27	9.581923	9.965772	9.616151	10.33
28	9.582229	9.965720	9.616509	10.33
29	9.582534	9.965668	9.616867	10.33
30	9.582840	9.965615	9.617224	10.33

Co-sine	Sine	Co-tang.	Ta

Degree 67.

Degree 22.

ne.	Co-sine		Tangent	Co-tang.	
2840	9.965615		9.617224	10.382776	30
3144	9.965563		9.617581	10.382418	29
3449	9.965511		9.617938	10.382061	28
3753	9.965458		9.618295	10.381705	27
4058	9.965406		9.618652	10.381348	26
4361	9.965353		9.619008	10.380992	25
4665	9.965301		9.619364	10.380635	24
4968	9.965248		9.619720	10.380279	23
5271	9.965195		9.620076	10.379924	22
5574	9.965143		9.620432	10.379568	21
5877	9.965090		9.620787	10.379213	20
6179	9.965037		9.621142	10.378858	19
6481	9.964984		9.621497	10.378503	18
6783	9.964931		9.621852	10.378148	17
7085	9.964878		9.622206	10.377793	16
7386	9.964825		9.622561	10.377439	15
7687	9.964772		9.622915	10.377085	14
7988	9.964719		9.623269	10.376731	13
8289	9.964666		9.623623	10.376377	12
8589	9.964613		9.623976	10.376024	11
8890	9.964560		9.624330	10.375670	10
9190	9.964507		9.624683	10.375317	9
9489	9.964454		9.625036	10.374964	8
9789	9.964400		9.625388	10.374612	7
0088	9.964347		9.625741	10.374259	6
0387	9.964294		9.626093	10.373907	5
0686	9.964240		9.626445	10.373555	4
0984	9.964187		9.626797	10.373203	3
1282	9.964133		9.627149	10.372850	2
1580	9.964080		9.627501	10.372499	1
1878	9.964026		9.627852	10.372148	0
e	Sine		Co-tang	Tangent	M

Degree 67.

Degree 23.

M	Sine	Co-sine	Tangent	Co-
0	9.591878	9.964026	9.627852	10.37
1	9.592175	9.963972	9.628203	10.37
2	9.592473	9.963919	9.628554	10.37
3	9.592770	9.963865	9.628905	10.37
4	9.593067	9.963811	9.629255	10.37
5	9.593363	9.963757	9.629606	10.37
6	9.593658	9.963703	9.629956	10.37
7	9.593955	9.963650	9.630306	10.36
8	9.594251	9.963596	9.630655	10.36
9	9.594547	9.963542	9.631005	10.36
10	9.594842	9.963488	9.631354	10.36
11	9.595137	9.963433	9.631704	10.36
12	9.595432	9.963379	9.632053	10.36
13	9.595727	9.963325	9.632401	10.36
14	9.596021	9.963271	9.632750	10.36
15	9.596315	9.963217	9.633098	10.36
16	9.596610	9.963102	9.633447	10.36
17	9.596903	9.963108	9.633795	10.36
18	9.597196	9.963054	9.634143	10.36
19	9.597490	9.962999	9.634490	10.36
20	9.597783	9.962945	9.634838	10.36
21	9.598075	9.962892	9.635185	10.36
22	9.598368	9.962836	9.635530	10.36
23	9.598660	9.962781	9.635879	10.36
24	9.598952	9.962726	9.636226	10.36
25	9.599244	9.962572	9.636572	10.36
26	9.59953	9.962617	9.636918	10.36
27	9.599827	9.962562	9.637205	10.3
28	9.600118	9.962507	9.637610	10.3
29	9.600409	9.962453	9.637956	10.3
30	9.600700	9.962398	9.638302	10.3
	Co-sine	Sine	Co-tang.	Ta

Degree 66.

Degree 23.

Sine	*Co-sine.*		Tangent	*Co-tang.*	
.600700	9.962398		9.638302	10.361698	30
.600990	9.962343		9.638647	10.361353	29
.601280	9.962288		9.638992	10.361007	28
.601570	9.962233		9.639337	10.360662	27
.601860	9.962178		9.639682	10.360318	26
.602149	9.962122		9.640027	10.359973	25
.602439	9.962067		9.640371	10.359624	24
.602728	9.962012		9.640716	10.359284	23
.603017	9.961957		9.641060	10.358940	22
.603305	9.961902		9.641404	10.358596	21
.603594	9.961846		9.641747	10.358253	20
.603882	9.961791		9.642091	10.357909	19
.604170	9.961735		9.642434	10.357566	18
.604457	9.961680		9.642777	10.357222	17
.604745	9.961624		9.643120	10.356980	16
.605032	9.961569		9.643463	10.356537	15
.605319	9.961513		9.643806	10.356194	14
.605606	9.961458		9.644148	10.355852	13
.605892	9.961402		9.644490	10.355510	12
.606179	9.961346		9.644832	10.355168	11
.606465	9.961290		9.645174	10.354826	10
.606750	9.961235		9.645516	10.354484	9
.607036	9.961179		9.645857	10.354142	8
.607322	9.961123		9.646199	10.353801	7
.607607	9.961067		9.646540	10.353460	6
.607892	9.961011		9.646881	10.353119	5
.608176	9.960955		9.647222	10.352778	4
.608461	9.960899		9.647562	10.352438	3
.608745	9.960842		9.647903	10.352097	2
.609029	9.960786		9.648243	10.351757	1
.609313	9.960730		9.648583	10.351417	0
ine	Sine		*Co-tang.*	Tangent	

Degree 66.

Degree 24.

M	Sine	Co-sine	Tangent	Co-tang.
0	9.609313	9.960730	9.648583	10.351417
1	9.609597	9.960674	9.648923	10.351077
2	9.609880	9.960617	9.649263	10.350737
3	9.610163	9.960561	9.649602	10.350398
4	9.610446	9.960505	9.649942	10.350058
5	9.610729	9.960448	9.650281	10.349719
6	9.611012	9.960392	9.650620	10.349380
7	9.611294	9.960335	9.650959	10.349041
8	9.611576	9.960279	9.651297	10.348703
9	9.611858	9.960222	9.651636	10.348364
10	9.612140	9.960165	9.651974	10.348026
11	9.612421	9.960109	9.652312	10.347688
12	9.612702	9.960052	9.652650	10.347350
13	9.612983	9.959995	9.652988	10.347012
14	9.613264	9.959938	9.653326	10.346674
15	9.613545	9.959881	9.653663	10.346337
16	9.613825	9.959824	9.654000	10.346000
17	9.614105	9.959768	9.654337	10.345663
18	9.614385	9.959710	9.654674	10.345326
19	9.614665	9.959653	9.655011	10.344989
20	9.614944	9.959596	9.655348	10.344652
21	9.615223	9.959539	9.655684	10.344316
22	9.615502	9.959482	9.656020	10.343980
23	9.615781	9.959425	9.656356	10.343644
24	9.616060	9.959367	9.656692	10.343308
25	9.616338	9.959310	9.657028	10.342972
26	9.616616	9.959253	9.657363	10.342637
27	9.616894	9.959195	9.657699	10.342301
28	9.617172	9.959138	9.658034	10.341966
29	9.617450	9.959080	9.658369	10.341631
30	9.617727	9.959023	9.658704	10.341296
	Co-sine	Sine	Co-tang.	Tang.

Degree 65.

Degree 24.

Sine	Co-sine	Tangent	Co-tang.	
9.617727	9.959023	9.658704	10.341296	30
9.618004	9.958965	9.659039	10.340926	29
9.618281	9.958908	9.659373	10.340627	28
9.618558	9.958850	9.659708	10.340292	27
9.618834	9.958792	9.660042	10.339958	26
9.619110	9.958734	9.660376	10.339624	25
9.619386	9.958677	9.660710	10.339290	24
9.619662	9.958619	9.661043	10.338957	23
9.619938	9.958561	9.661377	10.338623	22
9.620213	9.958503	9.661710	10.338290	21
9.620488	9.958445	9.662043	10.337956	20
9.620763	9.958387	9.662376	10.337623	19
9.621038	9.958329	9.662709	10.337291	18
9.621313	9.958271	9.663042	10.336958	17
9.621587	9.958212	9.663374	10.336625	16
9.621861	9.958154	9.663707	10.336293	15
9.622135	9.958096	9.664039	10.335961	14
9.622409	9.958038	9.664371	10.335629	13
9.622682	9.957979	9.664703	10.335297	12
9.622956	9.957921	9.665035	10.334965	11
9.623229	9.957862	9.665366	10.334634	10
9.623502	9.957804	9.665697	10.334302	9
9.623776	9.957745	9.666029	10.333971	8
9.624047	9.957687	9.666360	10.333640	7
9.624319	9.957628	9.666691	10.333309	6
9.624591	9.957570	9.667021	10.332979	5
9.624863	9.957511	9.667352	10.332648	4
9.625134	9.957452	9.667682	10.332318	3
9.625406	9.957393	9.668012	10.331987	2
9.625677	9.957334	9.668343	10.331657	1
9.625948	9.957276	9.668672	10.331327	0
Tang.	Sine	Co-tang.	Tangent	

Degree 65.

Degree 25.

M.	Sine	Co-sine	Tangent	Co tang.
0	9,625948	9,957276	9,668672	10,331328
1	9,626219	9,957217	9,669002	10,330998
2	9,626490	9,957158	9,669332	10,330668
3	9,626760	9,957099	9,669661	10,330339
4	9,627030	9,957040	9,669990	10,330009
5	9,627300	9,956981	9,670320	10,329680
6	9,627570	9,956922	9,670645	10,329355
7	9,627840	9,956862	9,670977	10,329023
8	9,628109	9,956803	9,671306	10,328694
9	9,628378	9,956744	9,671634	10,328366
10	9,628697	9,956684	9,671963	10,328037
11	9,628916	9,956625	9,672291	10,327709
12	9,629184	9,956565	9,672619	10,327381
13	9,629453	9,956506	9,672947	10,327053
14	9,629721	9,956449	9,673274	10,326726
15	9,629989	9,956387	9,673603	10,326397
16	9,630257	9,956327	9,673929	10,326071
17	9,630524	9,956267	9,674256	10,325744
18	9,630792	9,956208	9,674584	10,325416
19	9,631059	9,956148	9,674910	10,325090
20	9,631326	9,956088	9,675237	10,324763
21	9,631592	9,956029	9,675564	10,324436
22	9,631856	9,955969	9,675890	10,324110
23	9,632125	9,955909	9,676216	10,323784
24	9,632392	9,955849	9,676543	10,323457
25	9,632657	9,955789	9,676869	10,323131
26	9,632923	9,955735	9,677194	10,322806
27	9,633189	9,955665	9,677520	10,322480
28	9,633454	9,955609	9,677845	10,322155
29	9,633719	9,955548	9,678171	10,321829
30	9,633984	9,955488	9,678496	10,321504

Co-sine	Sine	Co-tang.	Tang.

Degree 64.

Degree 25.

ine	Co-sine	Tangent	Co-tang.	
3984	9.955488	9.678496	10.321504	30
4249	9.955428	9.678821	10.321179	29
4514	9.955367	9.679146	10.320854	28
4778	9.955307	9.679471	10.320529	27
5042	9.955246	9.679795	10 320205	26
5306	9.955186	9.680120	10 319880	25
5570	9.955125	9.680444	10.319556	24
5833	9.955065	9.680768	10.319232	23
6097	9.955004	9.681092	10.318908	22
6360	9.954944	9.681416	10.318584	21
6623	9.954883	9.681740	10.318260	20
6886	9.954823	9.682063	10.317937	19
7148	9.954762	9.682386	10.317613	18
7411	9.954701	9.682710	10.317290	17
7673	9.954640	9.683033	10.316967	16
7935	9.954579	9.683356	10.316644	15
8197	9.954518	9.683678	10.316321	14
8458	9.954457	9.684001	10.315999	13
8720	9.954396	9.684324	10.315676	12
8981	9.954335	9.684646	10.315354	11
9242	9.954274	9.684968	10.315032	10
9503	9.954213	9.685290	10.314710	9
9764	9.954152	9.685612	10.314388	8
0024	9.954090	9.685934	10.314066	7
0284	9.954029	9.686255	10.313745	6
0544	9.953968	9.686577	10.313423	5
0804	9.953906	9.686898	10.313102	4
1064	9.953845	9.687219	10.312781	3
1323	9.953783	9.687540	10.312460	2
1583	9.953722	9.687861	10.312138	1
1842	9.953660	9.688182	10.311818	0
ine	Sine	Co-tang	Tangent	M

Degree 64.

Degree 26.

M	Sine	Co-sine	Tangent	Co-tang
0	9.641842	9.953660	9.688182	10.311
1	9.642101	9.953598	9.688502	10.311
2	9.642360	9.953537	9.688823	10.311
3	9.642618	9.953475	9.689143	10.310
4	9.642876	9.953413	9.689463	10.310
5	9.643135	9.953351	9.689783	10.310
6	9.643393	9.953290	9.690103	10.309
7	9.643650	9.953228	9.690423	10.309
8	9.643908	9.953166	9.690742	10.309
9	9.644165	9.953104	9.691063	10.308
10	9.644423	9.953042	9.691381	10.308
11	9.644680	9.952980	9.691700	10.308
12	9.644936	9.952917	9.692019	10.307
13	9.645193	9.952855	9.692338	10.307
14	9.645449	9.952793	9.692656	10.307
15	9.645706	9.952731	9.692975	10.307
16	9.645962	9.952668	9.693293	10.306
17	9.646218	9.952606	9.693612	10.306
18	9.646473	9.952544	9.693930	10.306
19	9.646729	9.952481	9.694248	10.305
20	9.646984	9.952419	9.694566	10.305
21	9.647239	9.952356	9.694883	10.305
22	9.647494	9.952294	9.695201	10.304
23	9.647749	9.952231	9.695518	10.304
24	9.648004	9.952168	9.695835	10.304
25	9.648258	9.952105	9.696153	10.303
26	9.648510	9.952043	9.696470	10.303
27	9.648766	9.951980	9.696786	10.303
28	9.649020	9.951917	9.697103	10.302
29	9.949274	9.951854	9.697420	10.302
30	9.649527	9.951791	9.697738	10.302
	Co-sine	Sine	Co-tang.	Tang

Degree 63.

Degree 26.

ne	Co-sine		Tangent	Co-tang.	
9527	9.951791		9.697738	10.302264	30
9781	9.951728		9.698052	10.301947	29
0034	9.951665		9.698369	10.301631	28
0287	9.951602		9.698685	10.301315	27
0519	9.951539		9.699001	10.300999	26
0498	9.951476		9.699316	10.300684	25
1044	9.951412		9.699632	10.300368	24
1296	9.951349		9.699947	10.300052	23
1648	9.951286		9.700263	10.299737	22
1800	9.951222		9.700578	10.299422	21
2052	9.951159		9.700893	10.299107	20
2303	9.951095		9.701208	10.298792	19
2555	9.951032		9.701522	10.298477	18
2806	9.950968		9.701837	10.298163	17
3057	9.950905		9.702152	10.297848	16
3307	9.950841		9.702466	10.297534	15
3558	9.950777		9.702782	10.297216	14
3808	9.950714		9.703095	10.296905	13
4059	9.950650		9.703409	10.296591	12
4309	9.950589		9.703722	10.296277	11
4558	9.950522		9.704036	10.295964	10
4808	9.950458		9.704350	10.295650	9
5057	9.950394		9.704663	10.295337	8
5307	9.950330		9.704976	10.295023	7
5556	9.950266		9.705290	10.294710	6
5805	9.950202		9.705603	10.294397	5
6053	9.950138		9.705915	10.294084	4
6302	9.950074		9.706228	10.293771	3
6550	9.950009		9.706541	10.293459	2
6799	9.949945		9.706853	10.293146	1
7047	9.949881		9.707166	10.292834	0
Ta	e	Sine	Co-tang.	Tangent	

Degree 63.

Degree 27.

M	Sine	Co sine	Tangent	Co
0	9.657047	9.949816	9.707166	10.29
1	9.657295	9.949816	9.707478	10.29
2	9.657542	9.949752	9.707790	10.29
3	9.657790	9.949687	9.708102	10.29
4	9.658037	9.949623	9.708414	10.29
5	9.658284	9.949558	9.708726	10.29
6	9.658538	9.949494	9.709037	10.29
7	9.658777	9.949429	9.709349	10.29
8	9.659024	9 949364	9.709660	10.29
9	9 659271	9.949300	9.709971	10.29
10	9.659517	9.949235	9.710282	10.28
11	9.659763	9.949170	9.710593	10.28
12	9.660009	9.949105	9.710904	10.28
13	9 660255	9.949040	9.711214	10.28
14	9.660500	9.948976	9.711525	10.28
15	9.660746	9.948910	9.711836	10.28
16	9.660991	9.948845	9.712146	10.28
17	9.661236	9.948760	9.712456	10.28
18	9.661481	9.948715	9.712766	10.28
19	9.661726	9.948650	9.713076	10.28
20	9.661970	9.948584	9.713386	10.28
21	9.662214	9.948519	9.713695	10.28
22	9.662459	9.948453	9.714005	10.28
23	9.662702	9.948388	9.714314	10.28
24	9.662947	9.948323	9.714624	10.28
25	9.663190	9.948257	9.714933	10.28
26	9.663433	9.948191	9.715242	10.28
27	9.663677	9.948126	9.715550	10.28
28	9.663920	9.948060	9.715859	10.28
29	9.664163	9.947995	9.716168	10.28
30	9.664406	9.947929	9.716477	10.28
	Co-sine	Sine	Co-tang.	T

Degree 62.

Degree 27.

Sine	Co-sine.	Tangent	Co-tang.	M
64406	9.947929	9.716477	10.283523	30
64648	9.947863	9.716785	10.283215	29
64891	9.947797	9.717093	10.282907	28
65133	9.947731	9.717401	10.282598	27
65375	9.947665	9.717709	10.282290	26
65617	9.947599	9.718017	10.281983	25
65858	9.947533	9.718325	10.281673	24
66100	9.947467	9.718633	10.281367	23
66341	9.947401	9.718940	10.281060	22
66583	9.947335	9.719248	10.280752	21
66824	9.947269	9.719555	10.280445	20
67065	9.947203	9.719862	10.280138	19
67305	9.947136	9.720169	10.279831	18
67546	9.947070	9.720476	10.279524	17
67786	9.947004	9.720783	10.279217	16
68025	9.946937	9.721089	10.278911	15
68266	9.946871	9.721395	10.278604	14
68506	9.946804	9.721702	10.278298	13
68746	9.946738	9.722008	10.277991	12
68986	9.946671	9.722315	10.277685	11
69225	9.946604	9.722621	10.277379	10
69464	9.946537	9.722927	10.277073	9
69703	9.946471	9.723232	10.276768	8
69942	9.946404	9.723538	10.276462	7
70181	9.946337	9.723843	10.276156	6
70419	9.946270	9.724149	10.275851	5
70657	9.946203	9.724454	10.275546	4
70896	9.946136	9.724759	10.275240	3
71134	9.946069	9.725065	10.274935	2
71372	9.946002	9.725369	10.274630	1
71609	9.945935	9.725674	10.274326	0
Co-sine	Sine	Co-tang.	Tangent	M

Degree 62.

Degree 28.

M	Sine	Co-sine	Tangent	Co-
0	9,671690	9,945935	9,725674	10,274
1	9,671847	9,945868	9,725979	10,274
2	9,672084	9,945800	9,726284	10,273
3	9,672321	9,945733	9,726588	10,273
4	9,672558	9,945666	9,726892	10,273
5	9,672795	9,945598	9,727197	10,272
6	9,673032	9,945531	9,727501	10,272
7	9,673268	9,945463	9,727805	10,272
8	9,673505	9,945396	9,728109	10,271
9	9,673741	9,945328	9,728412	10,271
10	9,673977	9,945261	9,728716	10,271
11	9,674213	9,945193	9,729020	10,270
12	9,674448	9,945125	9,729323	10,270
13	9,674684	9,945058	9,729626	10,270
14	9,674919	9,944990	9,729929	10,270
15	9,675154	9,944922	9,730232	10,269
16	9,675389	9,944854	9,730535	10,269
17	9,675623	9,944786	9,730838	10,269
18	9,675859	9,944718	9,731141	10,268
19	9,676094	9,944650	9,731445	10,268
20	9,676328	9,944582	9,731746	10,268
21	9,676562	9,944514	9,732048	10,267
22	9,676796	9,944446	9,732351	10,267
23	9,677030	9,944377	9,732653	10,267
24	9,677264	9,944309	9,732955	10,267
25	9,677497	9,944241	9,733257	10,266
26	9,677731	9,944172	9,733558	10,266
27	9,677964	9,944104	9,733860	10,266
28	9,678197	9,944016	9,734162	10,265
29	9,678430	9,943967	9,734463	10,265
30	9,678663	9,943898	9,734764	10,265
	Co-sine	Sine	Co-tang.	Tang

Degree 61.

Degree 28.

ine	Ct-fine	Tangent	Co-tang.	
8663	9,943898	9,734764	10,265236	30
8895	9,943830	9,735666	10,264934	29
9128	9,943761	9,735362	10,264633	28
9360	9,943692	9,735668	10,264332	27
9592	9,943624	9,735968	10,264031	26
9824	9,94355	9,736269	10,263731	25
0056	9,943486	9,736570	10,263430	24
0288	9,943417	9,736870	10,263130	23
0519	9,943348	9,737171	10,262829	22
0750	9,943279	9,737471	10,262529	21
0982	9,943210	9,737771	10,262229	20
1213	9,943141	9,738071	10,261929	19
1443	9,943072	9,738371	10,261629	18
1674	9,943003	9,738671	10,261329	17
1904	9,942933	9,738971	10,261029	16
2135	9,942864	9,739271	10,260729	15
2365	9,942795	9,739570	10,260430	14
2595	9,942725	9,739870	10,260130	13
2825	9,942656	9,740169	10,259831	12
3055	9,942587	9,740468	10,259532	11
3284	9,942517	9,740767	10,259233	10
3514	9,942448	9,741066	10,258934	9
3743	9,942378	9,741365	10,258635	8
3972	9,942308	9,741664	10,258336	7
4201	9,942239	9,741962	10,258038	6
4430	9,942169	9,742261	10,257739	5
4658	9,942099	9,742559	10,257441	4
4887	9,942029	9,742858	10,257142	3
5115	9,941959	9,743156	10,256844	2
5343	9,941889	9,743454	10,256546	1
5571	9,941819	9,743752	10,256248	0
ine	Sine	Co-tang.	Tangent.	M

Degree 61.

Degree 29.

M	Sine	Co-sine	Tangent	Co-tang
0	9,688557	9,941819	9,743752	10,256
1	9,688579	9,941749	9,744050	10,255
2	9,688602	9,941679	9,744348	10,255
3	9,688625	9,941609	9,744645	10,255
4	9,688648	9,941539	9,744943	10,255
5	9,688670	9,941468	9,745240	10,254
6	9,688693	9,941398	9,745538	10,254
7	9,688716	9,941328	9,745835	10,254
8	9,688738	9,941257	9,746132	10,253
9	9,688761	9,941187	9,746429	10,253
10	9,688784	9,941116	9,746726	10,253
11	9,688806	9,941046	9,747023	10,252
12	9,688829	9,940975	9,747319	10,252
13	9,688852	9,940905	9,747616	10,251
14	9,688874	9,940834	9,747912	10,251
15	9,688897	9,940763	9,748209	10,251
16	9,689198	9,940693	9,748505	10,251
17	9,689421	9,940622	9,748801	10,251
18	9,689648	9,940551	9,749097	10,250
19	9,689873	9,940480	9,749393	10,250
20	9,690098	9,940409	9,749689	10,250
21	9,690323	9,940338	9,749985	10,250
22	9,690548	9,940267	9,750281	10,249
23	9,690772	9,940196	9,750576	10,249
24	9,690996	9,940125	9,750872	10,249
25	9,691220	9,940053	9,751167	10,248
26	9,691444	9,939982	9,751462	10,248
27	9,691668	9,939911	9,751757	10,248
28	9,691892	9,939840	9,752052	10,247
29	9,692115	9,939768	9,752347	10,247
30	9,692339	9,939697	9,752642	10,247
M	Co-sine	Sine	Co-tang.	Tang

Degree 60.

Degree 29.

ne	Co-sine	Tangent	Co-tang.	
3339	9.939697	9.752642	10.247358	30
2562	9.939625	9.752937	10.247063	29
2785	9.939554	9.753231	10.246769	28
3008	9.939482	9.753526	10.246474	27
3231	9.939410	9.753820	10.246180	26
3453	9.939339	9.754115	10.245885	25
3676	9.939267	9.754409	10.245591	24
3898	9.939195	9.754703	10.245297	23
4120	9.939123	9.754997	10.245003	22
4342	9.939051	9.755291	10.244709	21
4564	9.938980	9.755584	10.244415	20
4786	9.938908	9.755878	10.244122	19
5007	9.938835	9.756172	10.243828	18
5229	9.938763	9.756465	10.243535	17
5450	9.938691	9.756759	10.243241	16
5671	9.938619	9.757052	10.242948	15
5892	9.938547	9.757345	10.242655	14
6113	9.938475	9.757638	10.242362	13
6334	9.938402	9.757931	10.242069	12
6554	9.938330	9.758224	10.241776	11
6774	9.938257	9.758517	10.241483	10
6995	9.938185	9.758810	10.241190	9
7215	9.938112	9.759102	10.240898	8
7435	9.938040	9.759395	10.240605	7
7654	9.937967	9.759687	10.240313	6
7874	9.937895	9.759979	10.240021	5
8093	9.937822	9.760271	10.239728	4
8313	9.937749	9.760564	10.239436	3
8532	9.937676	9.760856	10.239144	2
8751	9.937603	9.761147	10.238852	1
8970	9.937531	9.761439	10.238561	0
ine	Sine	Co-tang.	Tangent.	M

Degree 60.

Degree 30.

M	Sine	Co-sine	Tangent	Co-
0	9.698970	9.937531	9.761439	10.2
1	9.699189	9.937458	9.761731	10.2
2	9.699407	9.937385	9.762023	10.2
3	9.699626	9.937312	9.762314	10.2
4	9.699844	9.937238	9.762606	10.2
5	9.700062	9.937165	9.762897	10.2
6	9.700280	9.937092	9.763188	10.2
7	9.700498	9.937019	9.763479	10.2
8	9.700716	9.936945	9.763770	10.2
9	9.700933	9.936872	9.764061	10.2
10	9.701151	9.936799	9.764352	10.2
11	9.701367	9.936725	9.764643	10.2
12	9.701585	9.936652	9.764933	10.2
13	9.701802	9.936578	9.765224	10.2
14	9.702019	9.936505	9.765514	10.2
15	9.702236	9.936431	9.765805	10.2
16	9.702452	9.936357	9.766095	10.2
17	9.702669	9.936284	9.766385	10.2
18	9.702885	9.936210	9.766675	10.2
19	9.703101	9.936136	9.766965	10.2
20	9.703317	9.936062	9.767255	10.2
21	9.703533	9.935988	9.767545	10.2
22	9.703748	9.935914	9.767834	10.2
23	9.703964	9.935840	9.768124	10.2
24	9.704179	9.935766	9.768413	10.2
25	9.704395	9.935692	9.768703	10.2
26	9.704610	9.935618	9.768992	10.2
27	9.704825	9.935543	9.769281	10.2
28	9.705040	9.935469	9.769570	10.2
29	9.705254	9.935395	9.769859	10.2
30	9.705469	9.935320	9.770148	10.2
	Co-sine	Sine	Co-tang.	T

Degree 59.

Degree 30.

	Co-sine		Tangent	Co-tang.	
3469	9.935320		9.770148	10.229852	30
3683	9.935246		9.770437	10.229563	29
3897	9.935171		9.770726	10.229274	28
4112	9.935097		9.771015	10.228985	27
4326	9.935022		9.771303	10.228697	26
4539	9.934948		9.771592	10.228408	25
4753	9.934873		9.771880	10.228120	24
4967	9.934798		9.772168	10.227832	23
5180	9.934723		9.772456	10.227543	22
5393	9.934649		9.772745	10.227255	21
5606	9.934574		9.773033	10.226967	20
5819	9.934499		9.773321	10.226679	19
6032	9.934424		9.773608	10.226391	18
6243	9.934349		9.773896	10.226104	17
6457	9.934274		9.774184	10.225816	16
6670	9.934199		9.774471	10.225529	15
6882	9.934123		9.774759	10.225241	14
7094	9.934048		9.775046	10.224954	13
7306	9.933973		9.775333	10.224666	12
7518	9.933897		9.775621	10.224379	11
7730	9.933822		9.775908	10.224092	10
7941	9.933747		9.776195	10.223805	9
8153	9.933671		9.776482	10.223518	8
8364	9.933596		9.776768	10.223232	7
8575	9.933520		9.777055	10.222945	6
8786	9.933444		9.777342	10.222658	5
8997	9.933369		9.777628	10.222372	4
9208	9.933293		9.777915	10.222085	3
9418	9.933217		9.778201	10.221799	2
9629	9.933141		9.778487	10.221513	1
9839	9.933065		9.778774	10.221226	0
	Sine		Co-tang.	Tangent	

Degree 59.

Degree 31.

M	Sine	Co-sine	Tangent	Co-tang
0	9.711839	9.933066	9.778774	10.221
1	9.712049	9.932990	9.779060	10.220
2	9.712259	9.932914	9.779346	10.220
3	9.712469	9.932838	9.779632	10.220
4	9.712679	9.932761	9.779918	10.220
5	9.712889	9.932685	9.780203	10.219
6	9.713098	9.932609	9.780489	10.219
7	9.713308	9.932533	9.780775	10.219
8	9.713517	9.932457	9.781060	10.218
9	9.713726	9.932380	9.781346	10.218
10	9.713935	9.932304	9.781631	10.218
11	9.714144	9.932227	9.781916	10.218
12	9.714352	9.932151	9.782202	10.217
13	9.714561	9.932074	9.782486	10.217
14	9.714769	9.931998	9.782771	10.217
15	9.714977	9.931921	9.783056	10.216
16	9.715186	9.931845	9.783341	10.216
17	9.715394	9.931768	9.783626	10.216
18	9.715601	9.931691	9.783910	10.216
19	9.715809	9.931614	9.784195	10.215
20	9.716017	9.931537	9.784479	10.215
21	9.716224	9.931460	9.784764	10.215
22	9.716431	9.931383	9.785048	10.214
23	9.716639	9.931306	9.785332	10.214
24	9.716846	9.931229	9.785616	10.214
25	9.717053	9.931152	9.785900	10.214
26	9.717259	9.931075	9.786184	10.213
27	9.717466	9.930998	9.786468	10.213
28	9.717672	9.930920	9.786752	10.213
29	9.717879	9.930843	9.787036	10.212
30	9.718085	9.930766	9.787319	10.212
	Co-sine	Sine	Co-tang.	Tang

Degree 58.

Degree 31.

Sine	Co-sine.		Tangent	Co-tang.	
718085	9.930766		9.787519	10.212681	30
718291	9.930688		9.787603	10.212397	29
718497	9.930611		9.787886	10.212114	28
718703	9.930533		9.788170	10.211830	27
718929	9.930456		9.788453	10.211547	26
719114	9.930378		9.788736	10.211264	25
719320	9.930300		9.789019	10.210981	24
719525	9.930223		9.789302	10.210698	23
719730	9.930145		9.789585	10.210415	22
719935	9.930067		9.789868	10.210132	21
720140	9.929989		9.790151	10.209849	20
720345	9.929911		9.790433	10.209565	19
720549	9.929833		9.790716	10.209284	18
720754	9.929755		9.790999	10.209001	17
720958	9.929677		9.791281	10.208719	16
721162	9.929599		9.791563	10.208436	15
721366	9.929521		9.791846	10.208154	14
721570	9.929442		9.792128	10.207872	13
721774	9.929364		9.792410	10.207590	12
721978	9.929286		9.792692	10.207308	11
722181	9.929207		9.792974	10.207024	10
722385	9.929129		9.793256	10.206744	9
722588	9.929050		9.793538	10.206462	8
722791	9.928972		9.793819	10.206180	7
722994	9.928893		9.794101	10.205899	6
723197	9.928814		9.794383	10.205617	5
723400	9.928736		9.794664	10.205336	4
723603	9.928657		9.794945	10.205054	3
723805	9.928578		9.795227	10.204773	2
724007	9.928499		9.795508	10.204492	1
724210	9.928420		9.795789	10.204211	0
Co-sine	Sine		Co-tang.	Tangent	M

Degree 58.

Degree 31.

M	Sine	Co-sine	Tangent	Co-tang.
0	9,724210	9,928420	9,795789	10,204211
1	9,724412	9,928341	9,796070	10,203930
2	9,724614	9,928262	9,796351	10,203649
3	9,724816	9,928183	9,796632	10,203368
4	9,725017	9,928104	9,796913	10,203087
5	9,725219	9,928025	9,797194	10,202806
6	9,725420	9,927946	9,797474	10,202526
7	9,725622	9,927867	9,797755	10,202245
8	9,725823	9,927787	9,798036	10,201964
9	9,726024	9,927708	9,798316	10,201684
10	9,726225	9,927628	9,798596	10,201404
11	9,726426	9,927549	9,798877	10,201123
12	9,726626	9,927469	9,799157	10,200843
13	9,726827	9,927390	9,799437	10,200563
14	9,727027	9,927310	9,799717	10,200283
15	9,727228	9,927231	9,799997	10,200003
16	9,727428	9,927428	9,800277	10,199723
17	9,727628	9,927628	9,800557	10,199443
18	9,727828	9,926828	9,800836	10,199164
19	9,728027	9,926027	9,801116	10,198884
20	9,728227	9,926227	9,801396	10,198604
21	9,728437	9,926751	9,801675	10,198325
22	9,728626	9,926671	9,801955	10,198045
23	9,728825	9,926591	9,802234	10,197766
24	9,729024	9,926511	9,802513	10,197487
25	9,729223	9,926431	9,802792	10,197208
26	9,729422	9,926351	9,803072	10,196928
27	9,729621	9,926270	9,803351	10,196649
28	9,729820	9,926190	9,803630	10,196370
29	9,730018	9,926110	9,803908	10,196092
30	9,730216	9,926029	9,804187	10,195813
M	Co-sine	Sine	Co-tang.	Tang.

Degree 57.

Degree 32.

ine	Co-sine		Tangent	Co-tang.	
30216	9,926029		9,804187	10,195813	30
30415	9,925949		9,804466	10,195534	29
30613	9,925868		9,804745	10,195255	28
30811	9,925787		9,805023	10,194977	27
31009	9,925707		9,805302	10,194698	26
31206	9,925626		9,805580	10,194420	25
31404	9,925545		9,805859	10,194141	24
31601	9,925464		9,806137	10,193863	23
31799	9,925384		9,806415	10,193585	22
31996	9,925303		9,806693	10,193309	21
32193	9,925222		9,806971	10,193028	20
32390	9,925141		9,807249	10,192751	19
32587	9,925060		9,807527	10,192433	18
32784	9,924978		9,807805	10,192195	17
32980	9,924897		9,808083	10,191917	16
33177	9,924816		9,808361	10,191639	15
33373	9,924735		9,808638	10,191362	14
33569	9,924653		9,808916	10,191084	13
33765	9,924572		9,809193	10,190807	12
33961	9,924491		9,809471	10,190529	11
34157	9,924409		9,809748	10,190252	10
34353	9,924328		9,810025	10,189975	9
34548	9,924246		9,810302	10,189697	8
34744	9,924164		9,810580	10,189420	7
34939	9,924083		9,810857	10,189143	6
35134	9,924001		9,811134	10,188866	5
35330	9,923919		9,811410	10,188589	4
35525	9,923837		9,811687	10,188313	3
35719	9,923755		9,811964	10,188036	2
35914	9,923673		9,812241	10,187759	1
36109	9,923591		9,812517	10,187483	0
-sine	Sine		Co-tang	Tangent	M

Degree 57.

Degree 33.

M	Sine	Co-sine	Tangent	Co-tang
0	9,736109	9,923591	9,812517	10,187
1	9,736309	9,923509	9,812794	10,187
2	9,736497	9,923427	9,813070	10,186
3	9,736692	9,923345	9,813347	10,186
4	9,736886	9,923263	9,813623	10,186
5	9,737080	9,923180	9,813899	10,186
6	9,737274	9,923098	9,814175	10,185
7	9,737467	9,923016	9,814452	10,185
8	9,737661	9,922933	9,814728	10,185
9	9,737854	9,922851	9,815004	10,184
10	9,738048	9,922768	9,815279	10,184
11	9,738241	9,922686	9,815555	10,184
12	9,738434	9,922603	9,815831	10,184
13	9,738627	9,922520	9,816107	10,183
14	9,738820	9,922438	9,816382	10,183
15	9,739013	9,922355	9,816658	10,183
16	9,739205	9,922272	9,816933	10,183
17	9,739398	9,922189	9,817209	10,182
18	9,739590	9,922106	9,817484	10,182
19	9,739783	9,922023	9,817759	10,182
20	9,739975	9,921940	9,818035	10,181
21	9,740167	9,921857	9,818310	10,181
22	9,740359	9,921774	9,818585	10,181
23	9,740550	9,921691	9,818860	10,181
24	9,740742	9,921607	9,819135	10,180
25	9,740934	9,921524	9,819410	10,180
26	9,741125	9,921441	9,819684	10,180
27	9,741316	9,921357	9,819959	10,180
28	9,741507	9,921274	9,820234	10,179
29	9,741698	9,921190	9,820508	10,179
30	9,741889	9,921107	9,820783	10,179

Co-sine	Sine	Co-tang.	Tang

Degree 56.

Degree 33.

	Co-sine	Tangent	Co-tang.	
889	9.921107	9.820783	10.179217	30
080	9.921023	9.821057	10.178943	29
271	9.920939	9.821332	10.178668	28
461	9.920855	9.821606	10.178394	27
652	9.920772	9.821880	10.178120	26
842	9.920688	9.822154	10.177846	25
032	9.920604	9.822429	10.177571	24
223	9.920520	9.822703	10.177297	23
412	9.920436	9.822977	10.177023	22
602	9.920352	9.823250	10.176739	21
792	9.920268	9.823524	10.176476	20
982	9.920184	9.823798	10.176202	19
171	9.920099	9.824072	10.175928	18
361	9.920015	9.824345	10.175655	17
550	9.919931	9.824619	10.175381	16
739	9.919846	9.824892	10.175108	15
928	9.919762	9.825166	10.174834	14
117	9.919677	9.825439	10.174560	13
306	9.919593	9.825713	10.174287	12
494	9.919508	9.825986	10.174014	11
683	9.919424	9.826259	10.173741	10
871	9.919339	9.826532	10.173468	9
059	9.919254	9.826805	10.173195	8
248	9.919169	9.827078	10.172922	7
436	9.919084	9.827351	10.172649	6
624	9.918999	9.827624	10.172376	5
811	9.918915	9.827897	10.172103	4
999	9.918830	9.828170	10.171830	3
187	9.918744	9.828442	10.171558	2
374	9.918659	9.828715	10.171285	1
562	9.918574	9.828987	10.171012	0
	Sine	Co-tang.	Tangent.	

Degree 56.

Degree 34.

M	Sine	Co-sine	Tangent	Co-tan
0	9.747562	9.918374	9.828987	10.17
1	9.747749	9.918489	9.829260	10.17
2	9.747936	9.918404	9.829532	10.17
3	9.748123	9.918318	9.829805	10.17
4	9.748310	9.918233	9.830077	10.16
5	9.748497	9.918147	9.830349	10.16
6	9.748683	9.918062	9.830621	10.16
7	9.748870	9.917976	9.830893	10.16
8	9.749056	9.917891	9.831165	10.16
9	9.749242	9.917805	9.831437	10.16
10	9.749429	9.917719	9.831709	10.16
11	9.749615	9.917634	9.831981	10.16
12	9.749801	9.917548	9.832253	10.16
13	9.749986	9.917462	9.832525	10.16
14	9.750172	9.917376	9.832796	10.16
15	9.750358	9.917290	9.833068	10.16
16	9.750543	9.917204	9.833339	10.16
17	9.750729	9.917118	9.833611	10.16
18	9.750914	9.917032	9.833882	10.16
19	9.751099	9.916945	9.834154	10.16
20	9.751284	9.916859	9.834425	10.16
21	9.751469	9.916773	9.834696	10.16
22	9.751654	9.916686	9.834967	10.16
23	9.751838	9.916600	9.835238	10.16
24	9.752023	9.916514	9.835509	10.16
25	9.752207	9.916427	9.835780	10.16
26	9.752392	9.916340	9.836051	10.16
27	9.752576	9.916254	9.836322	10.16
28	9.752760	9.916167	9.836593	10.16
29	9.752944	9.916080	9.836864	10.16
30	9.753128	9.915994	9.837134	10.16

Co-sine	Sine	Co-tang	T

Degree 55.

Degree 34.

Sine	Co-sine		Tangent	Co-tang.	
53128	9.915994		9.837134	10.162866	30
53312	9.915907		9.837405	10.162595	29
53496	9.915820		9.837675	10.162325	28
53679	9.915733		9.837946	10.162054	27
53862	9.915646		9.838216	10.161784	26
54046	9.915559		9.838487	10.161513	25
54229	9.915472		9.838757	10.161243	24
54412	9.915385		9.839027	10.160973	23
54595	9.915297		9.839297	10.160702	22
54778	9.915210		9.839568	10.160432	21
54960	9.915123		9.839838	10.160162	20
55143	9.915035		9.840108	10.159892	19
55325	9.914948		9.840378	10.159622	18
55508	9.914860		9.840647	10.159352	17
55690	9.914773		9.840917	10.159083	16
55872	9.914685		9.841187	10.158813	15
56054	9.914597		9.841457	10.158543	14
56236	9.914510		9.841726	10.158273	13
56418	9.914422		9.841996	10.158004	12
56600	9.914334		9.842266	10.157734	11
56781	9.914246		9.842535	10.157465	10
56963	9.914158		9.842804	10.157195	9
57144	9.914070		9.843074	10.156926	8
57326	9.913982		9.843343	10.156657	7
57507	9.913894		9.843612	10.156387	6
57688	9.913806		9.843882	10.156118	5
57869	9.913718		9.844151	10.155849	4
58049	9.913630		9.844420	10.155580	3
58230	9.913541		9.844689	10.155311	2
58411	9.913453		9.844958	10.155042	1
58591	9.913364		9.845227	10.154774	0
e	Sine		Co-tang.	Tangent	M

Degree 55.

Degree 35.

M	Sine	Co-sine	Tangent	Co-t
0	9.758591	9.913364	9.845227	10.15
1	9.758772	9.913276	9.845496	10.15
2	9.758952	9.913187	6.845764	10.15
3	9.759132	9.913099	9.846033	10.15
4	9.759312	9.913010	9.846302	10.15
5	9.759492	9.912921	9.846570	10.15
6	9.759672	9.912833	9.846839	10.15
7	9.759851	9.912744	9.847107	10.15
8	9.760031	9.912655	9.847376	10.15
9	9.760210	9.912566	9.847644	10.15
10	9.760390	9.912477	9.847913	10.15
11	9.760569	9.912388	9.848181	10.15
12	9.760748	9.912299	9.848449	10.15
13	9.760927	9.912210	9.848717	10.15
14	9.761106	9.912121	9.848985	10.15
15	9.751285	9.912030	9.849254	10.15
16	9.761464	9.911942	9.849522	10.15
17	9.761642	9.911853	9.849789	10.15
18	9.761821	9.911763	9.850057	10.15
19	9.761999	9.911674	9.850325	10.15
20	9.762177	9.911584	9.850593	10.15
21	9.762356	9.911495	9.850861	10.15
22	9.762534	9.911405	9.851128	10.15
23	9.762712	9.911315	9.851396	10.15
24	9.762889	9.911226	9.851664	10.15
25	9.763067	9.911136	9.851931	10.15
26	9.763245	9.911046	9.852199	10.15
27	9.763422	9.920956	9.852466	10.15
28	9.763599	9.910866	9.852731	10.15
29	9.763777	9.910776	9.853001	10.15
30	9.763954	9.910686	9.853268	10.15

Co-sine	Sine	Co-tang.	T

Degree 54.

Degree 35.

Sine	Co-sine	Tangent	Co-tang.	
563954	9.910686	9.853268	10.146732	30
564131	9.910596	9.853532	10.146465	29
564308	9.910506	9.853802	10.146198	28
564485	9.910415	9.854069	10.145970	27
564662	9.910325	9.854336	10.145664	26
564838	9.910235	9.854603	10.145397	25
565015	9.910144	9.854870	10.145130	24
565191	9.910054	9.855137	10.144863	23
565367	9.909963	9.855404	10.144596	22
565544	9.909873	9.855671	10.144329	21
565720	9.909782	9.855937	10.144063	20
565896	9.909691	9.856204	10.143795	19
566071	9.909601	9.856471	10.143529	18
566247	9.909510	9.856737	10.143268	17
566423	9.909419	9.857004	10.142996	16
566598	9.909328	9.857270	10.142730	15
566774	9.909237	9.857537	10.142463	14
566949	9.909146	9.857803	10.142197	13
567124	9.909055	9.858069	10.141931	12
567299	9.908964	9.858336	10.141664	11
567474	9.908873	9.858602	10.141398	10
567649	9.908781	9.858868	10.141132	9
567824	9.908690	9.859134	10.140866	8
567997	9.908599	9.859400	10.140600	7
568173	9.908507	9.859666	10.140334	6
568348	9.908416	9.859932	10.140068	5
568522	9.908324	9.860198	10.139802	4
568696	9.908233	9.860464	10.139536	3
568871	9.908141	9.860730	10.139270	2
569045	9.908049	9.860995	10.139005	1
569219	9.907958	9.861261	10.138739	0
Co-sine	Sine	Co-tang.	Tangent.	

Degree 54.

Degree 36.

M	Sine	Co-sine	Tangent	Co-tang.
0	9,769229	9,907958	9,861261	10,138
1	9,769392	9,907866	9,861527	10,138
2	9,769566	9,907774	9,861792	10,138
3	9,769740	9,907682	9,862058	10,137
4	9,769913	9,907590	9,862323	10,137
5	9,770087	9,907498	9,862589	10,137
6	9,770260	9,907406	9,862854	10,137
7	9,770433	9,907314	9,863119	10,136
8	9,770606	9,907223	9,863385	10,136
9	9,770779	9,907129	9,863650	10,136
10	9,770952	9,907037	9,863915	10,136
11	9,771125	9,906945	9,864180	10,135
12	9,771298	9,906852	9,864445	10,135
13	9,771470	9,906760	9,864710	10,135
14	9,771643	9,906667	9,864975	10,135
15	9,771815	9,906574	9,865240	10,134
16	9,771987	9,906482	9,865505	10,134
17	9,772159	9,906389	9,865770	10,134
18	9,772331	9,906296	9,866035	10,133
19	9,772503	9,906203	9,866300	10,133
20	9,772675	9,906111	9,866564	10,133
21	9,772847	9,906018	9,866829	10,133
22	9,773018	9,905925	9,867094	10,132
23	9,773190	9,905832	9,867358	10,132
24	9,773361	9,905738	9,867623	10,132
25	9,773533	9,905645	9,867887	10,132
26	9,773704	9,905552	9,868152	10,131
27	9,773875	9,905459	9,868416	10,131
28	9,774046	9,905365	9,868680	10,131
29	9,774217	9,905272	9,868945	10,131
30	9,774388	9,905179	9,869209	10,130

Co-sine	Sine	Co-tang.	Tan

Degree 53.

Degree 36.

Sine	Co-sine	Tangent	Co-tang.	
74388	9,905179	9,869209	10,130791	30
74558	9,905085	9,869773	10,130527	29
74729	9,904992	9,869337	10,130263	28
74899	9,904898	9,870001	10,129999	27
75070	9,904804	9,870265	10,129735	26
75240	9,904711	9,870529	10,129471	25
75410	9,904617	9,870793	10,129207	24
75580	9,904523	9,871057	10,128943	23
75750	9,904429	9,871321	10,128679	22
75920	9,904335	9,871585	10,128415	21
76090	9,904241	9,871849	10,128151	20
76259	9,904147	9,872112	10,127888	19
76429	9,904053	9,872376	10,127624	18
76598	9,903959	9,872640	10,127360	17
76768	9,903864	9,872903	10,127097	16
76937	9,903770	9,873167	10,126833	15
77106	9,903676	9,873430	10,126570	14
77275	9,903581	9,873694	10,126306	13
77444	9,903486	9,873957	10,126043	12
77613	9,903392	9,874220	10,125780	11
77781	9,903298	9,874484	10,125516	10
77950	9,903203	9,874747	10,125253	9
78119	9,903108	9,875010	10,124990	8
78287	9,903013	9,875273	10,124727	7
78455	9,902919	9,875536	10,124464	6
78623	9,902824	9,875799	10,124201	5
78792	9,902729	9,876063	10,123937	4
78960	9,902634	9,876326	10,123674	3
79129	9,902539	9,876589	10,123411	2
79295	9,902444	9,876851	10,123149	1
79463	9,902349	9,877114	10,122885	0
Sine	Sine	Co-tang.	Tangent	M

Degree 53.

Degree 37.

M	Sine	Co-sine.	Tangent	Co-tang.
0	9,779463	9,902349	9,877114	10,122
1	9,779631	9,902253	9,877377	10,122
2	9,779798	9,902158	9,877640	10,122
3	9,779965	9,902063	9,877903	10,122
4	9,780133	9,901967	9,878165	10,121
5	9,780300	9,901872	9,878428	10,121
6	9,780467	9,901776	9,878691	10,121
7	9,780634	9,901681	9,878953	10,121
8	9,780801	9,901585	9,879216	10,120
9	9,780968	9,901488	9,879478	10,120
10	9,781134	9,901391	9,879741	10,120
11	9,781301	9,901298	9,880003	10,119
12	9,781467	9,901202	9,880266	10,119
13	9,781634	9,901106	9,880528	10,119
14	9,781800	9,901010	9,880790	10,119
15	9,781966	9,900914	9,881052	10,118
16	9,782132	9,900828	9,881314	10,118
17	9,782298	9,900722	9,881576	10,118
18	9,782464	9,900626	9,881839	10,118
19	9,782690	9,900529	9,882101	10,117
20	9,782796	9,900433	9,882363	10,117
21	9,782961	9,900337	9,882625	10,117
22	9,783127	9,900240	9,882886	10,117
23	9,783292	9,900144	9,883148	10,116
24	9,783457	9,900047	9,883410	10,116
25	9,783623	9,899951	9,883672	10,116
26	9,783788	9,899854	9,883934	10,116
27	9,783953	9,899757	9,884195	10,115
28	9,784118	9,899660	9,884457	10,115
29	9,784282	9,899563	9,884719	10,115
30	9,784447	9,899467	9,884980	10,115
M	Co-sine	Sine	Co-tang.	Ta

Degree 52.

Degree 37.

	Co-sine		Tangent	Co-tang.	
4	79.899267		9.884980	10.115020	30
5	69.899370		9.885242	10.114758	29
7	69.899273		9.885503	10.114497	28
4	69.899175		9.885765	10.114235	27
0	59.899078		9.886029	10.113974	26
6	99.898981		9.886288	10.113712	25
8	39.898884		9.886549	10.113451	24
9	19.898784		9.886810	10.113190	23
6	19.898689		9.887072	10.112928	22
2	59.898592		9.887333	10.112667	21
8	89.898494		9.887594	10.112406	20
5	29.898397		9.887855	10.112145	19
1	69.898299		9.888116	10.111884	18
7	99.898201		9.888377	10.111623	17
4	29.898104		9.888638	10.111362	16
0	99.898006		9.888899	10.111101	15
1	99.897908		9.889160	10.110840	14
3	29.897810		9.889421	10.110579	13
9	59.897112		9.889682	10.110318	12
5	79.897614		9.889943	10.110057	11
2	09.897519		9.880204	10.109796	10
8	39.897418		9.890965	10.109535	9
4	59.897320		9.890725	10.109275	8
0	89.897222		9.890986	10.109014	7
7	09.897123		9.891247	10.108753	6
3	29.897025		9.891507	10.108493	5
9	49.896926		9.891768	10.108232	4
5	69.896828		9.892028	10.107972	3
1	89.896729		9.892289	10.107711	2
8	09.896631		9.892549	10.107451	1
4	29.896532		9.892810	10.107190	0
	Sine		Co-tang.	Tangent	M.

Degree 52.

Degree 38.

M	Sine	Co-sine	Tangent	Co-t
0	9.789342	9.896532	9.892810	10.107
1	9.789504	9.896433	9.893070	10.106
2	9.789665	9.896335	9.893330	10.106
3	9.789827	9.896236	9.893591	10.106
4	9.789988	9.896137	9.893851	10.106
5	9.790149	9.896038	9.894111	10.105
6	9.790310	9.895939	9.894371	10.105
7	9.790471	9.895840	9.894632	10.105
8	9.790632	9.895741	9.894892	10.105
9	9.790793	9.895641	9.895152	10.104
10	9.790954	9.895542	9.895412	10.104
11	9.791115	9.895443	9.895672	10.104
12	9.791275	9.895343	9.895932	10.104
13	9.791436	9.895244	9.896192	10.104
14	9.791596	9.895144	9.896452	10.104
15	9.791756	9.895045	9.896712	10.103
16	9.791917	9.894945	9.896971	10.103
17	9.792077	9.894846	9.897231	10.103
18	9.792237	9.894746	9.897491	10.103
19	9.792397	9.894646	9.897751	10.103
20	9.792557	9.894546	9.898010	10.102
21	9.792716	9.894446	9.898270	10.102
22	9.792876	9.894346	9.898530	10.102
23	9.793035	9.894245	9.898789	10.102
24	9.793195	9.894146	9.899049	10.102
25	9.793354	9.894046	9.899308	10.101
26	9.793513	9.893946	9.899568	10.101
27	9.793673	9.893845	9.899827	10.101
28	9.793832	9.893745	9.900086	10.101
29	9.793991	9.893645	9.900346	10.100
30	9.794149	9.893444	9.900605	10.100

Co-sine	Sine	Co-tang.	T

Degree 51.

Degree 38.

ne	Co-sine		Tangent	Co-tang.	
4149	9.893444		9.900605	10.099395	30
4308	9.893444		9.900864	10.099135	29
4467	9.893343		9.901124	10.098876	28
4626	9.893243		9.901383	10.098617	27
4784	9.893142		9.901642	10.098358	26
4942	9.893041		9.901991	10.098099	25
5101	9.892940		9.902160	10.097839	24
5259	9.892839		9.902419	10.097580	23
5417	9.892738		9.902678	10.097321	22
5575	9.892637		9.902937	10.097062	21
5733	9.892536		9.903196	10.096803	20
5891	9.892435		9.903455	10.096544	19
6049	9.892334		9.903714	10.096285	18
6206	9.892233		9.903973	10.096027	17
6364	9.892132		9.904232	10.095768	16
6521	9.892030		9 904491	10.095509	15
6678	9.891928		9.904750	10.095250	14
6836	9.891827		9.905008	10.094991	13
6993	9.891726		9.905267	10.094733	12
7150	9.891624		9.905526	10.094474	11
7307	9.891522		9.905784	10.094215	10
7464	9.891421		9.906043	10.093957	9
7621	9.891319		9.906302	10.093698	8
7777	9.891217		9.906510	10.093440	7
7934	9.891115		9.906819	10.093181	6
8091	9.891013		9 907077	10.092923	5
8247	9.890911		9.907336	10.092664	4
8403	9.890809		9.907594	10.092406	3
8560	9.890707		9.907852	10.092147	2
8716	9.890605		9.908111	10.091889	1
8872	9.890503		9.908369	10.091631	0
	Sine		Co-tang.	Tangent	N

Degree 51.

Degree 39.

M	Sine	Co-sine	Tangent	Co
0	9.798872	9.890503	9.908369	10.0
1	9.799028	9.890400	9.908627	10.0
2	9.799184	9.890298	9.908886	10.0
3	9.799336	9.890195	9.909144	10.0
4	9.799495	9.890093	9.909402	10.0
5	9.799651	9.889990	9.909660	10.0
6	9.799806	9.889888	9.909918	10.0
7	9.799961	9.889785	9.910176	10.0
8	9.800117	9.889682	9.910435	10.0
9	9.800272	9.889579	9.910693	10.0
10	9.800427	9.889476	9.910951	10.0
11	9.800582	9.889374	9.911209	10.0
12	9.800737	9.889271	9.911467	10.0
13	9.800892	9.889167	9.911724	10.0
14	9.801047	9.889064	9.911982	10.0
15	9.801201	9.888961	9.912240	10.0
16	9.801356	9.888858	9.912498	10.0
17	9.801510	9.888755	9.912756	10.0
18	9.801665	9.888651	9.913014	10.0
19	9.801819	9.888548	9.913271	10.0
20	9.801973	9.888444	9.913529	10.0
21	9.802127	9.888341	9.913787	10.0
22	9.802282	9.888237	9.914044	10.0
23	9.802435	9.888133	9.914302	10.0
24	9.802589	9.888030	9.914560	10.0
25	9.802743	9.887926	9.914817	10.0
26	9.802897	9.887822	9.915075	10.0
27	9.803050	9.887718	9.915332	10.0
28	9.803204	9.887614	9.915590	10.0
29	9.803357	9.887510	9.915847	10.0
30	9.803510	9.887406	9.916104	10.0
	Co-sine	Sine	Co-tang	

Degrée 50.

Degree 39.

Sine	Co-sine	Tangent	Co-tang.	
803510	9.887406	9.916104	10.083895	30
803664	9.887302	9.916362	10.083638	29
803817	9.887198	9.916619	10.083381	28
803970	9.887093	9.916876	10.083123	27
804123	9.886989	9.917134	10.082866	26
804276	9.886884	9.917391	10.082609	25
804428	9.886780	9.917648	10.082352	24
804586	9.886675	9.917905	10.082094	23
804734	9.886561	9.918162	10.081837	22
804886	9.886466	9.918420	10.081580	21
805038	9.886361	9.918677	10.081323	20
805191	9.886257	9.918934	10.081 66	19
805343	9.886152	9.919191	10.080809	18
805495	9.886047	9.919448	10.080552	17
805647	9.885942	9.919705	10.080295	16
805799	9.885837	9.919962	10.080038	15
805951	9.885732	9.920219	10.079781	14
806103	9.885627	9.920476	10.079524	13
806254	9.885521	9.920733	10.079267	12
806406	9.885416	9.920990	10.079010	11
806557	9.885311	9.921247	10.078753	10
806709	9.885205	9.921503	10.078496	9
806860	9.885100	9.921760	10.078240	8
807011	9.884994	9.922017	10.077983	7
807162	9.884889	9.922274	10.077726	6
807314	9.884783	9.922530	10.077469	5
807464	9.884677	9.922787	10.077213	4
807615	9.884572	9.923044	10.076956	3
807766	9.884466	9.923300	10.076699	2
807917	9.884360	9.923557	10.076443	1
808067	9.884254	9.923813	10.076186	0
Co-sine	Sine	Co-tang.	Tangent.	M

Degree 50.

Degree 40.

M	Sine	Co-sine	Tangent	Co-tang.
0	9,808067	9,884254	9,923813	10,076
1	9,808218	9,884188	9,924070	10,075
2	9,808368	9,884049	9,924327	10,075
3	9,808519	9,883936	9,924583	10,075
4	9,808669	9,883829	9,924839	10,075
5	9,808819	9,883723	9,925096	10,074
6	9,808969	9,883617	9,925352	10,074
7	9,809119	9,883510	9,925609	10,074
8	9,809269	9,883404	9,925865	10,074
9	9,809419	9,883297	9,926121	10,073
10	9,809569	9,883191	9,926378	10,073
11	9,809718	9,883084	9,926634	10,073
12	9,809868	9,882977	9,926890	10,073
13	9,810017	9,882871	9,927147	10,072
14	9,810166	9,882764	9,927403	10,072
15	9,810316	9,882659	9,927659	10,072
16	9,810465	9,882550	9,927915	10,072
17	9,810614	9,882443	9,928171	10,071
18	9,810762	9,882336	9,928427	10,071
19	9,810912	9,882228	9,928683	10,071
20	9,810061	9,882121	9,928940	10,071
21	9,811210	9,882014	9,929196	10,070
22	9,811358	9,881907	9,929452	10,070
23	9,811506	9,881799	9,929708	10,070
24	9,811655	9,881692	9,929964	10,070
25	9,811804	9,881584	9,920219	10,069
26	9,811952	9,881477	9,930475	10,069
27	9,812100	9,881369	9,930731	10,069
28	9,812248	9,881261	9,930987	10,068
29	9,812396	9,881153	9,931243	10,068
30	9,812544	9,881045	9,931499	10,068
	Co-sine	Sine	Co-tang.	Tang

Degree 49.

Degree 40.

ine	Co-sine		Tangent	Co-tang.	
2544	9,881045		9,931499	10,068501	30
2692	9,880937		9,931755	10,068245	29
2840	9,880829		9,932010	10,067989	28
2988	9,880721		9,932266	10,067734	27
3135	9,880613		9,932522	10,067478	26
3283	9,880505		9,932778	10,067222	25
3430	9,880397		9,933033	10,066967	24
3578	9,880289		9,933289	10,066711	23
3725	9,880180		9,933545	10,066455	22
3872	9,880072		9,933800	10,066200	21
4019	9,879963		9,934056	10,065944	20
4166	9,879855		9,934311	10,065688	19
4313	9,879746		9,934567	10,065433	18
4460	9,879637		9,934822	10,065177	17
4607	9,879529		9,935078	10,064922	16
4753	9,879420		9,935333	10,064666	15
4900	9,879311		9,935589	10,064411	14
5046	9,879202		9,935844	10,064156	13
5193	9,879093		9,936100	10,063900	12
5339	9,878984		9,936355	10,063645	11
5485	9,878875		9,936610	10,063389	10
5631	9,878766		9,936866	10,063134	9
5777	9,878656		9,937121	10,062879	8
5923	9,878547		9,937376	10,062623	7
6069	9,878438		9,937632	10,062368	6
6215	9,878328		9,937887	10,062113	5
6361	9,878219		9,938142	10,061858	4
6506	9,878109		9,938397	10,061602	3
6652	9,877999		9,938653	10,061347	2
6797	9,877890		9,938908	10,061092	1
6943	9,877780		9,939163	10,060837	0
ine	Sine		Co-tang.	Tangent	M

Degree 49.

Degree 41.

M	Sine	Co-sine.	Tangent	Co-tang.
0	9,816943	9,877780	9,939163	10,0601
1	9,817088	9,877670	9,939418	10,0601
2	9,817233	9,877560	9,939673	10,0603
3	9,817378	9,877450	9,939928	10,0600
4	9,817523	9,877340	9,940183	10,0598
5	9,817668	9,877230	9,940438	10,0595
6	9,817813	9,877120	9,940693	10,0593
7	9,817958	9,877009	9,940948	10,0590
8	9,818103	9,876899	9,941203	10,0588
9	9,818247	9,876789	9,941458	10,0585
10	9,818392	9,876678	9,941713	10,0583
11	9,818536	9,876568	9,941968	10,0580
12	9,818681	9,876457	9,942223	10,0577
13	9,818825	9,876347	9,942478	10,0575
14	9,818969	9,876236	9,942733	10,0572
15	9,819113	9,876125	9,942988	10,0570
16	9,819257	9,876014	9,943243	10,0567
17	9,819401	9,875904	9,943498	10,0565
18	9,819545	9,875793	9,943752	10,0562
19	9,819689	9,875682	9,944007	10,0560
20	9,819832	9,875571	9,944262	10,0557
21	9,819976	9,875459	9,944517	10,0555
22	9,820119	9,875348	9,944771	10,0552
23	9,820263	9,875237	9,945026	10,0550
24	9,820406	9,875125	9,945281	10,0547
25	9,820549	9,875014	9,945535	10,0545
26	9,820693	9,874903	9,945790	10,0542
27	9,820836	9,874791	9,946045	10,0540
28	9,820979	9,874679	9,946299	10,0537
29	9,821122	9,874568	9,946554	10,0535
30	9,821264	9,874456	9,946808	10,0532
	Co-sine	Sine	Co-tang.	Tan

Degree 48.

Degree 41.

	Co-sine	Tangent	Co-tang.	
	9.874456	9.946808	10.053192	30
	9.874344	9.947065	10.052935	29
	9.874232	9.947317	10.052682	28
	9.874120	9.947572	10.052428	27
	9.874008	9.947826	10.052173	26
	9.873896	9.948081	10.051919	25
	9.873784	9.948335	10.051664	24
	9.873672	9.948590	10.051410	23
	9.873560	9.948844	10.051156	22
	9.873447	9.949099	10.050901	21
	9.873335	9.949353	10.050647	20
	9.873223	9.949607	10.050393	19
	9.873110	9.949862	10.050138	18
	9.872998	9.950116	10.049884	17
	9.872885	9.950370	10.049630	16
	9.872772	9.950625	10.049375	15
	9.872659	9.950879	10.049121	14
	9.872546	9.951133	10.048867	13
	9.872434	9.951388	10.048612	12
	9.872321	9.951642	10.048358	11
	9.872208	9.951896	10.048104	10
	9.872094	9.952150	10.047850	9
	9.871981	9.952404	10.047595	8
	9.871868	9.952659	10.047341	7
	9.871755	9.952913	10.047087	6
	9.871641	9.953167	10.046833	5
	9.871528	9.953421	10.046579	4
	9.871414	9.953675	10.046325	3
	9.871301	9.953929	10.046071	2
	9.871187	9.954183	10.045817	1
	9.871073	9.954437	10.045563	0
	Sine	Co-tang.	Tangent.	M

Degree 48.

Degree 42.

M	Sine	Co-sine	Tangent	Co-
0	9.825511	9.871073	9.954437	10.0
1	9.825651	9.870960	9.954691	10.0
2	9.825791	9.870846	9.954945	10.0
3	9.825931	9.870732	9.955199	10.0
4	9.826071	9.870618	9.955453	10.0
5	9.826211	9.870504	9.955707	10.0
6	9.826351	9.870390	9.955961	10.0
7	9.826491	9.870271	9.956215	10.0
8	9.826631	9.870161	9.956469	10.0
9	9.826770	9.870047	9.956723	10.0
10	9.826910	9.869933	9.956977	10.0
11	9.827049	9.869818	9.957231	10.0
12	9.827189	9.869704	9.957485	10.0
13	9.827328	9.869589	9.957739	10.0
14	9.827467	9.869474	9.957993	10.0
15	9.827606	9.869360	9.958246	10.0
16	9.827745	9.869245	9.958500	10.0
17	9.827884	9.869130	9.958754	10.0
18	9.828023	9.869015	9.959008	10.0
19	9.828162	9.868900	9.959262	10.0
20	9.828301	9.868785	9.959515	10.0
21	9.828439	9.868670	9.959769	10.0
22	9.828578	9.868555	9.960023	10.0
23	9.828716	9.868439	9.960277	10.0
24	9.828855	9.868324	9.960530	10.0
25	9.828993	9.868209	9.960784	10.0
26	9.829131	9.868093	9.961038	10.0
27	9.829269	9.867978	9.961291	10.0
28	9.829407	9.867862	9.961545	10.0
29	9.829545	9.867747	9.961799	10.0
30	9.829683	9.867631	9.962052	10.0
	Co-sine	Sine	Co-tang.	T

Degree 47.

Degree 42.

	Co-sine		Tangent	Co-tang.	
818	9.867631		9.962052	10.037947	30
821	9.867515		9.962306	10.037694	29
959	9.867399		9.962560	10.037440	28
096	9.867283		9.962813	10.037187	27
234	9.867167		9.963067	10.036933	26
372	9.867051		9.963320	10.036680	25
509	9.866935		9.963574	10.036426	24
646	9.866819		9.963827	10.036173	23
784	9.866703		9.964081	10.035919	22
921	9.866586		9.964335	10.035665	21
058	9.866470		9.964588	10.035412	20
195	9.866353		9.964842	10.035158	19
332	9.866237		9.965095	10.034905	18
469	9.866120		9.965348	10.034652	17
606	9.866004		9.965602	10.034398	16
742	9.865887		9.965855	10.034144	15
879	9.865770		9.966109	10.033891	14
015	9.865653		9.966362	10.033638	13
152	9.865536		9.966616	10.033384	12
288	9.865419		9.966869	10.033131	11
425	9.865302		9.967122	10.032878	10
561	9.865185		9.967376	10.032624	9
697	9.865068		9.967629	10.032371	8
833	9.864950		9.967883	10.032117	7
969	9.864833		9.968136	10.031864	6
105	9.864716		9.968389	10.031611	5
241	9.864598		9.968643	10.031357	4
376	9.864480		9.968896	10.031104	3
512	9.864363		9.969146	10.030851	2
648	9.864245		9.969403	10.030597	1
783	9.864127		9.969656	10.030344	0
	Sine		Co-tang.	Tangent	M.

Degree 47.

Degree 45.

M	Sine	Co-sine	Tangent	Co
0	9.833783	9.864127	9.969656	
1	9.833918	9.864010	9.969909	
2	9.834054	9.863892	9.970162	
3	9.834188	9.863774	9.970416	
4	9.834324	9.863656	9.970669	
5	9.834460	9.863537	9.970923	
6	9.834595	9.863419	9.971178	
7	9.834730	9.863301	9.971432	
8	9.834865	9.863183	9.971687	
9	9.834999	9.863064	9.971943	
10	9.835134	9.862946	9.972198	
11	9.835269	9.862827	9.972454	
12	9.835503	9.862709	9.972709	
13	9.835538	9.862590	9.972965	
14	9.835672	9.862471	9.973221	
15	9.835806	9.862353	9.973477	
16	9.835941	9.862234	9.973733	
17	9.836075	9.862115	9.973989	
18	9.836209	9.861996	9.974245	
19	9.836343	9.861877	9.974501	
20	9.836477	9.861757	9.974757	
21	9.836611	9.861638	9.975013	
22	9.836745	9.861519	9.975269	
23	9.836878	9.861399	9.975526	
24	9.837012	9.861280	9.975782	
25	9.837146	9.861161	9.976039	
26	9.837279	9.861041	9.976295	
27	9.837412	9.860922	9.976552	
28	9.837545	9.860802	9.976809	
29	9.837679	9.860682	9.977065	
30	9.837812	9.860562	9.977322	

Co-sine	Sine	Co-tang	T

Degree 46.

Degree 43.

Sine	Co-sine	Tangent	Co-tang.	
3781	9.860562	9.977250	10.022750	30
3794	9.860448	9.977302	10.022698	29
3807	9.860322	9.977354	10.022646	28
3821	9.860202	9.977406	10.022594	27
3834	9.860082	9.977458	10.022542	26
3847	9.859962	9.977510	10.022490	25
3860	9.859842	9.977562	10.022438	24
3874	9.859721	9.977614	10.022386	23
3887	9.859601	9.977666	10.022334	22
3900	9.859480	9.977718	10.022282	21
3914	9.859360	9.977770	10.022230	20
3927	9.859239	9.977822	10.022178	19
3940	9.859118	9.977874	10.022126	18
3953	9.858998	9.977926	10.022074	17
3966	9.858877	9.977978	10.022022	16
3980	9.858756	9.978030	10.021970	15
3993	9.858635	9.978082	10.021918	14
4006	9.858514	9.978134	10.021866	13
4019	9.858392	9.978186	10.021814	12
4032	9.858272	9.978238	10.021762	11
4045	9.858150	9.978289	10.021711	10
4059	9.858029	9.978341	10.021659	9
4072	9.857908	9.978393	10.021607	8
4085	9.857786	9.978445	10.021555	7
4098	9.857665	9.978497	10.021503	6
4111	9.857543	9.978549	10.021451	5
4124	9.857421	9.978600	10.021400	4
4138	9.857300	9.978652	10.021348	3
4151	9.857178	9.978704	10.021296	2
4164	9.857056	9.978756	10.021244	1
4177	9.856934	9.978807	10.021193	0
Sine	Sine	Co-tang.	Tangent	M

Degree 46.

Degree 44.

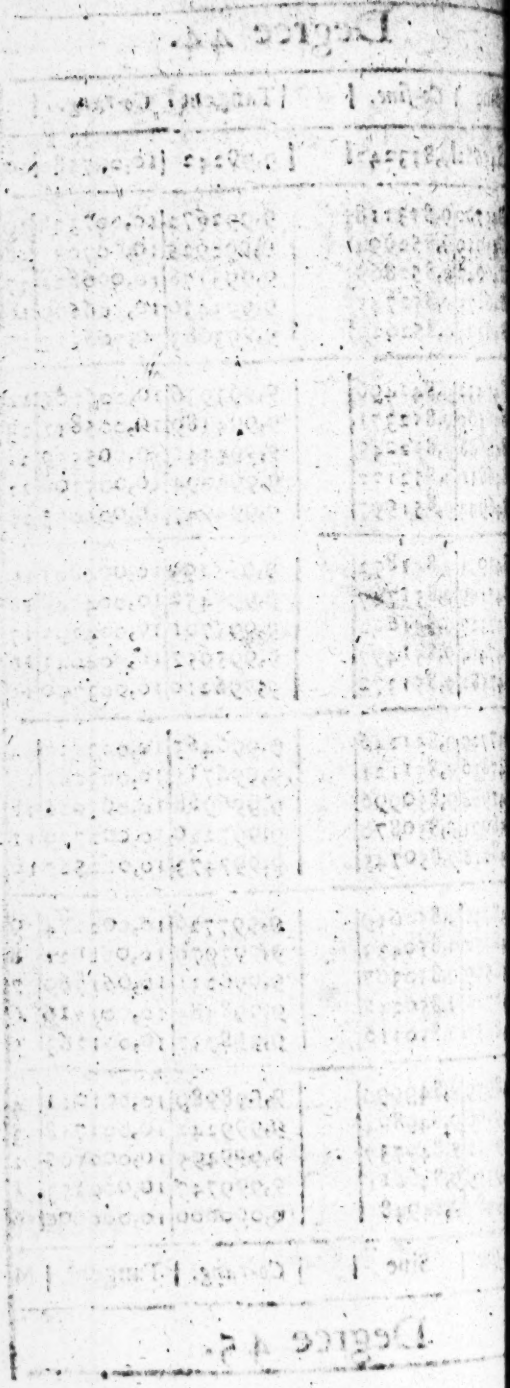
M	Sine	Co-sine	Tangent	Co-tang
0	9,841771	9,856934	9,984837	10,015163
1	9,841902	9,856811	9,985090	10,014910
2	9,842033	9,856690	9,985343	10,014657
3	9,842163	9,856568	9,985596	10,014404
4	9,842294	9,856445	9,985848	10,014152
5	9,842424	9,856323	9,986101	10,013899
6	9,842555	9,856201	9,986354	10,013646
7	9,842685	9,856078	9,986607	10,013393
8	9,842815	9,855956	9,986859	10,013141
9	9,842945	9,855833	9,987112	10,012888
10	9,843076	9,855710	9,987365	10,012635
11	9,843206	9,855588	9,987618	10,012382
12	9,843336	9,855465	9,987871	10,012129
13	9,843465	9,855342	9,988123	10,011877
14	9,843595	9,855218	9,988376	10,011624
15	9,843725	9,855096	9,988629	10,011371
16	9,843855	9,854973	9,988887	10,011113
17	9,843984	9,854850	9,989134	10,010866
18	9,844114	9,854727	9,989387	10,010613
19	9,844243	9,854603	9,989640	10,010360
20	9,844372	9,854480	9,989893	10,010107
21	9,844502	9,854356	9,990145	10,009855
22	9,844631	9,854233	9,990398	10,009602
23	9,844760	9,854109	9,990651	10,009349
24	9,844889	9,853986	9,990903	10,009097
25	9,845018	9,853862	9,991156	10,008844
26	9,845147	9,853738	9,991409	10,008591
27	9,845276	9,853614	9,991662	10,008338
28	9,845404	9,853590	9,991914	10,008086
29	9,845533	9,853266	9,992167	10,007833
30	9,845662	9,853242	9,992420	10,007580
M	Co-sine	Sine	Co-tang.	Tang

Degree 45.

Degree 44.

Sine	Co-sine.		Tangent	Co-tang.	
9,845661	9,853242		9,992420	10,007580	30
9,845790	9,853118		9,992672	10,007328	29
9,845919	9,852994		9,992925	10,007075	28
9,846047	9,852869		9,993178	10,006822	27
9,846175	9,852745		9,993430	10,006569	26
9,846304	9,852620		9,993683	10,006317	25
9,846431	9,852496		9,993936	10,006064	24
9,846560	9,852371		9,994189	10,005811	23
9,846688	9,852246		9,994441	10,005559	22
9,846816	9,852122		9,994694	10,005306	21
9,846944	9,851997		9,994947	10,005053	20
9,847071	9,851872		9,995199	10,004801	19
9,847199	9,851747		9,995452	10,004548	18
9,847327	9,851622		9,995701	10,004295	17
9,847454	9,851497		9,995957	10,004043	16
9,847582	9,851372		9,996210	10,003790	15
9,847709	9,851246		9,996463	10,003537	14
9,847836	9,851121		9,996715	10,003285	13
9,847964	9,850996		9,996968	10,003032	12
9,848091	9,850870		9,997220	10,002779	11
9,848218	9,850745		9,997473	10,002527	10
9,848345	9,850619		9,997726	10,002274	9
9,848472	9,850493		9,997979	10,002021	8
9,848599	9,850367		9,998231	10,001769	7
9,848726	9,850242		9,998484	10,001516	6
9,848852	9,850116		9,998737	10,001263	5
9,848979	9,849990		9,998989	10,001011	4
9,849106	9,849864		9,999242	10,001758	3
9,849232	9,849737		9,999495	10,000505	2
9,849359	9,849611		9,999747	10,000253	1
9,849485	9,849485		10,000000	10,000000	0
Co-sine	Sine		Co-tang.	Tangent.	M

Degree 45.



THE
Young Gentleman's
MECHANICKS:

CONTAINING

he more Uſeful and Eaſy
ELEMENTS of *Mechanicks*,
more properly ſo called, of *Sta-*
icks, and *Hydroſtaticks*.

EDWARD WELLS, D. D.
Rector of *Cotesbach* in *Leiceſter-*
ſhire.

LONDON,
ed for *James Knapton*, at the *Crown*
St. *Paul's* Church-Yard. 1713.

THE
REFACE.

Othing more needs be ſaid, to ſhew the great Uſefulneſs of the Science, denoted by the Name of Mechanicks, than teaches, and explains in a ical Manner, thoſe Principles, is founded the Power or the ſeveral Machines or En- ade uſe of either in Civil or Affairs; and alſo that it ex- n like manner the various Mo- heavy Bodies, conſider'd either ir or in the Water. The bare of Machins is indeed an illi- ſervile Employ, and fit only ns of the meaner Sort; whence ndy-craft Perſons are ſome- lled, by way of Degradation,

Mechanicks. *But the Knowled*
the Principles, whereon depend m
nical Arts or Trade, is justly est
a Liberal Science, not below the
dy of Persons of the highest Ab
and Rank. Forasmuch as this
ence both gives a very considera
sight into the Works of Nature;
only with respect to Sublunary B
but even to the Frame, Order
Preservation of the Celestial Sys
and also may be infinitely serv
to the Concerns of Humane Li
the Invention of new Machines
struments: especially if it be c
ted by Persons, who have both
ty of Parts to invent, *and al*
lity of Pocket actually to expe
their Inventions, *and by this*
to compleat and perfect them.
is to be fear'd, that many a
Invention has unfortunately mi
because the Inventer has not
good Purse, *as well as a good*

THE
ung Gentleman's
ECHANICKS.

CHAP. I.

in are explain'd ſuch Terms as ate to Mechanicks in general.

1. Mechanicks, *what, and why ſo called.*

THAT Art or Science, which teaches how to move any given Weight, by any given Power, we call in one (†) *Mechanicks*; becauſe it is ſerviceable to the contriving of Machines or Engines, which are us'd

(†) The Word *Mechanicks* is deriv'd from the Greek μηχανή; from which is made likewiſe the corre- Latin Word *Machina*; and from this is form'd ſh Word *Machine*. From the ſame original Greek deriv'd alſo the Word *Mechaniſm*, i. e. the Arti- rivance of a Thing.

in the Concerns of Life, whether or Military.

2. *A Weight, and a Power, and Motion, what in Mechanicks.*

By *a Weight* is understood in M nicks any heavy Body; by a *Po* whatever can move an heavy and by *Motion*, Local Motion Change of Place.

3. *The Axis and Center of Motion, what.*

The *Axis of Motion* is that righ round which a Body being mov'd, Point of the Body describes whose Centers are all in the said Line; and the Mid-point of the called *the Center of Motion.* Thus the Sphere or Bowl ADBE, being co to move round the right Line A said AB will be the Axis, and the the Center of its Motion.

4. *The Direction of Motion, what.*

That right Line, wherein any B Power endeavours to move, is ca *Line of Direction*, or simply, the *on* of the Motion of the respectiv or Power. And if a Body mov Curve Line, then the Direction of tion in each Point of the Curve Tangent of the said Point. Thus AB is the Direction of the We and BC the Direction of the Pow

(*) Hence any Weight or heavy Body is or steem'd a Power, in reference to any other Wei vy Body, which it do's or can move.

Fig. 3, the Tangents TA, TB, TC, ſpectively the ſeveral Directions of mov'd round the Curve ABC, at eral Points A, B, and C.

t-Force, which is in Bodies mov'd, hereby they continually move, or our to move, is call'd their *Ma-*, and ariſes from the Weight of uantity of Matter in) the Body , and the Velocity or Swiftneſs with it moves.

5. *The* Momentum *of a Body, what.*

different Weight of Bodies ariſes he different Quantity of Matter in d Bodies. Whence the Quantity ter in any Body may be eſtimated Weight. For inſtance, if a cubi- of Lead be ſix times heavier than cal Inch of Wood, it may thence r'd, that the Quantity of Matter a Piece of Lead is ſix times as as the Quantity of Matter in a like f Wood.

6. *The Weight of Bodies is* proportional *to their Matter.*

t Gravity or Weight, which ariſes he more or leſs Compactneſs of naturally belonging to the ſeveral or Sorts of Bodies, is called their *Gravity.* Thus, although a of Feathers be equal in Weight ound of Lead, yet Lead is ſaid to *ifically* heavier than Feathers, be- here is not ſo much Matter, nor uently ſo much Weight, in a Fea-

7. Specifick Gravity, *what.*

ther, as in a Piece of Lead of the Bulk with the Feather.

8. Centrum Gravium, *what.* Gravity or Weight being that, causes in Bodies a Tendency to mo not hinder'd by some external F downwards, or towards the Center Earth; hence the Center of the E ſtil'd *Centrum Gravium*, or the Ce which heavy Bodies tend.

9. *The* Center of Gravity, *what.* The *Center of Gravity* in any B that Point, through which a Plan ſing divides the Body into two Seg or Parts of equal Weight. It is th ter of Gravity, which is of chief in Bodies, a Body being not pr ſaid to deſcend or aſcend, but its Center of Gravity deſcends cends.

10. Velocity, *what.* Velocity (which is the other I ent, that together with Weight ma the Momentum of a Body) is that tion of Motion, which is meaſu comparing together the Length an of Motion. Thus *Equal* Velocity whereby equal Length is paſs'd ov qual Time: *Greater* Velocity i whereby either a greater Length is over in equal Time, or an equal in leſs Time: *Leſs* Velocity, wh trariwiſe. Further, a Body that an hundred Yards, may be ſaid t with *double* the Velocity of a Bod

s but fifty Yards in the ſame Time ;
with *triple* or thrice the Velocity of
ner Body, that moves but thirty-
Yards and one third in the ſame
; and with *quadruple* or four-
the Velocity of a Body that moves
wenty-five Yards in the ſame Time.
ſo on.

e Momentum of a Body, ariſing (as
een obſerv'd §. 5,) from its Weight
elocity, may be known by multi-
one of its Ingredients into the o-
And conſequently the different Mo-
ms of different Bodies may be
by multiplying the Weight of
Body into its reſpective Velocity.
ſuppoſe the Weight of a Body A to
Pound, and of any other Body B
four Pound ; and the Velocity of
e five-times as much as that of A.
omentum of A will be 10 ; *viz.*
nd, the Weight of A, being mul-
into 1 for its Velocity, (the Velo-
f A to B being ſuppos'd as 1 to 5,)
oduct will be 10 for the Momen-
A. And in like manner, 4 Pound,
ight of B, being multiply'd into 5
Velocity of B, (for the Velocity of
is ſuppos'd as 5 to 1) the Product
20 for the Momentum of B.
it appears that the Momentum of
B is

11. *The Momentum of a Body, how to be known.*

B is double to that of A, being as 20

12. *The* Fundamental Theorem *of Mechanicks.*

Hence is deduced the following rem, which may be ſtiled the Fund tal Theorem of Mechanicks, *viz. Bodies have their Velocities reciproc portional to their Weights, their tum's will be equal.* Let the Wei a great Body A be 500 Pound, the Weight of a little Body B Pound; and let their reſpective V ties be reciprocally proportional, (the Weight of A is to that of B as 1 ſo let the Velocity of B be to that 10 to 1.) It will hence follow, t Momentum of B will be juſt equal of A. Namely, 500 Weight × 1 ty = 500 the Momentum of A; Weight × 10 Velocity = 500 t mentum of B.

13. *The general* End *and* Effect *of Mechanical Engin's.*

Hence it appears, not only t little *may have* its Momentum equa Momentum of any great Body mov'd with a given Velocity; what Method is to be taken, in make it *actually have* ſo; namely, triving that the Velocity of th may be ſo much greater than the ty of the Body to be mov'd, Weight of the Body is greater Weight (or ſomewhat equivale to) of the Power. On which it is, that the forementiond Theo

ly esteem'd the Fundamental Theo-
f Mechanicks; forasmuch as there-
founded the principal Mechanism
ntrivance of Machin's or mechanical
's, made use of to draw or raise
Bodies; the whole End and Ef-
these being to give such a Veloci-
the Power, or (which comes to
ne) so to diminish the Velocity of
eight or heavy Body, as that the
ntum of the Power may be equal,
ewhat exceed, the Momentum of
dy so to be drawn or rais'd.

this is what will be illustrated in
following Chapters, in reference
six simple and therefore primary
's, or (as they are otherwise cal-
echanical Powers; to some one or
f which may be reduc'd all o-
echanical Engin's. And I shall
of them in the Order, wherein
re follow (*) reckon'd up, *viz.*

14. *The simple mechanical Powers, six in Number.*

1 *Vectis*

reas some reckon but five simple Machines or
Powers, comprehending the Balance under
and others that reckon six simple Machines, u-
the Balance as the first, and the Leaver as the
ave judg'd it more proper to agree with the
ckoning six simple or primary Machines, and
far with the former, as to reckon the Leaver
and the Balance as the second simple Machine,
the Leaver is a more simple Machine than the
Balance;

1. *Vectis* or the *Leaver.*
2. *Libra* or the *Balance.*
3. *Axis in Peritroehio* or the *A*
   the Wheel.
4. *Trochlea* or the *Pulley.*
5. *Cochlea* or the *Screw.*
6. *Cuneus* or the *Wedge.*

Balance; and the Balance is no other than a p
Kind of Leaver. In like manner, whereas som
the Wedge as the fifth, and the Screw as the si
Machine, I have rather follow'd them, who have
the ſaid Order, foraſmuch as the Screw partake
the Faculty of the Leaver than the Wedge.

CHAP. II.

Of the Leaver.

1. *A* Leaver, *what.*

THE *Leaver* is ſo call'd fr
principal Uſe thereof, whi
vare, i. e. to lift or raiſe up he
dies. For the more eaſy explai
Mechaniſm thereof, it is here c
as a Line, which will not bend,
be turn'd round about an unm
Point; which is therefore the C

otion, and rests upon somewhat
call'd the *Fulcrum*, i. e. Prop of
aver. Thus *Fig.* 4, AB is the Lea-
the Center of its Motion, and F
ulcrum or Prop.

2. *Theorem the first.*

main Doctrine of the Leaver may
nprised under two Theorems, where-
first is this: *The Space (or Arch)*
d by each Point of a Leaver, and
ently the Velocity of each Point of a
, is, as its Distance from the Ful-
r Prop. For instance, the Laver
Fig. 4.) being mov'd from ACB to
the Point A will describe the Arch
nd the Point B will describe the
Bb; and by reason of the like
ACa and BC*b*, A*a* is to B*b* as AC
that is, the Spaces describ'd by
ints A and B are as their Distances
the Center of Motion, or (which
to the same) from F the Fulcrum.
Powers being apply'd to the Points
B, which draw the Arms CA and
the Leaver AB perpendicularly,
aces describ'd by them will be (not
ches A*a* and B*b*, but (the Perpen-
s AD and EB, these being the
es of the Progress made by the
s according to their respective
ion. But the Triangles ACD
CE being equiangular, therefore

(*) AD

(*) AD is to BE as AC to BC or *b*C, that is, the Lengths gone ov the Powers according to their resp Directions, are as their Distances C the Center of Motion, or (w comes to the same) from F the Ful Now Velocity being (according to 1. §. 10) that Affection of Mo which is measur'd by comparing to Length and Time, and consequently Sort of) greater Velocity being whereby a greater Length is past o equal Time, as in the Case befo hence it follows, that the Velocity of the Point B, or of the Power a thereto, is so much greater than th locity of the Point A or the Powe ply'd thereto; (as the Arch B*b* o Line BE is greater than the Arch the Line AD; and consequently so greater) as BC or *b*C is than AC o that is, the Velocity of each Point Leaver, or of any Power apply'd th is as its Distance from (C the Cen Motion, or from) F the Fulcrum (

(*) By Theorem 9, of the Young Gentlem metry.

(†) For the Sake of such as may not readily a the foregoing Geometrical Demonstration of the t rem, the same is sensibly demonstrated in *Fig.* 4, ally dividing BC, *b*C, B*b*, and BE into six equal P of the same Length or Compass as AC, *a*C, A*a*, a spectively.

e Truth of the first Theorem being 2.
nstrated and apprehended, it will *Theorem the second.*
y to apprehend the Truth of the
l Theorem, which is this: *In a*
the Power, which is to the Weight
dy) to be rais'd, as the Distance of
eight (according to its (||) Line of
ion) from the Fulcrum is to the Di-
of the Power (according to its Line
ction) from the Fulcrum, will coun-
e or hold up the Weight; and there-
ing a little increas'd, will raise the
. For it has been shewn under
n 1, that the Spaces describ'd by
owers apply'd to each End of the
, *i. e.* by) the Power apply'd
End and the Weight apply'd to
her, are proportional to their
istances from the Fulcrum; and
uently that their Velocities, be-
oportional to the said Squares,
o proportional to the said Di-
Wherefore if (*Fig.* 5) CA the
e of the Weight from the Ful-
e to CB the Distance of the Power
e Fulcrum, as the Power P is to
eight W, it will follow, that
e the Velocity of the Weight

---

s *Fig.* 5, B*b* the Line of Direction of the Power
t from C to A*a*, the Line of the Weight's Di-

will

will be to the Velocity of the Powe
the Power is to the Weight. And
ſequently the Momentum of the P
will (by the Fundamental Theorem
1. §. 12) be equal to the Momentu
that Weight; and therefore the P
will be able to ſuſtain the Weight;
if it be but a little increas'd, (eith
the Addition of ſome new Force, o
by increaſing its Diſtance from the
crum) it will be able to raiſe
Weight.

4. *Theorem the ſecond illuſtrated by an Example.*

An Example will illuſtrate the M
Let W the Weight of the Body
rais'd be 120 Pound, and the For
the given Power P be equivalent o
20 Pound, and conſequently the W
be to the Power as 6 to 1. Where
the Power being ſo plac'd at B (R
and W the Weight at A, as that B
be ſix times as long as AC, in ſuch
ſition the Power will have a Force
ent to counterpoiſe, and ſo hold
Weights. For the Velocity of P t
of W being proportional to their
ces from C, will be in this Caſe
1; and conſequently the Momen
P, *viz.* 20 Pound × 6 Velocity =
120 Pound × 1 Velocity, the Mom
of W. Wherefore the Power P (ha
little new Force added to its ow
being remov'd a little farther

C, viz. to *b*, and ſo having its Ve-
a little more increas'd, it will raiſe
Weight. For ſuppoſing *b*C to be
as 7 to 1, the Momentum of P will
Pound × 7 Velocity = 140; which
derably greater than the Momen-
f W at A, this being (as afore
but 120.

ng thus ſhewn the Truth of the
egoing Theorems, and theſe be-
licable to the other following me-
l Powers, as well as to other Ca-
eſpect of the Leaver, it may be of
obſerve here, that the Import of
nd Theorem here laid down will
future be expreſs'd in ſhort thus;
: CW : CP, that is, As much
Force of the Power P is leſs than
ight W, ſo much leſs muſt be
Diſtance of the Weight from the
, than CP the Diſtance of the
rom the Fulcrum.

as this has been ſhewn to hold
reſpect of the Leaver (as it is cal-
ome) of the firſt Kind, that is,
e Fulcrum is between the Power
Weight; ſo does it hold good al-
ſpect of the Leaver of the ſecond
hen the Weight is between the
nd the Fulcrum; and alſo in the
f the third Kind, when the Pow-
twe.n the Weight and the Ful-

5. *Theorem the 2d expreſs'd in ſhort by Letters.*

6. *The Kinds of Leavers.*

crum. For in a Leaver of the
Kind, the Weight W at D (*Fig.*
(as is obvious) juſt the ſame Effe
would have on the other Side of t
crum and at the ſame Diſtance f
viz. at E. And in like manner th
er P at D (*Fig.* 7.) has juſt the ſa
fect, as it would have at E, CE b
qual to CD. Wherefore the Le
the ſecond and third Kind being
effect no other than that of the fi
Mechaniſm of theſe depends on t
Theorem or Rule, as the Mecha
that, *viz.* P : W : CW : CP.

7. *Sciſſers, Pinchers, &c. Kinds of Leavers.*

To the Leaver of the firſt Kind
ferr'd Sciſſers, Pinchers, Snuffe
ſuch-like Inſtruments, they conſi
it were of two ſuch Leavers join
ther, but acting contrary Ways;
be eaſily apprehended by *Fig.* 8
To the Leaver of the ſecond Kin
ferr'd the Oars of a Boat, as a
Cutting-Knives as are fix'd at o
To the Leaver of the third Kind
referr'd a Ladder, lifted up betw
two Ends, in order to rear it a
Wall. Laſtly, an Hammer dr
Nail is referr'd to a fourth Kind o
call'd a *Bended* Leaver. For th
as the Weight at one End A,
Hand of him that draws the N
the Power at the other End B of

nd the Point C (*Fig.* 10) is as the
m. And this fourth Kind of Lea-
likewiſe reducible to the Firſt, the
t or Nail at A having the ſame Ef-
s if it were at *a*, *Fig.* 10.

Weight carried on a Pole or Stick 8.
n two Perſons may alſo be referr'd *How to place a Weight or Body on a Stick or the like, ſo as to make the Burden* proportio-nal *to the different Strengths of them that carry the Stick.*
aver of the ſecond Kind, ſo it may
ood Uſe to obſerve, that agreea-
the foremention'd Theorem P :
W : CP, the Weight may be ſo
as that the Burden thereof ſhall
portion'd to the different Strengths
two Perſons ; Namely, by taking
note the Strongeſt, W the Weak-
C the Weight. For then the Im-
the ſaid Theorem will be this :
h as the Strength of the Strongeſt
reater than the Strength of the
W, ſo much muſt CW the Di-
of the Weight from the Weakeſt,
ter than CP the Diſtance of the
from the Strongeſt. For Exam-
ppoſe the Leaver or Stick PW
1) to be divided into 18 equal
nd the Perſon at P to be twice as
s the other at W, then C the Bo-
carried muſt be plac'd at 6. For
Diſtance CW will contain 12 of
l Parts, into which the Stick is
and the Diſtance CP will con-
6 of them ; and ſo CW = 2CP,

and consequently the weakest Per
W will sustain but half so much
strongest at P. In like manner, if t
dy *c* were placed at 3, then the Per
P would sustain 5 times so much,
other at W; because then the D
portion of the Distances would be
to 3, *c*W containing 15 of the
Parts, whereof *c*P would contain
Lastly, if the Body *c* were place
that is, in the Mid-point of the S
Pole, then each Person P and W
bear equal Burden, because the D
9P = Distance 9W.

9. *The like explain'd as to drawing any Weight.*

And the same Theorem holds g
respect of Drawing (as well as Ca
any Burden. For (*Fig.* 12) let V
present the Pole of any Wag
Coach, and AB the Cross-bar, at
Extremities are Spring-trees DE a
to which are to be fasten'd the Ho
the like. Because A and B are equ
from C, therefore the Horses at A
will draw equal Weight. Whe
one of the Spring-trees be fasten'
then the said Horse would draw a
Part more of the Weight W, than
ther Horse at B; forasmuch as
stance C1 is a fourth Part less th
Distance CB. And so if one Ho
fasten'd to 2, he would draw t
much as the other at B; beca

the Half of CB. And ſo

he Mechaniſm of the Leaver may 10.
o account for the Reaſon, why it *Other Particulars ſolv'd.*
difficult to lift up a Staff (or the
f any Length, at or near one of
s, than nearer the Middle: Name-
uſe the Momentum of any Thing
in'd increaſes proportionably to
nce from its Fulcrum or Prop.
ſame is the Reaſon, why it is
fficult to lift up a long Staff or
, at Arm's Length, or keeping the
rm from Hand to Shoulder ex-
out, than when we bend the Arm
lbow. Namely, becauſe in the
Caſe the Shoulder is the Ful-
Prop, in the latter Caſe the El-
nd therefore in the former the
of the Weight from the Prop, is
han in the latter. And ſo much
Leaver.

CHAP. III.

Of the Balance.

1. *The Beam of a Balance no other than a Sort of Leaver.*

THE Beam AB (*Fig.* 13 and the principal Part of a B and is no other than a Leaver of Kind, which (instead of resting Fulcrum at C the Center of M tion) is held up from above by what fastened to C its Center tion.

2. *The Mechaniſm of the Balance explain'd in general.*

Hence the Mechaniſm of the depends on the ſame Theorem as the Leaver, *viz.* P : W :: C Where, taking P to denote the Weight, W the unknown Weig C the Center of the Balance's Mo Import of the Theorem is this; *as the known Weight is leſs than known Weight*, (i. e. *the Weight the Body weigh'd*) *ſo much will ſtance of the unknown Weight from ter of Motion be leſs than the Di the known Weight from the Center on, where the ſaid two Weights wi poiſe one the other; and conſequ known Weight ſhew the Quantity o known Weight.*

Inſtance, on the (*) Roman Ba-
or Steel-yard, (*Fig.* 13) the un-
Weight or Body to be weigh'd be-
ply'd to A, and the known Weight
g mov'd up and down, nearer and
(from the Point C) upon the Arm
nd being found at the Diſtance 5
nterpoiſe W the unknown Weight,
it follows, (*viz.* becauſe the Di-
CP is 5 times greater than the Di-
CW, or which is the ſame, CA)
e Quantity of the unknown Weight
times as much as the Quantity of
known Weight. And therefore,
ſing P to be one Pound Weight, W
five Pound Weight; or ſuppoſing
e two or three Pound Weight, W
reſpectively ten or fifteen Pound
t.

3. *The Mechaniſm of the Roman Balance, or Steel-yard particularly explained.*

now, becauſe in a common Balance
(†) Pair of Scales, (*Fig.* 14) the
rms CA and CB of the Beam AB
ought to be) always of an equal
; hence the Quantity of the

4. *The Mechaniſm of the* common Balance *or of a* Pair of Scales, *explain'd.*

Sort of Balance is call'd the *Roman* Balance, from
uch us'd at *Rome*. And it is commonly call'd a
, as being a Sort of Yard made of Steel.

the Latins call this *Libra* or Balance by the pe-
me of *Bilanx* (namely *à binis Lancibus*, i. e.
two Scales belonging thereto) whence is made
h Word *Balance*.

known Weight P in one Scale mu
ſelf be equal to the Quantity of t
known Weight W put in the other
And therefore the Quantity of t
known Weight on this Sort of Bal
found out, (not as on the Steel-Ya
moving *one* and the ſame Weight
or farther from C the Center of M
but) by putting in *various* Weig
known Quantity, 'till the known W
counterpoiſes the unknown; and
by ſhews the Quantity of the un
Weight.

5. For it is to be obſerved, th
known Weight then ſhews the
true Quantity of the unknown W
when the one counterpoiſes the
For the uſual Cuſtom of maki
Weight of the Thing ſomewh
than counterpoiſe the known We
which it is weigh'd, (even ſo as
the Scale, wherein the Thing bo
put, to the Ground or the like) h
from Tradeſmen being willing to
ſtomers, by allowing them ſomewh
than ſtrictly true, or (as it is comm
(*) properly enough call'd) *bare*
And conſequently, when we ſ

Bare *Weight is ſtrictly* true *Weight.*

(*) Becauſe hereby is denoted true Weigh
barely as to Weight, without any reſpect to C

ſman, who is more free than ordi-
n what he allows over bare Weight,
e makes *good* Weight, or *better*
t than another, that makes only
Weight; thereby is to be under-
not that the Weight he makes is
or truer than the other's, in reſpect
intrinſick Quantity; but that it is
or better than the other's, in reſpect
Advantage accruing to the Buyer,
hat is allow'd over bare or true
t.

alſo not to be omitted, that as one
is ſaid to counterpoiſe (or to be of
Weight with) another, when they
ſeverally apply'd to the two diffe-
rms of the Balance, the Beam lies
rizontally or equally level; ſo this
d in our common Speech (not on-
e Weight for the Reaſon afore aſ-
but alſo) *ſtanding* Weight, foraſ-
s in this Caſe the Beam ſtands e-
neither Arm aſcending or deſcend-
re than the other; and for the
Reaſon the Bodies in this Caſe
d by the Learned to be *in Æquili-*

6. Standing *Weight, and to be in Æquilibrio, what.*

---

is ſhewn by the Piece of Iron CE, call'd the
being in a Line or directly between TX and

Laſtly,

7. *How to discover a false Balance or Pair of Scales.*

Laſtly, it is well to be obſerv'd, the Truth or Exactneſs of (a com Balance or) a Pair of Scales great pending on its two Arms being of Length, as well as of equal Weig themſelves, hence it follows, that Arm be longer than the other, it w a falſe Balance, or not ſhew the Quantity of the unknown Weight (which is the ſame) the true W of the Thing weigh'd. For inſt ſuppoſe *Fig.* 14, the Arm AC to be ded into ten equal Parts, and the Arm BC to contain but nine ſuch Parts; and conſequently AC to be than BC by a tenth Part. It will by the Fundamental Theorem (C §. 12, or which comes to the ſame this Theorem, P : W : : CW : CP,) a known Weight of nine Poun ply'd to A, will have its Moment qual to a Body of ten Pound Weig ply'd to B; for 9 Pound × 10 V =90 Momentum = 10 Pound × 9 city,) and conſequently will counte one the other in ſuch a falſe-B though their Weights be not eq themſelves. The Way to diſcov falſe Balances, is by changing the and unknown Weight into the co Scales. For upon ſuch a Change, equality of the Weights will m

) appear; whereas in a true Balance
ir of Scales, ſuch a Change of the
s will make no Difference, there be-
o Difference in the Length of the
, and conſequently no Difference in
elocity of the known or unknown
ht, to which ever Arm they be ap-

B 2

Namely, the Body being of 10 Pound Weight in it
d when apply'd to A, its Velocity being in reſpect
elocity of the known Weight of 9 Pound in it ſelf,
o 9, the Momentum of the ſaid Body will be (10
* 10 Velocity =) 100; whereas the Momentum of
wn Weight will be (9 Pound * 9 Velocity =) 81.
nſequently the known Weight will not counterpoiſe
y by a great deal put in the Scale of the ſhorter
o though it did ſo in the Scale of the longer Arm

CHAP. IV.

Of the Axle in the Wheel.

HIS Machine is ſo called, as con- I.
ſiſting principally of a Wheel BGHI, *This Machine, why ſo called.*
a Cylinder or Axle ADEF. A-
which Axle goes the Rope, which
the Weight W, when the Axle is
round by means of the Wheel; as
5.

A 3

2. *It is in effect no other than a Leaver, capable of being perpetually turn'd round its Fulcrum.*

As the Balance is no other than a
ver, particularly apply'd to findin
the Quantity of an unknown Weig
this mechanical Power is no other t
Leaver, apply'd to the ſame Uſe a
Leavers are, (*viz.* to the raiſing t
great Weights) with this particular
vantage added thereunto: Namely, w
as in the Leaver it ſelf the Motion c
continu'd only for ſo ſhort a Spac
(*Fig.* 4 and 5) as is anſwerable t
little Diſtance CA between the Ful
and the Weight; in this Inventio
Inconvenience is remedy'd, foraſm
by continual turning round the W
and ſo (together with it) the Axle,
Weight may be rais'd to any Heigh
Occaſion ſhall require.

3. *Its Mechaniſm explain'd.*

As this Machine is no other th
Leaver AB capable of being perpe
turn'd round, ſo the Fulcrum of
Center of its Motion is C, and the
B is that to which the Power P is
apply'd, and A the other End, to w
is to be apply'd the Weight W to b
ſed. Hence the Mechaniſm of thi
gine depends on the ſame Theorem
that of the Leaver, *viz.* P : W ::
CP; that is with particular refere
this Machine; If as much as the P
is leſs in it ſelf than the Weight, ſo
the Radius of the Axle be leſs tha

R

of the Wheel, (and consequent-
uch the Circumference of the Axle
than the Circumference of the
; and consequently so much the
elocity of the Weight apply'd to
le be less than the Velocity of the
apply'd to the Wheel) by the Fun-
tal Rule the Momentum of the
will be equal to that of the
, and consequently the Power will
rpoise and so hold up the Weight;
erefore the Power being a little in-
will raise the Weight.

Momentum of the Power thus in- 4.
g according to the Disproportion *An Obser-*
n the Circumference of the Wheel, *vation.*
e Circumference of the Axle;
it follows, that the Line of the
Direction must always touch the
ference of the Wheel, so as to
(*)right Angle with the Ray, that
same Distance between the Pow-
he Center of Motion, and conse-
the same Momentum of the Pow-
be every where alike preserv'd;

---

the Weight, as is evident from the Make of this
is rais'd up no more than is answerable to the
the Rope that goes round the Axle in one turn
of the Wheel.

it is, so as to be the Tangent of the Point of the
nce it touches, according to *Chap.* 1. §. 4.

which

which otherwise would not be so, supposing (*Fig.* 16) the Power P so apply'd to any Spoke, *viz.* G, the Line of its Direction be GH, the Distance of the Power is to be CI IC, and so less than BC, and consequently the Momentum of the Power so much lessen'd, as IC is less Whereas if the Line of the Power rection were , then the Mom of the Power at C would be th as at

5. *Machines and Instruments referable thereunto.*

To this third simple Machine ferr'd the Crane *Fig.* 17, and all which consist of Wheels with Te *Fig.* 18, and also the Machine rep ed *Fig.* 19, and the several other ments represented *Fig.* 20, 21, 24 and 25. In each of which A sents the Place of the Weight, B th of the Power, and C the Center on or Fulcrum.

CHAP. V.

Of the Pulley.

HE *Pulley* is a Machine consisting of one or more little Wheels, a- which a Rope being put and (*) at one End, the said Rope makes id Wheels turn round upon their Axles; and thereby at the same oves the Weight or Body to be or rais'd, as *Fig.* 26, 27, 28.

1. *A Pulley, what.*

he Pulley it self be fasten'd up to Place or Mark H, and the Weight apply'd to one End of the Rope, e Power P to the other, (as *Fig.* hen it is evident, that as much as ight W ascends, just so much the P descends; and consequently that oth move with the same Velocity. ore in this Case the Force of the must in it self be equivalent to the of the Body rais'd, in order to

2. *An* upper *Pulley, what, and its Use explain'd.*

(*) Effect of this Machine being thus produc'd by Rope, hence it takes the Name of the *Pulley* a-

make

make its Momentum equal to the
mentum of the Body. And hence
lows, that ſuch a Pulley does not
duce to the increaſing of the Mome
of the Power, or (which comes
ſame) to the decreaſing the Mome
of the Body or Weight to be rais'd
only to the keeping of the Cords
fretting; and ſo to the making
move more eaſily than otherwiſe
would. And, becauſe when there
caſion to apply ſeveral Pulleys tog
ſuch as are of this Sort, (or whi
Ropes go over,) are plac'd upper
hence theſe are diſtinguiſh'd by the
of *Upper* Pulleys.

3. *A lower Pulley, what, and its Uſe explain'd.*

But if (not the Pulley it ſelf, bu
End of the Rope be faſten'd to an
H or the like, and the other End
led by the Power P, and the We
hangs on to the Pulley, (as *Fi*
then it is evident, that in order t
the Weight W one Foot, each P
and *Bb* of the Rope muſt be ſhorte
Foot, (reckoning downwards fr
Hook H and its oppoſite Point
conſequently that in the ſame T
Power P muſt move two Foot.
fore in this Caſe the Velocity of th
er will be double to that of the W
and conſequently if the Power
duple to the Weight, that is, as

mentum will be equal to that of
eight, and ſo will ſuſtain the
t. And therefore if the Power be
increaſed, it will be able to raiſe
ight. And the ſame holds when
re two Wheels or Pulleys, one
and one Lower, as *Fig.* 28.

s *Fig.* 29) there be three Wheels, 4.
the Rope goes from Wheel to *Further of the ſame.*
three times, *viz.* from A to B, and
to D, and from E to F; and
ently in order to raiſe the
W one Foot, each of the three
AB, CD, and EF of the Rope
e ſhorten'd one Foot, which
e done without the Power P mo-
orward three Foot in the ſame
in this Caſe the Velocity of the
will be triple or thrice as much,
Velocity of the Weight. And
e, though the Power be to the
but as 1 to 3, yet its Momentum
equal to the Momentum of the
and ſo will ſuſtain it; and if the
e but a little increaſed, will raiſe
ght.

e manner it is evident from *Fig.* 5.
if there be four Wheels, and ſo *The ſame yet further exemplify'd.*
e goes four times from Wheel to
then, while the Weight W is
e Foot, the Power P will move
four Foot; and conſequently

(by the fundamental Theorem) a
er P, which is to the Weight W b
to 4, will have its Momentum e
the Momentum of the Weight
therefore being a little increas'd, w
it.

6. *The general Theorem relating to the Pulley.* Hence the Mechanism of the Pu
pends on this universal Theorem
If a Power is to the Weight, as
Number of the Ropes (*i. e.* of th
of the Rope, *viz.* AH and Bh,
and 28; and AB, CD, EF, *Fig.*
AB, CD, EF, GH, *Fig.* 30,) ap
the lower Pulleys, then the Power
able to hold up the Weight; an
fore being a little increas'd, will r
Weight.

7. *Examples.* For Instance, Suppose the We
be rais'd 360 Pound, and the Pow
self to be equivalent only to 180
(*i. e.* the Power to be to the Wei
to 2.) therefore one lower Pul
ther with, as *Fig.* or without
per Pulley) will render the Mo
of the Power equal to that
Weight, namely, by rend'ring th
city of the Power double to tha
Weight. For 180 p. * 2 Velocity
Momentum = 360 p. * 1 Velocity
Power be to the Weight as 1 to
quivalent only to 120 Pound, th
must be three Wheels, that so th

the Power may be triple to that Weight; Namely, 120 p. × 3 Ve- = 360 Momentum = 360 p. × 1, y. And ſo on.

CHAP. VI.

Of the Screw.

HE *Screw*, which makes here the fifth ſimple mechanical Power, is Cylinder, having ſeveral Riſings llows alternately winding round ſpiral manner. It conſiſts proper- wo Parts, one call'd the *outward* the other the *inward*; namely, the former receives the latter in- Whence the inward Screw is o- e call'd the *Male*, and the out- e *Female* Screw. The Male or is denoted *Fig.* 31, by MS, the or outward by TS.

1. *The* Screw, *what.*

Machine is chiefly us'd to ſqueeze Things, and by that Means, in aſes, to break Things. It is alſo nes uſed to raiſe up any Weight, ruſt it forward.

2. *Its Uſes.*

3. *This in some sort a* compound *Machine.*

For the more eaſie Uſe of thi
chine, (*i. e.* for the more eaſie m
the Male Screw through the Fema
the Female along the Male) it is u
apply thereto an Handle or round
AB; and ſo to the Power of the
to adjoin alſo the Power of the Ax
the Wheel: The Handle or Stick
its turning round, ſupplying the P
a Wheel, and the Cylinder of the
being as the Axle to the ſaid Whe
that this Machine is in ſome ſort
pound Machine.

4. *The fundamental* Theorem *particularly apply'd to this Machine.*

Now the Effect of this Machi
pends upon the ſame fundamental
rem, as the other; which funda
Theorem particularly apply'd to t
brick or Make of this Machine,
thus: *If, as the Compaſs deſcrib'
Power in one Turn of the Screw,
Interval or Diſtance between any tw
diate ſpiral Windings (meaſur'd a
to the Length of the Screw) ſo is the
or Reſiſtance to the Power; then th
and the Reſiſtance will be eqnivalen
the other, and conſequently the P
ing a little increaſed will move t
ſtance.*

5. *The ſame further illuſtrated or prov'd.*

For it is evident from the bar
ction of this Machine, that in o
of the Screw, the Weight P (*Fi
ſo much lifted up, or the Reſiſ

and 32) is ſo much remov'd, or
ing to be preſs'd is ſqueez'd ſo
loſer together, as is EG the Di-
between two immediate Spirals;
the ſame Time the Power (ap-
A or B) to be mov'd ſo much,
Compaſs AB, deſcrib'd by the
wer in one Turn of the Screw.
re the Velocity of the Weight (or
r anſwers thereto) will be to the
of the Power, as is the ſaid Di-
etween the Spirals to the Com-
rib'd by the Power, in one Re-
or Turning round of the Screw.
efore by the fundamental Theo-
s the Power is to the Weight, ſo
ance between the Spirals to the
deſcrib'd by the Power, the
ill have its Moment equal to
ment of the Weight or Re-
And therefore the Power, be-
le increas'd, will overcome the
e.

CHAP. VII.

Of the Wedge.

1. *A Definition of a* Wedge, *and its several Parts.*

THE *Wedge* is commonly m
Iron or ſome harder Sort o
ter, in Shape of a Priſm not ver
whoſe oppoſite Baſes are Iſoſcela
gles. The Height of either of th
angles is eſteem'd the Height
Wedge, and the Baſis of either
is eſteem'd the Thickneſs of the
and the right Line which joins
the Vertexes of both the Triang
led the Edge of the Wedge; and
rallelogram which joins together
of the two Triangles, is call'd
of the Wedge.

2. *Its* Effect *accounted for by the fundamental Theorem.*

The Effect of this Machine
likewiſe upon the ſame fun
Theorem as the former; whi
particularly apply'd hereto w
thus: *If the Power directly appl
Back of the Wedge be to the Reſiſt
overcome by the Wedge, as the Th
the Wedge is to its Height; then
Power will be equivalent to the R
and therefore being increas'd will ov

t Tenaciouſneſs or Firmneſs, where- 2.
Parts of the Wood (or the like) *The ſame further illuſtrated or prov'd.*
one to the other, is the Reſiſtance
overcome by the Wedge. Now it
lent, that while the Wedge is drove
he Wood, the Way or Length
it has gone according to its own
nfity, is (*Fig.* 34) BA; and in
anner DC is the Way or Length
in the ſame Time by the Impedi-
i. e. the Parts C and D of the
are ſo far divided aſunder. And
ingly as the Wedge is drove down
and further along its Height, ſo
rts C and D of the Wood are divi-
ore and more along the Thickneſs
Wedge; and in the whole Pro-
roportionably, as is evident from
ature of a Triangle. Whence it
to paſs, that if as the Thickneſs
Wedge (*i. e.* the Way of the Im-
nt, and conſequently its Veloci-
o the Height of the Wedge (*i. e.*
y, and conſequently the Velocity
Power,) ſo the Power to the
ment or Reſiſtance, then the Mo-
n of the Power and the Impe-
will be equal the one to the o-
and conſequently the Power be-
reas'd will overcome the Reſiſt-

4. *Other Instruments reducible to the Wedge.*

To the Wedge may be refer Edge-Tools, and Tools that sharp Point, in order to cut, slit, chop, pierce, bore, or the lik Knives, Hatchets, Swords, B &c. And thus I have gone throu six simple Machines, or primary nical Powers, so far forth as see quisite to the Design of this tise.

5. *Mechanicks what, in the more restrain'd Sense of the Word.*

It remains only to observe here as the Word *Mechanicks* in its Acceptation comprehends the wh ctrine or Science of Local Motion a restrain'd Sense it is taken parti to denote the Doctrine of Mot apply'd to Machines or mechani gines, the more simple of which here describ'd.

C

CHAP. VIII.

Of Staticks.

HE Word *Staticks* denotes in the Greek Language the Science or Do- of Weights, and conſequently in geſt Acceptation, is of the ſame Ex- ith Mechanicks in the largeſt Senſe f, both comprehending the whole ine or Science of Motion with refe- to the Weights of Bodies; and ſo ſynonymous Words, or Words of me Import. But as the Word *Me-* *ks* in a reſtrain'd Senſe is taken to more particularly the Doctrine of n and Weights in reference to Me- al Engines; ſo the Word *Staticks* reſtrain'd Senſe is taken to denote eſpecially that Part of the Doctrine tion, which ariſes from the Weight ural Bodies conſider'd in themſelves par'd together, and which conſe- y has not at leaſt ſo immediate a to mechanical Engines. In this r therefore I ſhall take notice of articulars as relate to Staticks in its d Senſe, and are agreeable to the of this Treatiſe.

1. Staticks, *what.*

It

2. *The Motion of heavy Bodies downwards is* uniformly accelerated.

It is then obſervable in the firſt
that the natural Motion of heavy
(*i. e.* That Motion of them down
which ariſes from the natural Princ
their Gravity or Weight) is fou
Experience to be a *Motion uniform
lerated*, i. e. a Motion which acqu
equal Times equal Degrees of Ve
For when an heavy Body deſcend
ſame Gravity, which gave it a
downwards at firſt, will all along
nue to act alike upon it, and ſo
it in the ſecond Inſtant of Time
on equal to what it gave it at the fi
again in the third Inſtant or Minute
it likewiſe a Motion equal to that
So that at the third Minute the Sum
whole Motion will be the Triple
firſt; and ſo at the fourth Minute t
of the whole Motion will be Qu
to what it was at the firſt. And
Degrees of Motion, or Velocity
Body moved, will always incre
does the Time wherein it moves.

3. *The* Spaces *thro' which Bodies fall, are as the Squares of the Time wherein they fall.*

Hence it comes to paſs, that t
ces, through which Bodies deſ
their Fall, are as the Squares of th
they take up in Falling, always a
ing from the Beginning of the Fa
Inſtance, if a Body be ten Minut
ling, and the firſt Minute it falls
the ſecond Minute it will have fa

and the third Minute nine Miles; and
: According to which Proportion at
enth Minute, the Body will have
an hundred Miles. Whence it ap-
that if the Times be taken in the
netical Progreſſion of natural Num-
viz. 1, 2, 3, 4, 5, *&c.* the Spa-
eſcrib'd in thoſe Times, reckoning
the Beginning of the Motion, will
1, 4, 9, 16, 25, *&c.* This is wont
demonſtrated or at leaſt illuſtrated,
nſidering (*Fig.* 35.) AB to repre-
he firſt equal Part of Time, and BC
reſent the firſt Degree of Velocity
'd in the firſt Part of Time: And
ſe BD (=AB) to repreſent the ſe-
equal Part of Time, and FE (=BC
) the ſecond Degree of Velocity, ac-
in the ſecond Part of Time, and e-
n it ſelf to that Degree of Velocity
'd in the firſt Part of Time: and con-
tly AD to repreſent two equal Parts
ne, and DE to repreſent two equal
es of Velocity. Wherefore the Tri-
ABC will repreſent the Space gone
n AB, the firſt equal Part of Time,
he Velocity BC; and the Triangle
will repreſent the Space gone thro'
two ſuch equal Parts of Time, with
elocity DE (the Double of BC.)
e ſaid ſimilar Triangles ABC and
are (by *Theor.* 10 of *Young Gen-*

tleman's

tleman's Geometry) as the Squares their homologous Sides AB and A that is, the Spaces gone through in Times AB and AD are as the Square the ſaid Times. Thus (*Fig.* 35.) A is actually divided into four Parts, equal to ABC.

4. *The Spaces deſcrib'd by the Fall of heavy Bodies, conſider'd in each equal Part of Time diſtinctly by it ſelf.*

Hence it is obvious, that the Sp deſcrib'd or gone through by a Bod its Fall, in each (equal Part or) Mi of Time, are according to the Seri the natural Numbers, 1, 3, 5, 7, 9, theſe being the Differences of the Sq 1, 4, 9, 16, *&c.* Thus if a Body one Mile in the firſt Minute, and Miles in the ſecond Minute, it will t fore, during the ſecond Minute, der'd diſtinctly from the firſt, fall Miles. And in like manner, if a falls at the End of two Minutes Miles, and at the End of three Mi nine Miles, it will, during the thir nute conſider'd by it ſelf, fall five and ſo five Times as far as it did th Minute.

5. *How to find, what Space a Body will go thro' in a given Time.*

Hence it is obvious, that the gone through in a determinate Ti any Body being known, thereby be found out, what Space it w through in a given Time; *viz.* b ing a fourth Proportional. For ple, ſuppoſe a Body to fall twen

in one Minute, and it be requir'd
How far the ſame Body will fall
e ſame Medium in Three Minutes;
nſwer will be, 216 Foot. For as 1
i. e. as the Square of one Minute
he Square of three Minutes) ſo is
he Space gone through in the firſt
e) to 216, which therefore is the
the Body will go through in three
es.

ke Manner, the Time being known, in an heavy Body deſcends through minate Space, it is obvious to find, long Time it will deſcend thro' an- given Space, *viz.* by finding a fourth tional. For Example, ſuppoſe a as taken up one Minute in falling four Foot, and it be requir'd to hat Time it will take up in falling oot in the ſame Medium; the An- ill be three Minutes. For as 24 (*i. e.* as the firſt Space given is to ond Space given) ſo is 1 the of the given Time to 9 the Square Time requir'd, which therefore is Minutes.

6. *To find, what* Time *a Body will take up in deſcending thro' a given Space.*

Body in falling acquires equal of Velocity in equal Times, ſo contrary, in riſing it loſes equal of Velocity in equal Times. For vity of the Body thrown up con- drawing it downwards, its Moti-

7. *Of the* Aſcent *of* Bodies.

on

on upwards will continually decre
proportion to the Force downwar
duc'd in equal Times by the Gra
the Body. But it has been ſhewn
(§. 2.) that the Motion downward
niformly accelerated; and therefo
Motion upwards muſt be *unifor*
tarded. Whence it follows, that
Body goes through the ſame Space
qual Times, in riſing as it does in
but in an inverted Order. Nam
the Body be five Seconds in riſing
Height of twenty-five Foot, and th
aſcended through the firſt Sec
Time be nine Foot, the ſaid Body
the ſecond Second of Time, will
through a Space of ſeven Foot;
the third Second a Space of fi
the fourth a Space of three, an
fifth and laſt a Space of one Fo
it will be as it were in Æquilibr
the Impetus upwards being n
equal to the Impetus downwar
Body will neither riſe nor fall f
Time. After which it will begi
in the like inverſe Proportion,
that the firſt Second it will fall
the ſecond three Foot, the third
fourth ſeven, and the fifth and
Foot, thus taking up five Seconds
to deſcend twenty five Foot, as i
the ſame Time to aſcend twenty-

the ſame Principles it comes to
that the Force which a Body ac-
in falling, will make it riſe up a-
o the ſame Height which it fell
For Inſtance, ſuppoſe (*Fig.* 36)
) Pendulum, *i. e.* the Body or
faſten'd to one End A of the
AB, whoſe other End is faſten'd
ix'd Point C, as the Center of its
n: Suppoſe, I ſay, the Body to
m D to A, it will riſe again to E,
s of an equal Height with D. For
DA into any Number of equal
ſuppoſe three, markt *a*, *b*, *c*, and
as many markt *d*, *e*, *f*, E. By
egoing Principles it is obvious,
atever Force or Degree of Velo-
Body acquires by its Gravity in
ing from D to *a*, it will acquire
much in deſcending from *a* to *b*,
ce as much in deſcending from *b*
ith which triple Force it will be-
ſcend from *c* towards *d*. But in
g from *c* to *d* the Body will by
ity drawing downwards loſe one
ree acquir'd Degrees of Velocity
; and in aſcending from *d* to *f*
ſe two of the ſaid acquir'd De-

8. *Why Bodies riſe up again to the ſame Height, from which they fell.*

ſo call'd from the Weight's *hanging* on the
ire, to which it is faſten'd; the Latin Word
oting any Thing that thus hangs.

grees

grees, the Force of Gravity dow
at *f* being double to its Force at
ſo deſtroying two of the acquir'd
of Velocity or Force, wherewith
dy at *c* began to aſcend. And like
E, the Force of Gravity downwar
be triple to that at *d*, and ſo deſ
the three acquir'd Degrees of
wherewith the Body began at *c*
cend: And therefore the Body
cend no higher, but begin to fall
wards again towards *c* or A.

9. *The* Reſiſtance of the Air *not conſider'd in the foremention'd Caſes.*

But now it is to be obſerv'd,
this and the foregoing Caſes, the
ance of the Air, wherein the Bo
conceiv'd to move, is not con
which is the Cauſe why the fo
Theorems will not exactly agree w
periments; and particularly as
Pendulum, which, by means of
ſiſtance of the Air, does not re
actly to the ſame Height from wh
fell, and conſequently have not
tual Vibration or Motion forwa
backwards, but ſtand ſtill in ſome

10. *Pendulums aſcend and deſcend thro' equal Spaces in equal Times,* &c.

Abſtracting from or ſetting a
Reſiſtance of the Air, it follo
the foremention'd Principles, t
Pendulum does aſcend and deſce
the equal Spaces DA and AE i
Times. To which I ſhall here
worth remarking, that the Lo

ndulums are as the Squares of the
s, wherein they perform their Vibra-

Gravity is that natural Principle
n causes Bodies to descend, so no
will descend, but when its Center
avity can descend. Thus the in-
Body ABDE, (*Fig.* 37.) which is
the Horizontal Plane HP, cannot
ward the Part D, to which it in-
because by such a Change of Situ-
its Center of Gravity C would rise;
y be known by describing from A
Center) the Arch CF, which is
me, as C the Center of Gravity
make about the Point A, if the
ABDE could fall; for it is evident
ne Part of that Arch rises above
int C, and consequently the Cen-
Gravity in such a Case must for
Time ascend of its own Accord,
is contrary to the Nature of heavy

II. *How the Fall of Bodies depends on their Center of Gravity.*

on the contrary, the inclin'd Body
(*Fig.* 38) must of necessity fall
s the Part D, which it inclines to,
e its Center of Gravity C may de-
n such a Situation; as will appear
wing as afore) from A through C
ch CF, which is the same that will
rib'd by C the Center of Gravity
A, when the Body ABED falls:)

II. *The same further illustrated.*

For all the Points of the ſaid Atch
below the Point C.

13. *Of Bodies ſtanding firmly.*

Hence it appears, that to have a
keep firm upon any Thing, that ſup
it, and is not it ſelf inclin'd, the
Line CF drawn from the Center of
vity in the ſaid Body toward the C
of the Earth, and wherein the ſaid
endeavours to deſcend (*i. e.* in ſho
Line of Direction) muſt neceſſaril
in ſome Part of the Baſe or Foot
the ſaid Body; as *Fig.* 37. Othe
if the Line of Direction CG falls wi
the Baſis, the Body will neceſſaril
down, as in *Fig.* 38.

14. *Further of the ſame.*

Whence it follows, that by how
leſs the Baſis of a Body is, even th
it be not inclin'd, ſo much more
will it be mov'd out of its Place; b
the leſſer Change of Poſition is capa
make the Line of Direction be with
Baſis or Foot. Which is the Reaſo
a Bowl rolls eaſily upon a Plane,
that a Needle will not ſtand up
Point. It is obvious, that agreea
what has been ſaid, it follows alſo
the wider the Foot of a Body is
more firmly will it ſtand, becau
much the greater Change is neceſſ
cauſe the Line of Direction to fal
out the Foot of the ſaid Body.

ppears alſo from what has been 15.
that if the Plane OP, which ſuſtains *Of the Sliding and Falling of Bodies.*
Body ABDE (*Fig.* 39) be inclin'd,
Body will ſlide, when its Line of
ction CG falls upon any Part of its
AD. And that a Body *abde* will fall,
its Line of Direction *eg* falls with-
he ſaid Baſe *ed*.

Hence it follows, that a Bowl ſet up- 16.
n inclining Plane, as the Roof of an *Further of the ſame.*
e, will roll off it, becauſe its Line
irection not being perpendicular to
lane, (but to the Horizon,) can-
ſs through its Foot where it touches
me, it being almoſt an indiviſible

is remarkable, that we naturally 17.
e this Law of Staticks, we are *Why a Man bends, when he riſes from a Seat.*
peaking of, to keep our ſelves from
Thus, when we would riſe from
, we bend our Body, by thruſt-
orward our Head and Back HB,
tring backwards our Feet and Legs
Fig. 40) ſo as the Line of Directi-
of our Center of Gravity may paſs
h our Footing F or Baſis. And it
ious, that if we bend not enough,
backwards or towards B; but if
d too much, we fall forwards or
s F or K.

18. *Why a Man leans forward when he goes up Stairs.*

In like Manner, when we go up S
as we lift one Foot up, ſo we ben
Body forward, that we may (both
eaſier, and alſo) may preſerve our
from falling beckwards, by havin
Line of Direction of our Cente
Gravity too far from our Footing,
in this Caſe is chiefly the Foot
up upon the Stair, when we are g
up.

19. *Other common Poſtures accounted for.*

On the ſame Account it is, that
a Man carries a Burthen on his
he leans forward; but if he car
Burthen before him, he leans bac
If he carries it with his right Han
leans to his Left; if with his left
he leans to his Right. Namely
hereby he (may not only more
carry the Burden, but alſo) may
ſerve himſelf from falling that
which the Burden he carries,
him.

20. *The like Rules obſerv'd by irrational Creatures.*

And that we learn this, not
Reaſon, but rather from natural l
is evident, not only from Mens d
without knowing the Reaſon of
alſo from irrational Creatures doi
like. For a Gooſe going through
or other Door, be it never ſo
does thruſt forth, and ſo downwa
Head, not becauſe he is afraid of

ainſt what is ſo much above it, but)
onformity to this Law of Staticks.
ely, at a Barn Door there is uſually
reſhold or the like, to be gone o-
Wherefore the Gooſe having put
Foot on the Threſhold, thruſts
ard his Head, that his Center of
ity may not be too far behind his
and ſo pull him backwards; but
by thruſting his Head and Neck ſo
is requiſite beyond the Threſhold,
Direction of his Center of Gravity
e brought ſo near to his Footing,
t he may go over the Threſhold
eaſily, and without falling either
ard or forward. So that it appears,
ooſe is not to be laugh'd at for ſo
, but rather they, who neither
the Reaſon of his ſo doing, nor at
hat he does no other than they do
elves in like Caſes. And this is ſuf-
to our Deſign, to have taken no-
f concerning Staticks, ſo call'd in a
reſtrain'd or proper Senſe.

CHAP. IX.

Of Hydroſtaticks.

1. Hydroſtaticks, *what.*

THO' the Word *Statick*, both in its own literal Import, and al its largeſt Acceptation, denotes the ence of the Weights of Bodies in ral, whether fluid (or liquid) or no id, and whether weigh'd in the A Water; yet it is uſual to reſtrain Sta in a more appropriated Senſe to deno Science of the Weights only of ſolid not fluid) Bodies, and that too weigh'd in the Air; and to give th culiar Name of (*) *Hydroſtaticks* to Part of Staticks largely taken, treats of the Weight of fluid or l Bodies, eſpecially of Water, and o lid Bodies put into (†) liquid Bodie ſpecially into Water.

(*) It is a Greek Word, compounded of *Science of Weights*, and ὕδωρ, Water.

(†) There is this Difference between a *fluid* and Body. A Fluid is a Body, whoſe Parts are eaſily ted, and being ſeparated join together immedia

he primary Property of heavy Flu- 2.
are theſe; that if an heavy Fluid *The primary Theorems or Properties of Fluids.*
D (*Fig.* 41) be, either not preſs'd
, or equally preſs'd from above, its
r Surface AB will lie horizontal or
And if the ſaid Fluid be diſturb-
it will by its own Gravity return to
ame Level of its upper Surface. For
ower Particles of the Fluid will not,
eaſon of their Gravity, aſcend up-
of themſelves, to raiſe any Part of
aid Surface above the other; nor
he upper Particles of the Fluid de-
, ſince there is no Room for them,
lower Part of the Veſſel, it being
up by other Particles of equal
ut. And therefore if there be no
re at all from above, or an equal
re, then there is nothing to diſturb
er the Level of the upper Surface.
it be diſturb'd, the fluid will re-
o its Level, becauſe the higher Part
(*Fig.* 41) being heavier, will partly
ſs the Particles under it, and will
run down into the lower Part;

---

ame, Water, Oil, and other Liquors. A Liquid is whoſe Parts are alſo eaſily ſeparated, and again gether immediately, and withal will continually ſpread themſelves, 'till their upper Surface becomes s Water, Oil, but not Flame. Hence it appears, Liquids are Fluids, but all Fluids are not Liquids.

and that ſo long, as any one Part
Surface is higher than the others.

3. *Further of the ſame.* If the Fluid (*Fig.* 42) be from
unequally preſs'd, it is obvious, th
Part thereof AE, which is moſt p
will deſcend, the Particles ſo p
thruſting out of their Places the
Particles, that are either not pre
all, or leſs preſs'd; which therefo
aſcend to HI in proportion to th
ſcent FG of the Particles preſs'
Namely, as the Part preſs'd AE is
to the Part not or leſs preſs'd E
the Aſcent EH or BI will be aqual
Deſcent AF or EG.

What has been ſaid of the upp
face AB, holds good of any other
lel Surface within the Fluid, as E
43. Namely, if it be equally preſ
the upper Part of the Fluid, and
by any Thing elſe ſwimming eit
the Top or within the Fluid, it w
tain its horizontal Situation or
But if it be any where preſs'd mor
in other Parts of it, there it will
the Parts of it which are leſs pre
cending as that deſcends. And
what has been hitherto ſaid, are i
the following Propoſitions, as ſo
Corollaries.

n heavy Body EFHB (*Fig.* 44.) upon a Fluid AB, be of an equal t with the Air that takes up an e- pace AHEG, the Surface AB of the will retain its Level, as being eve- re equally preſs'd. 4. *Corollary the firſt.*

e Body EFIK (*Fig.* 45) be lighter he Air which takes up the like then the ſaid Body, and the Par- f the Fluid under it, will aſcend, Aggregate (of the Body IKEF and Parts EFHB of the Fluid ſo aſ- *viz.*) IHBK be of equal Weight e Air, that takes up the like Space For then the Surface AB will be preſs'd every where, both in AH 5. *Corollary the ſecond.*

e Body EFAB, (*Fig.* 46,) be hea- n the Air of a like Bulk, then y will ſwim in the Fluid, de- g ſo far into it, 'till the Aggre- f the Air GEIK and the Fluid *viz.*) AHEG, which take up to- ike Space with the ſaid Body, be l Weight with the Body. For face AB will become equally both in AH and HB. 6. *Corollary the third.*

e Body EGDF (*Fig.* 47) be hea- t only than the Air, but alſo) e Water (or other Fluid) that a like Space HCGE, then the ill ſink down quite to the Bottom 7. *Corollary the fourth.*

CD,

CD, there being nothing to coun
or bear it up, till it comes to t
tom. For the Body being suppos
put on the upper Surface AB, a
upon the Part IB thereof, it bein
er than a like Bulk of Air, the
the Fluid IBDE will be more
thereby, than the other Part of
id AIEC by the Air, and there
Particles in IBDE will descend.
like Manner when the Body is
GF, it being heavier also tha
Bulk AHGI of Water, it will
the Parts of the fluid Water G
than are those that are under G
therefore will cause the Parts u
still to give way, till it comes to
tom CD.

8. *Corollary the fifth.* Upon the same Principles, if
GHIK (*Fig.* 48) be immerg'd
Fluid, and be (of equal Weight
much EFGH of the Fluid as tak
like Space, *i. e.* in short, be) of
specifick Gravity with the Flu
the said Body will stay where it
in the Fluid, without either risin
ing. For the Surface EI being
press'd all along, the said Bod
rise. And the Surface FK being
equally press'd, neither can the
sink.

f the ſaid Body be ſpecifically than the Fluid, and conſequently k GHIK be lighter than the Bulk it evidently follows, that the FK will be more preſs'd in FH HK, and conſequently the Body will be thruſt up and riſe ſtill high-higher for the ſame Reaſon, till it o ſuch a Situation, as that it is of Gravity with ſo much Air (which Caſe ſuppos'd ſpecifically lighter e Body) and ſo much of the Flu-ake up equal Space with it; (as ſented *Fig.* 46) according to §. laſtly, if the Body be of the ſame Gravity with the Air, then it juſt to the very Top of the Flu-is repreſented *Fig.* 44. according

what has been ſaid, it is eaſie to for ſeveral Occurrences in Flu-particularly in Water; of which e remarkable or common ſhall be otice of. Hence then it is eaſie why we don't feel the Weight Bucket and the Water in it, we draw Water, 'till it begins out of the Water. Namely, ſuppoſing the Bucket with ater in it to be repreſented, by GHIK, it being of the ſame

9. *Why a Bucket of Water has* little *or* no Weight *under Water.*

† Gra-

(†) Gravity with the like Bulk o
EFGH, the Surface FK will be
preſs'd, both in FH, and HK; an
fore the Bucket of Water can't
but is held up by the Water un
and conſequently the Weight of
more felt, than is a Weight w
counterpois'd by an equal Weig
a Balance. And the ſame holds
while the Bucket is drawing up
Surface AB. Where it, and th
in it, coming into the Air ſo mu
ſically lighter than the Water o
&c. of the Bucket, the Air can
it, and ſo the Weight of it muſt
tain'd then by him that draws
Bucket.

10. *Why the Depths of ſome Seas cannot be* fathom'd.

On the ſame Principles the
why one cannot fathom the Dept
Sea in ſome Places, is this: Bec
the Plummet or Lead made uſe o
cifically heavier than Water, yet
to which the Plummet is faſten'd
this Caſe) ſpecifically lighter t

(†) The Water in the Bucket is certainly o
ſpecifick Gravity with the Reſt of the Water.
Bucket, if all of Wood, is much lighter, An
wiſe, the ſpecifick Lightneſs of the Wood o
the ſpecifick Gravity of the Iron about the Buc
to an *Æquilibrium* with the Water.

Wherefore when the Depth is
that ſuch a Quantity of Line is
as that the Weight of the Line
together is no greater than the
of a like Bulk of Water, then the
Plummet will ſink no deeper;
ſequently will not ſhew the Depth
ea in that Place.

rding to the ſame Principles we
ly tell, why one Liquor will ſwim
other, or remain at Top without
if it be pour'd on gently; as Oil
of Water; and Water in re-
Quickſilver. Namely, becauſe
cifically lighter than Water, and
han Quickſilver.

likewiſe it ceaſes to be a Won-
t a Ship, which has ſail'd in the
s, has ſunk and been loſt in the
of a River of freſh Water; name-
auſe Sea-water is much heavier
ſh Water. And therefore though
with its Burden was lighter than
ulk of Salt-water, and ſo was
by the Salt-water in the Sea, yet
eavier than a like Bulk of freſh
and therefore ſunk in the River.
vious, that for the ſame Rea-
Ship or Boat will draw more
(that is, though it does not ſink
wn to the Bottom, yet will ſink
in freſh Water than in Salt-wa-

11. *Why ſome Liquors ſwim upon others.*

12. *Why Ships, that ſail in the Ocean, ſink in freſh Water.*

More-

13. *Why Wood, that will ſwim for ſome time, ſinks afterwards.*

Moreover the Reaſon, why Wood, that has been ſwimming time upon Water, will ſink at this, *viz.* becauſe ſuch a Piece of may be of the ſame or greater Gravity than Water, ſetting a Pores of the Wood; but by ra Pores, the Air which fills the ſ and the Wood together, make Whole lighter than an equal Bulk ter, and therefore the Wood wi at firſt. But after ſome Time th being got into the Pores, and ſ excluded, then the Water in t and the Wood make up one Wh vier than the Water of a like B ſo muſt ſink according to the P of Hydroſtaticks aforemention'd.

14. *Why hollow Veſſels of Braſs or Iron will ſwim, when Lumps of the ſame Metal will ſink.*

From which Principles it follo that although a Lump of Braſs or other Metal will ſink in Wate ing ſpecifically heavier; yet a the like of Braſs or Iron (*&c.*) ſink but ſwim; becauſe the Ke the Air contain'd in its Hollow, one Whole, which is much ligh an equal Bulk of Water.

15. *A Body will weigh leſs in Water, than in Air.*

It follows alſo, that a Body w leſs in Water than in Air, by much as is the Weight of an equ of Water. Whence it comes to p two Pieces of Metal of different

s, ſuppoſe one of Gold and the
f Silver, which ſhall be of equal
in the Air, will not be ſo in the
that Metal whoſe ſpecifick Gra-
greateſt, (*viz.* the Gold) lo-
of its Weight in the Water
other (*viz.* the Silver,) be-
takes up the Room of a leſs
Water.

Fig. 49.) a Fluid, particularly
or) Quickſilver contain'd in
CD preſſes on the Part C of the
B, more than the Air preſſes on
Parts, C will deſcend, and the
t run down out of the Tube,
is left CI in the Tube preſſes C,
uch as the other Parts are preſs'd
ir. And then no more will run
e Tube, becauſe of the equal
n all Parts of the Surface AB.
is to be noted, that the Weight
uickſilver CI is equivalent to the
f a Column of Air, extended
Surface AB up to the Top of
ſphere, and of the ſame Baſis
ter of the Tube CD.

16. *Why a Fluid* in a Tube *will ſtand up higher than the ſame Fluid* without the Tube.

is obſervable (*Fig.* 50) that
kſilver will riſe up juſt to the
ght in the oblique or ſloping
, as it does in the upright Tube
ough the Quick-ſilver EF in the
be more, and conſequently
heavier,

17. *Why a Fluid will riſe to the ſame Height in a* ſloping *Tube, as in an* upright

heavier, than the Quickſilver C
Tube CD. The Reaſon whereo
that as much as the Quickſilve
more than the Quickſilver CI,
greater is the Baſis E than the
(this being equal to the Baſis of
linder EK, wherein the Quickſi
will be found to be juſt ſo mu
Quickſilver EF, by reaſon of
Tubes EK and EG being Cyli
Equal Height upon the ſame Ba
therefore Equal one to the other.
it is obvious that ſuppoſing the
be Twice as big as the Baſis
conſequently the Quickſilver E
Tube EK (or the Quickſilver
Tube EG) to be Twice as muc
Quickſilver CI in the Tube CD
dently follows that as the Q
CI do's equiponderate to a C
Air of the ſame Baſis C, ſo th
ſilver EH (or EG) which is t
ble Quantity of CI, do's but
derate to a Double Quantity
i. e. to a Column of Air, wh
E is the Double of the Baſis C.

18. *Further of the ſame.* Since according to the Rule
ticks, Bodies gravitate or we
vier in a Perpendicular, tha
Oblique or ſloping Situation,
be ask'd, why the Quickſilver
tating or preſſing upon the Baſi

do's the Quickſilver EF, (that be-
erpendicular to E, this oblique,)
uickſilver do's not deſcend lower in
ube EK, and aſcend higher in the
EG. The Reaſon then is this,
much as EH preſſes downwards
han EF, ſo much do's the Force
which (ariſes from the Preſſure of
on the other Parts of the Surface
nd) ſuſtains the Quickſilver in each
preſs upwards more in the up-
han in the oblique Tube; and
the ſame Reaſon, *viz.* becauſe
ſſure, whether upward or down-
s in the Tube EK perpendicular,
in the Tube EG it is oblique,
erefore not ſo great. Now the
t E, which holds up the Quick-
n the upright Tube, being as to
o much greater than the Force at
holds up the Quickſilver in the
, as the Weight of the Quick-
the upright Tube, is greater
e Weight of the other; it fol-
at the ſaid Force at E acting
according to the Perpendicular
as able to ſuſtain the Quickſilver
upright Tube, as is the ſame
ting upwards according to the
Line EF to ſuſtain the ſame
of Quickſilver in the oblique
And therefore the Quickſilver in

the upright Tube will not descend
er by Running any of it out of
Tube, nor will the Quickſilver
oblique Tube aſcend any high
any more Running into it:
Quickſilver in both Tubes will ke
equal Height.

19. *The* Shape *of the Veſſel makes no Difference as to the* Riſing *of the Fluid.*

And the ſame will hold as to
ther Tube (or Veſſel) of any
Shape: Namely, the Height of the
ſilver will be the ſame in any Ve
it is in a Cylinder of the ſame
and Baſis with the Veſſel. Th
Inſtance, *Fig.* 51, the Height
Quickſilver is the ſame, both
round-headed Tube DEHF, a
ſharp-headed KLB, as it is in t
linder inſcrib'd in the former,
or the Cylinder BLOP, circumſc
bout the latter. For as to the
headed Tube DEHF, as much
Quickſilver as lies without the C
CI, is ſuſtain'd not ſo much by C
the Parts MT and NS of the Tube
which are under them, and no
the Quickſilver is properly or
ſuſtain'd by C, than what lies wi
Cylinder CI. And conſequently th
will be the ſame, as if the Quickſi
put in the Cylinder CI.

o the ſharp-headed Veſſel LBK, 20.
e Cylinder BLOP, the Quickſilver *Further of the ſame.*
ewiſe riſe as high in the one as
er. For as to ſo much of the
ſel, as allows a free Aſcent from
face AB, the Quickſilver will there
its juſt Height CI, for the ſame
as it will riſe to the ſame Height
Cylinder BLOP. As to the other
f the ſaid Veſſel, where the
ver is hinder'd by the Sides of
el to riſe up to the Height CI,
as is there wanting in the Weight
Quick-ſilver, which is over the
e ſubjacent Parts of the Surface
much is ſupply'd by the Sides of
ſel. For although the Sides of
el do not preſs downwards as a
Force, yet they keep the Quick-
om being thruſt Higher by the
at preſſes upwards. And conſe-
the ſeveral Parts of the Sides
look'd on as ſo many Impedi-
hich are equivalent to the Weight
uch Quickſilver, as would be
between the reſpective Parts of
, and the Parallel to I. And ſo
ce AB will be equally preſt all
ſer the Veſſel B.

21. *Why the Fluid rises higher in a Tube at the* Bottom, *than at the* Top *of a Mountain.*

The Preſſure of the Air bein
which makes the Quickſilver riſe
Tube; it follows, that the greate
Preſſure of the Air, the higher
Quickſilver riſe; which is the
that the Quickſilver will riſe hig
the ſame Tube at the Bottom of a
tain or conſiderable Hill, than at t
Namely, becauſe the Column of A
ſing the Surface AB will be ſo
longer at the Bottom, than at t
of the Mountain, as is the Height
Mountain; and conſequently t
Column containing ſo much m
will preſs ſo much more.

22. *Of the* Weather-Glaſs.

So as to the (*) *Barometer* or *W*
Glaſs, the Quickſilver riſes and
the Tube, as the Air is hea
lighter, and ſo more or leſs pre
Quickſilver that is in the Well
call'd) of the Barometer. And
is agreed by all: but what it
cauſes the Air to be ſometimes
ſometimes lighter, is not ſo
That the Vapours do not, (as is
been, commonly ſuppos'd,) incre
Gravity of the Air, ſeems mo

(*) It is ſo call'd as being a *Meaſure of the W* Air, the Word βάρος ſignifying *Weight*, an *Meaſure*, in the *Greek* Tongue.

ble on this Account: Namely, that the Vapours are in the lower Re- of the Air, as in rainy Weather, the Air is lighteſt, as appears by lling of the Mercury (or Quick- in the Barometer. And when the s are in the middle Region of the e. when the Clouds are high,) not increaſe the Preſſure of the or although the Air be heavier is not becauſe the Clouds are but the Clouds are high, becauſe is heavy. For when the Air Earth is more denſe than uſual, becomes heavier than the Va- hich therefore muſt aſcend, and ttle in that Region of the Air, of the ſame Gravity with them- If the Vapours were the Cauſe aſing the Air's Gravity, there as many Vapours in the Air at a s are equal in Weight to three f Mercury, for ſo much we find ury riſes or falls in the Weather- Now Mercury is about 14 Times than Water, and conſequently ſt be in the Air at once ſo many as are equal in Weight to a f Water of 42 Inches in Height, e Baſis is equal to the Surface rth; which is much more than in Rain, during a whole Year.

For a whole Year's Rain do's no
Veſſel above 14 or 15 Inches hig
obſerv'd in the *Hiſtory of the Roy*
ty at Paris.

23. *The probable Cauſe of the* Riſing *and* Falling *of the* Mercury *in a Weather-Glaſs.*

The Reaſon then, why the
heavier at one Time than another
more probably from there bein
Air on that Part of the Earth's
when the Air grows heavier, A
proceeds from Winds. For Exa
the Wind, which is nothing but
of Air, ſhould blow on any Pl
the Air thus mov'd ſhould be
that Place by Mountains or H
if two contrary Winds ſhould
the ſame Place; in either Caſe
will be heaped up in the Mid
conſequently there being more
Gravity will be increas'd. B
aforeſaid Circumſtances, or the
not happen in any particular
then the Air which is over it, w
leſs in Quantity, and conſequen
ter. Whence it is probable,
Winds are the Cauſes of the Va
the Air's Gravity.

24. *Of the* Siphon, *or Inſtrument us'd to draw off Wine or other Liquors out of Veſſels.*

It was afore (§. 16.) intima
what has been ſaid in that and
lowing Paragraphs concerning
or Quickſilver, holds true li
reſpect of other Fluids. But
mention'd Affections have bee

particularly as to Quickſilver, foraſ-
as it is that which is us'd in Wea-
Glaſſes, wherein the ſaid Affections
re frequently ſeen by Us: It be-
he Deſign of this Treatiſe by the
e Principles of Mechanicks to ex-
chiefly ſuch Things as more fre-
y occur in common Uſe. On
Account I ſhall add here the Ex-
on of the Effect of the *Siphon*, (or
ment us'd by *Vintners*, &c. in
g off Wine, or the like,) accor-
o the forementioned Principles of
nicks, or more particularly of)
aticks. It is then obvious, that
2.) the Air being ſuck'd out of
hon at E, the Wine (or other Li-
in the Veſſel ABFG will be thruſt
he Preſſure of the Air upon the
AB, into the other End H of the
apply'd to the ſaid Surface. And
ne will be thruſt upwards in the
'till the Weight thereof comes
Equilibrium with the Preſſure of
rnal Air, ſuppoſe 'till it comes
leight CI. If therefore the Top
he Siphon be not higher than I,
ne will riſe to the ſaid Top of it;
re finding a Vent or Paſſage down
er Leg DE of the Siphon, it will
way, and ſo run out at E; con-
ſo to do, the Cauſes ſtill conti-

nuing, 'till the Veſſel is emptied.
is to be obſerv'd, that the Mou
the Siphon muſt be lower th
Mouth H. For elſe, E and H
qually preſs'd by the Atmoſphere,
Leg DE be but of an equal Leng
DH, and conſequently the Fluid
Leg of equal Gravity, with the
the other; the Fluid would ſta
Siphon, not moving one Way
other, without ſome other extern
beſide the Air. And by Parity
ſon, if DE be ſhorter than DH,
the Fluid contain'd in DE of leſs
than that in DH, the Fluid wo
(not from H to E, but) from
the Air coming up at E, and
the Fluid forward the contra
This would happen, except at
Siphon be ſo wide, as that the
aſcend by the Sides of the Fluid
the Fluid it ſelf deſcends in DE.

25. *The Mechaniſm of a common Pump, explain'd.*

Proceed we next to explain
chaniſm of a Common Pump (
by the Principles of *Hydroſtatick*
Handle KL being thruſt down, th
GF is drawn upwards from E, a
ther with it the Air that lies
Wherefore the Water at the Bott
ing without the Hollow of the
A and B preſs'd by the Air, bu
the Hollow of the Pump at C b

aſt, not ſo much) preſs'd by the will therefore aſcend from C to D, hruſting open the Valve E, (as ha- no Weight or Preſſure upon it,) will l ſtill higher to the Bottom of the , as meeting with no Preſſure from , and being continually preſs'd up- from Beneath. The Water ha- thus fill'd the hollow Part EF of mp, and the Sucker FG (by lifting Handle KL) being forced down- , do's force alſo downwards the under it between F and E, and s Means forces down and keeps own the Valve E. Wherefore the between E and F being thus by the Sucker; and having no downwards through E, it lifts up lve G of the Sucker FG, this be- only Way it can make for it ſelf. Vater thus ariſing above the Sucker, ther with it (the Valve G being own by the Weight of the Water pon it, ſo that the Water can't ck that way again; and the Han- being thruſt downwards again) upwards: where finding an open , it runs out at the Mouth H of out, other Water aſcending up in ean while through D, (as afore,) he Sucker is drawn up. And thus ally, as long as the Handle is

lifted

lifted up and down, and consequently the Sucker is moved up and down the same Causes being thus continued.

26. *Why some Wells will not be made Pump-Wells.*

It is obvious from the Principles of *Hydrostaticks* afore laid down, that the Distance between the Water at the Well, and the Valve E, (or Bottom of the Sucker when at the Lowest, at E,) must not be greater than the Altitude, whereto the Pressure of the Air is sufficient to make the Water rise, which is found by Experience to be about 32 Foot. Hence such Wells, as have the Water lying so deep, as that (it can't be made, but) the Distance is greater than the aforesaid Altitude, cannot be *Pump-Wells*, but must be *Draw-Wells*, i. e. can't have their Water drawn up by a Pump, but by a Bucket and Rope.

27. *The Tide explain'd by the Principles of Hydrostaticks.*

I shall conclude this Chapter with shewing how that Rising and Falling of the Sea-Water, which we call the *Tide*, (and which has been by old observ'd, but is by none of the old Philosophers, as I know of, rightly accounted for,) may be very naturally accounted for by the Principles of *Hydrostaticks*, after such a manner as is very easy to be comprehended.

Moon's Orbit being over those
of the terraqueous Globe, which
tween the Tropicks, and where are
gest Seas, as she passes over
Seas, she presses (those Particles of
which are next under Her, and
the next underneath, and so the
intermediate Air, and thereby at
he Water of the Sea, and so causes
id Water to rise, on each side of
Part of the Sea which lies under her
that is, toward the North and
Parts of the World. Hence to us
ive in the northern Parts of the
, the Tide always comes from the
because the Moon's Orbit is South
And on the like Account, the
comes from the North, to those
ive in the southern Parts of the
, because the Moon's Orbit is
of them. And hence it is evident
hat the Tide must come so much
or later to any Place, as it lies
r less distant from the Tropicks,
t Tract of the Sea which is under
oon's Orbit.

28. *Why the Tide always comes to Us from the South.*

Tide being thus caus'd by the
e of the Moon, it plainly follows,
cause the Moon comes every Day
Meridian of any Place fifty Mi-
later than the Day before; there-
e Tide likewise must fall out fifty
Minutes

29. *Why the Tide comes every Day fifty Minutes Later than the Day afore.*

Minutes later every Day, than the before, in Reſpect of any part Place.

30. *Why the Tide is* ſix Hours *coming in, and* ſix Hours *going out.*

Alſo, becauſe the Moon is ſix in coming from the Horizon of Place to its Meridian, therefore (it vious, that) the Tide muſt be ſix coming in. And in like manner, be the Moon is ſix Hours in going fro Meridian to the Horizon of any therefore the Tide muſt be the Time in going out. Where it is ous, that as the Preſſure of the begins in reſpect of any particular ſo ſoon as ſhe comes to its Horizo her Preſſure continually increaſes comes to the Meridian of that Place conſequently the Tide riſes ſtill and higher, 'till ſhe comes to the Meridian. Which as the Moon again, ſo her Preſſure decreaſes, conſequently the Tide ſinks lower lower.

31. *Why the Tides are* Bigger *at the Change and Full of the Moon, than at other Parts of the Month.*

Moreover it being agreeable to Laws of Staticks, (as is found by riments,) for heavy Bodies to gra ſo much more, as they are nearer t Center of the Earth; hence the Re why the Tides are Bigger at the and Full Moons, than at the Qua is eaſily to be aſſign'd; namely, Moon's being at her Change and

to the Earth, than at her Quar-

in it is obſerv'd, that as the Tides Change and Full of the Moon are than at any other Time of the ; ſo the Tides at the Change and f the Moon about the Equinox's ger, than thoſe at any other Time Year. Which may be accounted the Moon's being then over the Part of the Ocean, which is un- e Celeſtial Equator, and conſe- by its Preſſure thereon making r Quantity of Water recede to ide, *viz.* Northward and South- than when ſhe preſſes on one Side Equator, and ſo not in the Middle Ocean.

32. *Why the Tides at the Change and Full of the Moon about the Equinox's are* bigger, *than at other Parts of the Year.*

mains to be obſerv'd, that it is trable (†) by the Principles of , that the Moon do's cauſe (the well, or in one Word) the Tide, y in that Part of the Sea which ver, but alſo in the Part oppoſite And hence it comes to paſs,

33. *Why there be* Two *Tides in every twenty-five Hours.*

---

as would have a more accurate Account of the him conſult *Newton's Phyſ. Mathem. Lib.* 3. 37, and *Numb.* 16. of the *Philoſoph. Tranſact.* ; as alſo *Gregory's Aſtron. Phyſ.* and *Geom. Lib.* . I have here contented my ſelf with a more count, as being more Eaſy to be under-

that

that there be two Tides every
five Hours; namely, becauſe
Time the Moon going from
dian, returns to it again;
quently cauſes one Tide, whil
the upper Hemiſphere, and
while ſhe is in the lower Hemi

34. And thus I have taken No
much of *Mechanicks* (in gen
cluding *Mechanicks*, *Statick*
droſtaticks properly ſo called)
ſwerable to the Deſign of th
tiſe.

FINIS.

Fig. 1.
D
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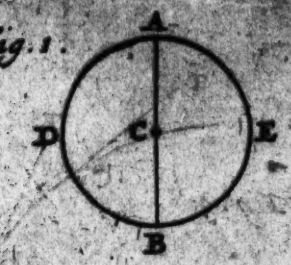


Fig. 1.

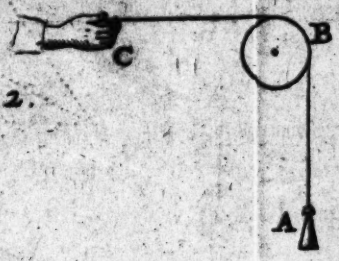
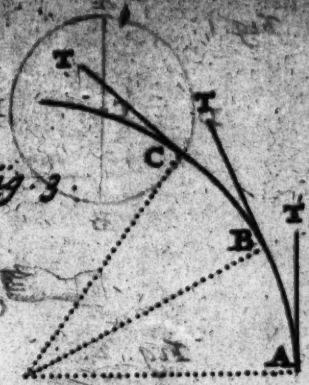


Fig. 2.



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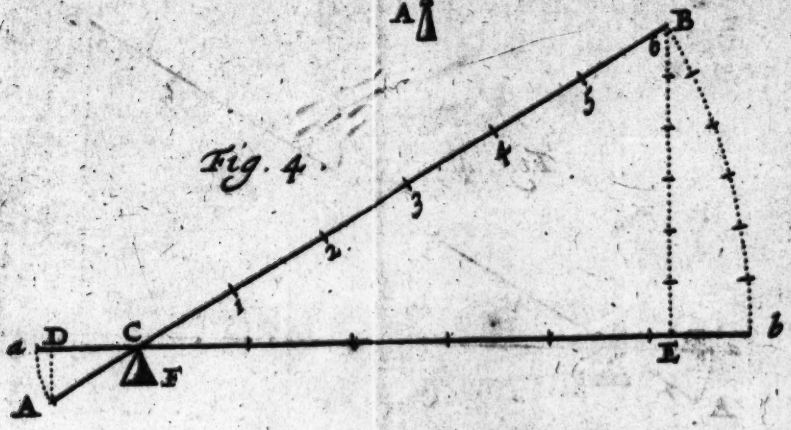


Fig. 4.

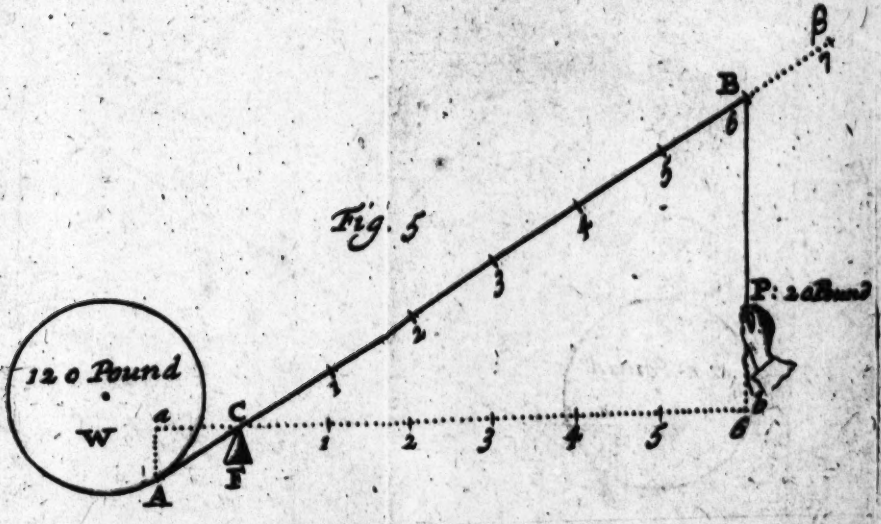


Fig. 5

Fig. 7

Fig. 1

Fig. 6

Fig. 7

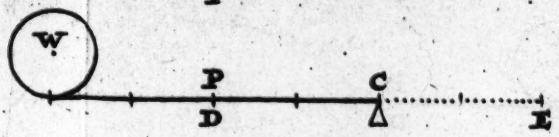


Fig. 8

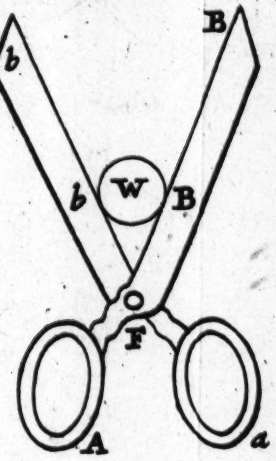


Fig. 9

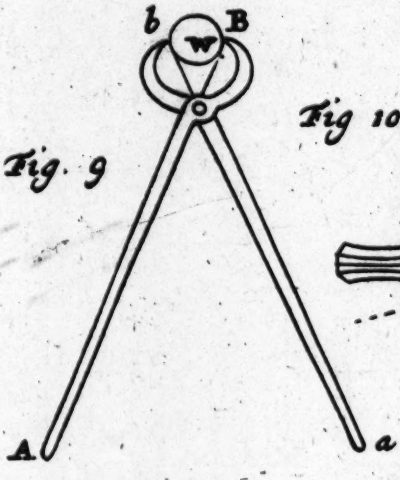


Fig 10

Fig. 11

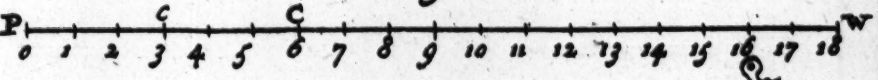


Fig. 12

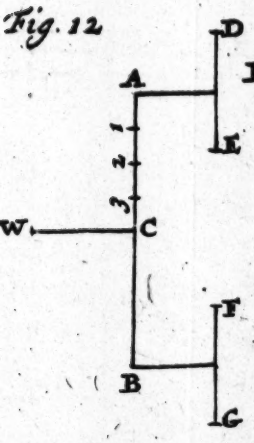


Fig. 13

Fig. 14

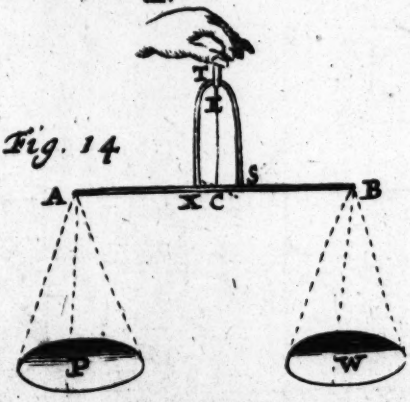


Fig. 2

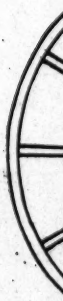


Fig. 15

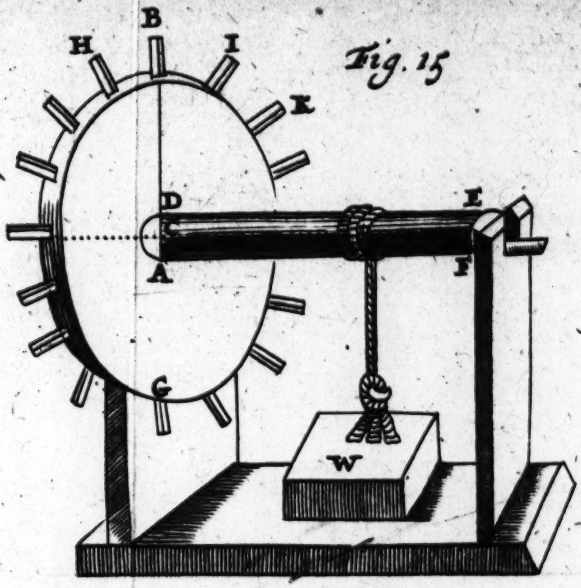


Fig. 16.

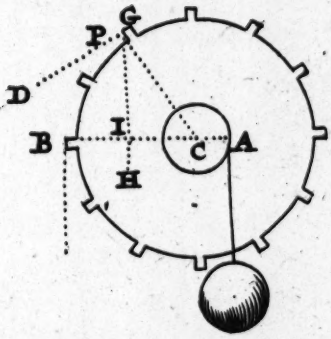
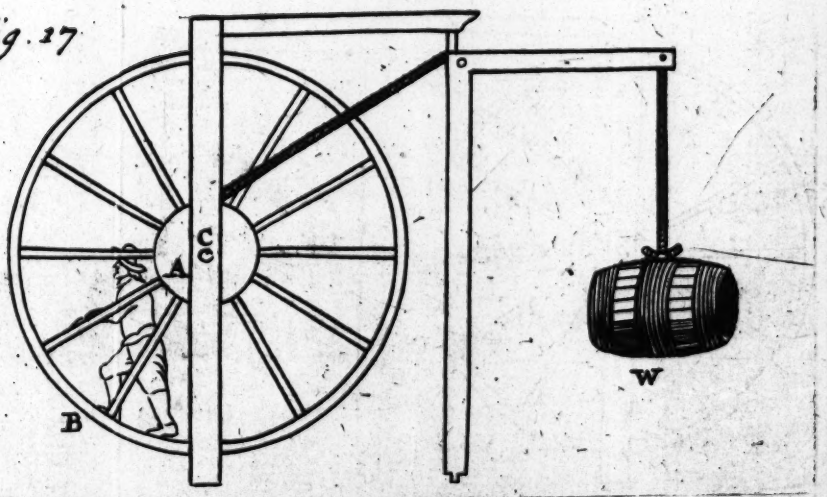


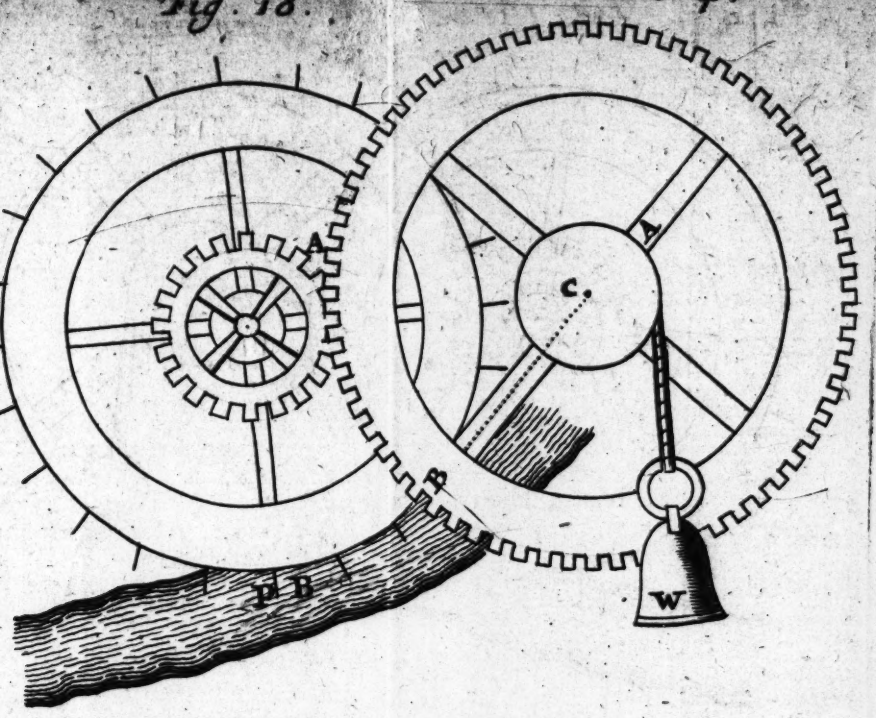
Fig. 17



W
A
A

Fig. 18.

Mechan. Plate 4.



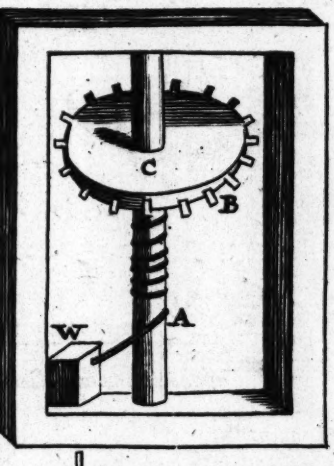


Fig. 19

Fig. 20

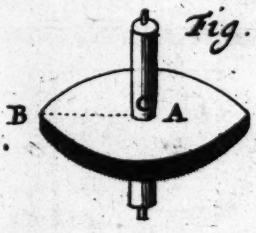


Fig. 21

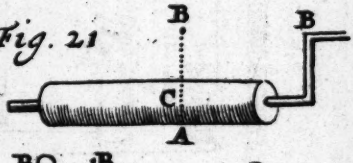


Fig. 22

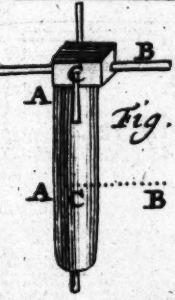


Fig. 23

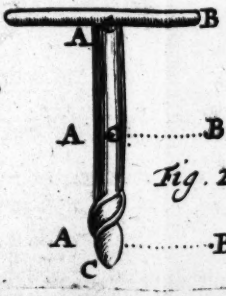


Fig. 24

Fig. 2

P

Fig

B

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Fig. 25

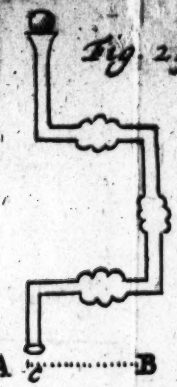


Fig. 26

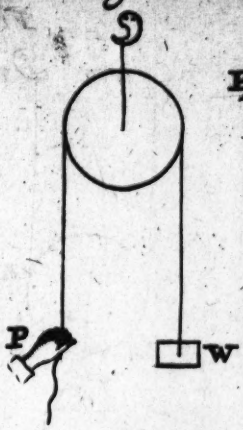


Fig. 27

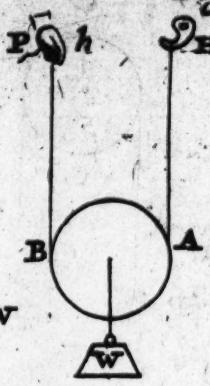


Fig. 28

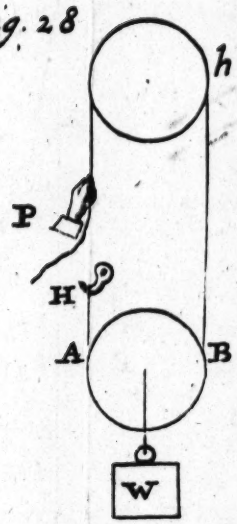


Fig. 29

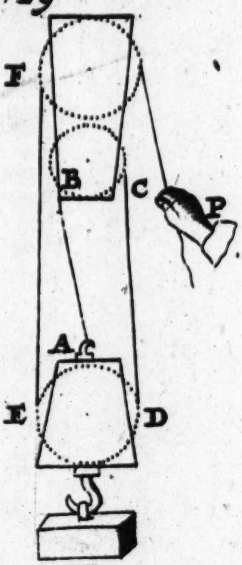


Fig. 30

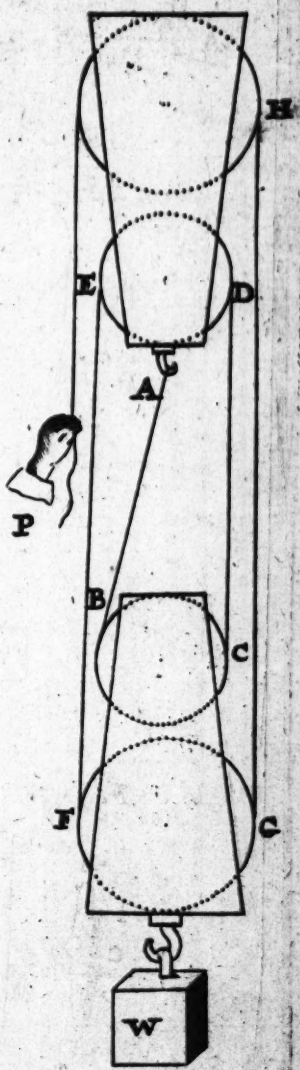
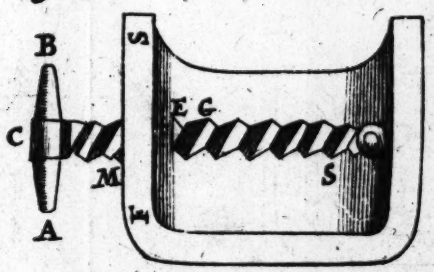


Fig. 31



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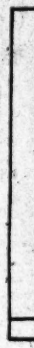


Fig. 3

Fig.

Fig. 32

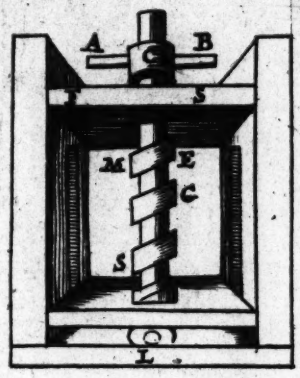


Fig. 33

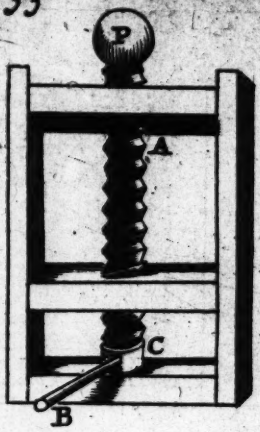


Fig. 34

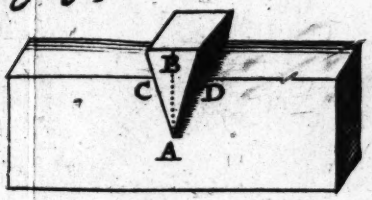
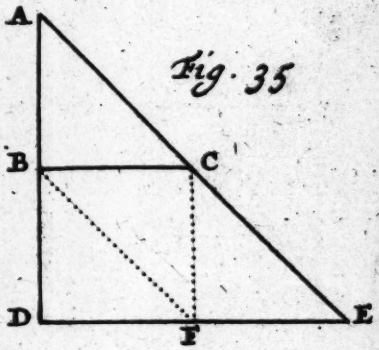


Fig. 35



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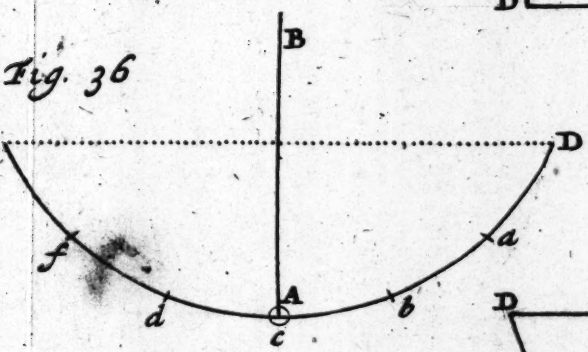

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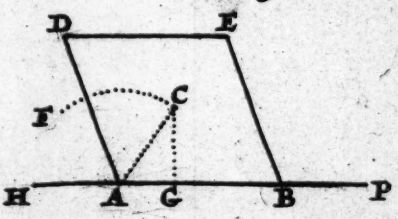


D
A
C

Fig. 38

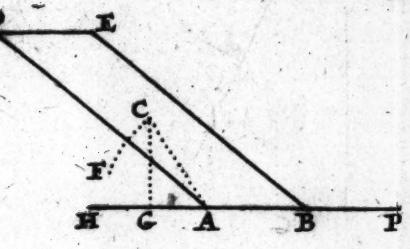
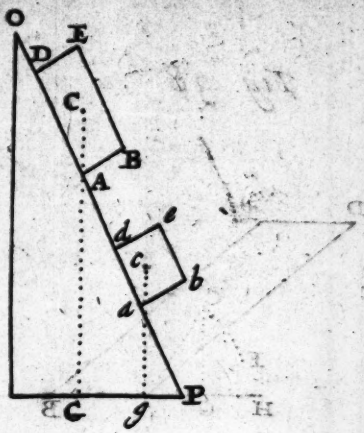


Fig. 39



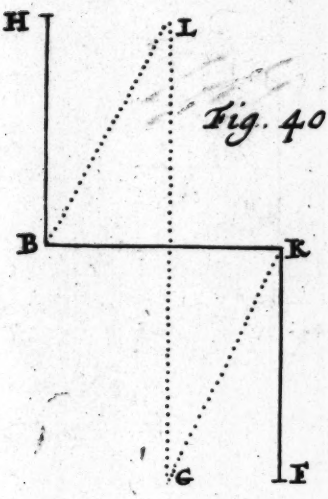


Fig. 40

Fig. 41

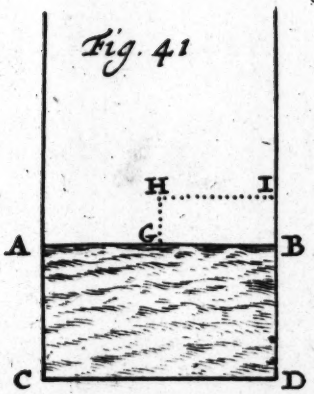
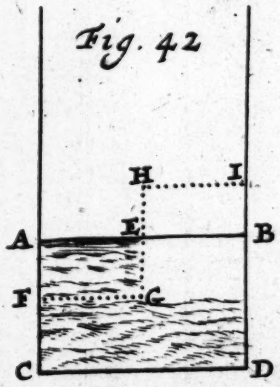


Fig. 42



A
E
C

C
I
A
C

A

Fig. 43

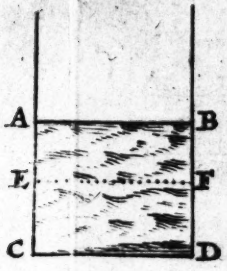


Fig. 44

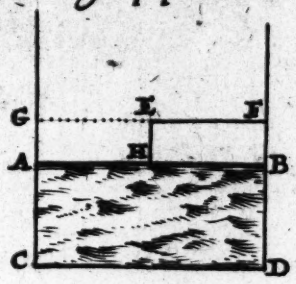


Fig. 45

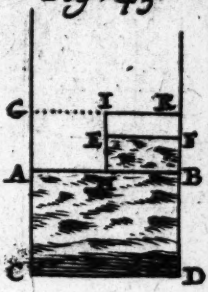


Fig. 46

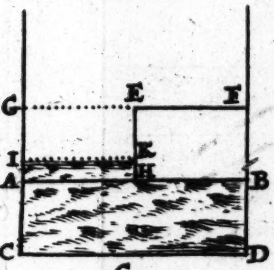


Fig. 47

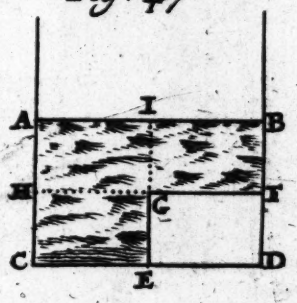
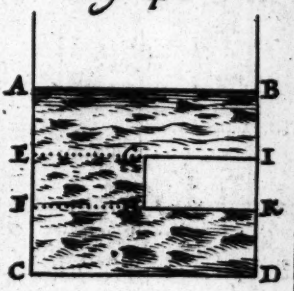


Fig. 48



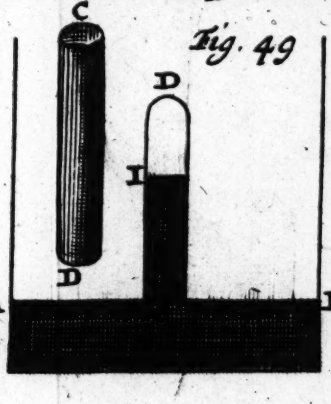


Fig. 49

Fig. 50

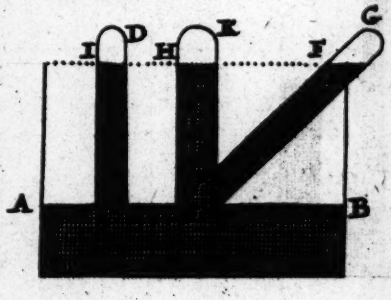
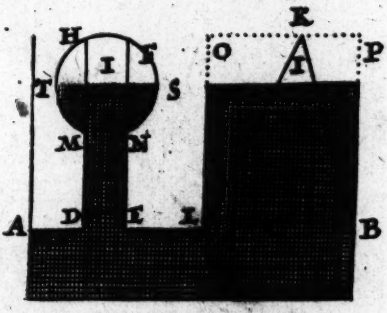


Fig. 51



Fig

Fig. 52.

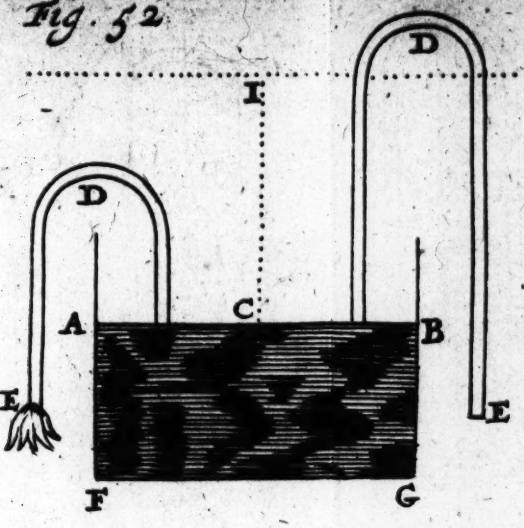
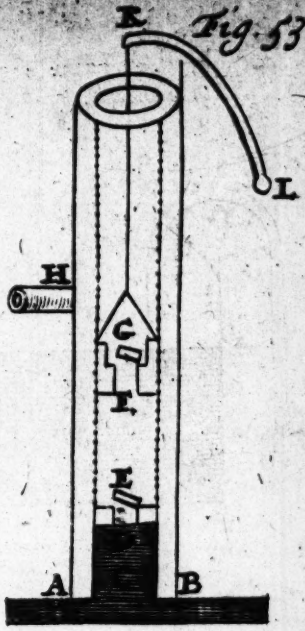


Fig. 53.



THE

Young Gentleman's OPTICKS:

CONTAINING

The more Useful and Easy ELEMENTS of *Opticks*, more properly so called, of *Catoptricks*, *Dioptricks*, and *Perspective*.

EDWARD WELLS, D.D. Rector of *Cotesbach* in *Leicestershire*.

LONDON,

Printed for *James Knapton*, at the *Crown* in St. *Paul's* Church-Yard. 1713.

THE PREFACE.

AS Vision or Sight is the most useful and pleasant Sensation of the Body, so Optick*s must consequently be* very *useful and pleasant, as not* only explaining the Manner of Vision *in a Geometrical Way, but also by the same Principles* shewing *how to* supply or remedy the several Defects of Sight, *by the Help of proper Instruments, such as* Spectacles,

tacles, Microſcopes, T
copes, &c.

Add hereto, that to this
ence is owing the Inventio
ſeveral Inſtruments, which ſ
to pleaſe *and* divert *the M*
by curious and ſurpriſing
rations ; ſuch as are the D
Chamber, *the* Magick-L
tern, &c. *All which are*
counted for in the follo
Treatiſe.

THE Young Gentleman's OPTICKS.

CHAP. I.

Of Opticks *in general.*

THE Word *Opticks*, as it literally ſignifies in the Greek Tongue any Thing that relates to the Sight; ſo in its largeſt ...tation it is uſed by the Learned, to ...e the whole Compaſs of Science or ...ledge (Phyſical as well as Mathe...al) relating to Viſion or Sight.

1. *Opticks in general, what.*

...s now agreed by the Learned, that ...is caus'd by the Eye's receiving in... Rays, which iſſue from the Object ... Theſe Rays, if they paſs from the ... to the Eye through a Medium all

2. *Opticks properly ſo call'd, what.*

along of the ſame Nature, diffuſe
ſelves all along after the ſame Man
right (becauſe the ſhorteſt) Lines. W
this Sort of Viſion is call'd *Direct*
and the Word *Opticks* is taken in a
reſtrain'd Senſe, to denote that P
Opticks in general, which treats o
Viſion.

3. Catoptricks, *what.* If the Rays come not directl
the Objects to the Eye, but fall f
on ſome impenetrable Body, a
thence reflected or beaten back
Eye, then the Viſion is called *Re*
ſion; and not the Object it ſelf,
Image is ſeen. A rough Surface ſc
and ſcatters the Rays, that they
no Image at all, or at beſt but a v
perfect Image, of the Object; an
fore thoſe Bodies, which are cap
being moſt poliſh'd, or made moſt
make the beſt Reflexion. Such
us'd therefore now-a-days for L
glaſſes; but Glaſs being of it ſe
cid or tranſparent, there is uſuall
the Back-ſide of Looking-glaſſes,
ſilver to render it impenetrable
Rays. Before Looking-glaſſes
vented, the Ancients made uſe of
Metal; and ſuch a Piece of Met
uſe of to ſee one's ſelf in, the G
led Κάτοπτρον; and from thence
of Opticks in general, which

Vision, is distinguish'd by the
e of *Catoptricks.*

the Rays of the visible Object pass 4.
gh different Mediums, so that they Dioptricks
ore or less diffus'd in the one than *what.*
other, then each Ray will be re-
d or broken in one Medium, from
right Line wherein it diffus'd it self
e other Medium; whence this is cal-
efracted Vision. Thus a Ray passing
Water into the Air appears refrac-
e. broken or bent, from that right
it made in the Water. And because
nowledge of refracted Vision is most
in order to the making of Telesc-
, and such other Instruments; by
g through which, Objects are more
seen; hence that Part of Opticks
eral, which treats of refracted Vi-
distinguish'd by the Name of *Diop-*
, διοπτρον being a Word capable of
ing in the Greek Language any In-
nt made for to *look through.*

the forementionn'd Parts of Opticks 5.
eral, may be added *Perspective*, or Perspec-
t of representing visible Objects in tive, *what.*
ure, just as they appear to the
This Art is so call'd, because
ficture is to be suppos'd between the
nd the Object, and transparent
o the Object may be seen through
he Word *perspicere* in the Latin
Tongue

Tongue ſignifying *to ſee through* a T

I ſhall ſpeak of each of the fore
on'd ſeveral Parts of Opticks, ſo
is requiſite to the Deſign of this
tiſe; after that I have here adjoi
Definitions or Explications of ſuch
or Words, as are of principal Uſe
ticks; as alſo ſuch Theorems, as
greeable both to the Principles of
metry and Experience, and therefo
us'd as Axioms in demonſtrating th
orems of this Treatiſe relating to
tricks and Dioptricks.

DEFINITIONS

1. *A Radiant* is whatever ſend
Rays from its ſeveral Points. He
only the Object, ſuppoſe a Can
called the Radiant; but alſo its
ſuppoſe the Image of a Candle ſe
Glaſs, is call'd ſo likewiſe. As
the Image of an Image, and
Where note, that for Diſtinction
the Object is uſually ſpecify'd
Name of the *prime* (or *primary*) R
And ſo the immediate Image of
ject may be diſtinguiſh'd by the N
a *ſecond* Radiant; the Image of
Image, by the Name of the *third*
ant, and ſo on; that which is
the Eye being always diſtingu

re there are many Images) by the
e of the *last* Radiant.

Parallel Rays are ſuch, as keep all
g at an equal Diſtance one from the

Diverging Rays are ſuch, as being
uced both Ways, concur or meet in
Point that Way, which is contrary
eir Motion. Thus *Fig.* 10, the Rays
AB, and AF, which are ſuppos'd to
from A, are diverging Rays.

Converging Rays are ſuch, as being
uced both Ways, concur the ſame
they move. Thus *Fig.* 10, the Rays
OB, and HF, which are conceiv'd
ve toward *a*, are converging Rays.

B. Paralleliſm, Divergency, and
rgency are to be underſtood of Rays
ſame Point.

The *Focus* is that Point, where the
of the ſame Point being produced
. Thus *Fig.* 10, A is the Focus of
verging Rays AD, AB, and AF;
is the Focus of the converging
ED, OB, and HF. Whence it is
s, that the Focus of parallel Rays
e conceiv'd to be at an *infinite* Di-
, *i. e.* at ſo great a Diſtance, as that
ays of the ſame Point, though they
rictly diverging or converging, yet
e or converge ſo very little, (at
reſpect of that Length and Porti-

on of them, wherein they are confi
as that they may be look'd upon a
rallel.

6. The *Angle of Incidence* is nat
and ſo properly the Angle ABC (F
comprehended between the incide
AB and the Surface BC which it (i
or) falls upon. But the Angle AB
ing always the Complement of th
gle ABP to a Right; and becauſe
is to be prov'd of the Angle ABC
be primarily, and ſo more eaſily p
of the Angle ABP, hence for Conv
cy ſake Writers of Opticks do u
take ABP for the Angle of Inci
and agreeably define it to be the
comprehended under the inciden
AB, and a right Line PB perpen
to B the Point of Incidence. Agr
alſo hereto,

7. A *Reflex Angle* is that, wh
comprehended under the reflex B
and the foreſaid Perpendicular P
the Angle EBP *Fig.* 8: Though
properly or naturally EBG be the
of Reflexion.

8. A *Refracted Angle* is that, w
comprehended under the refracte
BG and the Perpendicular KF
(*Fig.* 28.) *viz.* the Angle GBF.

By *Inflection*, as a general Word, is
ied either Reflection or Refraction.
consequently both the reflecting Sur-
BC (*Fig.* 8) and the refracting Sur-
BD (*Fig.* 28.) are ſtiled inflecting
ces. And both a reflected Ray BE
8) and a refracted Ray BG (*Fig.* 28.)
mprehended under the Name of in-
d Rays.

AXIOMS.

A Ray of Light falling perpendicu-
on an inflecting Surface, either goes
rd in a right Line, or is reflected in-
ſelf. Thus (*Fig.* 9.) the Ray AC
be conceiv'd as reflected by the re-
g Surface BC into it ſelf. And the
AD (*Fig.* 28.) is to be conceiv'd to
orward beyond the refringent Sur-
D in a right Line DE or DN.

The Inflection of any Ray is per-
d in a Surface perpendicular to the
ing Surface. Thus (*Fig.* 8.) the Ray
reflected into BE in the Surface
E perpendicular to the reflecting Sur-
BC. And likewiſe (*Fig.* 28.) the Ray
s refracted into BG in the Sur-
BGND perpendicular to the re-
ut Surface BD. And note, that
id Surface wherein any Ray is in-
flected,

flected, is thence ſtil'd the *Plane* [illegible] *flection.*

CHAP. II.

Of Opticks properly *ſo call'd*, Direct *Viſion.*

THEOREM I.

Fig. 1. THE Eye is always placed [illegible] Center of its own Sphere [illegible] ſion.

Demonſtration. At what Diſtan[illegible] ever, ſuppoſe O*a*, the Eye O can [illegible] Way, it is ſelf-evident, that th[illegible] Eye, if by no means hinder'd, ca[illegible] the ſame Diſtance any other Way towards *b* it can ſee at the Diſtanc[illegible] towards *c* at the Diſtance O*c*, [illegible] quite round. Wherefore the ſeve[illegible] ſtances O*a*, O*b*, O*c*, &c. being eq[illegible] to the other, and all proceeding fr[illegible] ſame Point O, it follows that t[illegible] the Rays of a Circle, whoſe Cent[illegible] and Circumference *abcdefgh*. Wh[illegible] ſo it follows, that the ſaid right Li[illegible] O*b*, O*c*, &c. are the Rays likew[illegible]

re generated by the Turning of the mention'd Circle round about any s Diameters, ſuppoſe *ae*, as an Axand conſequently that O the Eye be the Center of the ſaid Sphere. Q. D.

ol. 1. As it is evident from what has ſaid, that could the Eye be placed e it might ſee every Way, the Space, h would then fall within its Sight, ld repreſent an entire Sphere; ſo it ws, that by the Interpoſition of the n, the Space or Compaſs of actual n, even when the Eye has a full , is equal but to an Hemiſphere. ly, ſuppoſing *ae* to repreſent the upurface of the Earth whereon the Eye ed, the Extent of actual Viſion will ual to the Hemiſphere *Oabcde*, the EF denoting the Horizon or the dary of the Sight downwards.

ol. 2. From this Theorem it follows that whatever Portion of the Hea- (whether All *abcde*, or only Part as *bd*, or *bc*, &c.) is ſeen by us, it rs as a Portion of the concave Surf a Sphere.

ol. 3. Becauſe the Sun and Moon ars are beyond the Extent of diViſion, and conſequently the Eye diſcern, how far one is more diſtant it than the other; hence it comes to paſs,

pass, that the Sun, Moon, and
do appear to us, as all situated i
Surface *aceg* of one and the same S
O*aceg*, and so equally distant fro
though they are in reality at sever
stances from us.

Corol. 4. So much of the Heaven
seen by us, *viz.* *abcde*, appearin
spherical Figure only for the
contain'd in this Theorem; it then
lows, that the Body of the H
may be of a Quadrilateral Figu
ABCD, or of any besides a S
cal.

Scholium. This first Theorem w
Corollaries is of good Use to
young Students for Astronomy.
have chosen to place this Theorem
direct Vision, partly because it wi
the more easily understood, and
because, (though it be not one
geneous Matter contain'd in the
or Space which we call Heaven, a
sequently what we see therein
considerable Distance, we see no
rect, but refracted Vision, yet) i
to the same in respect as to the
fore us, whether Things in the
are visible to us by direct or refra
sion.

T

THEOREM II.

Eye can ſee at one View (i. e. at Fig. 2.
Time and Place) no more Objects,
whoſe Rays fall within the Com-
of a Semicircle, to which the Eye,
her its Pupil, is the Center.
. For the Eye can ſee only
Objects, whoſe Rays, paſſing thro'
Pupil, reach to its Retina or Optick
placed at the Bottom of the Eye.
ow it is evident from the bare In-
on of *Fig.* 2. that no Ray can en-
Pupil *O*, but what falls within
micircle DHE, whoſe Center is *O*.
ppoſing a Ray to paſs along ED
ameter of the Semicircle, it will
ver the Pupil *O*, and not enter it.
onſequently all Rays falling with-
e ſaid Semicircle, will much leſs
the Pupil. Q. E. D.

. Rays of Light (as well as other
, whether natural or artificial) ha-
greater or leſs Force, as they are
more oblique, it follows that ſuch
s paſs through the very Middle of
micircle DHE, are of moſt Efficacy
ing Viſion, as entring the Pupil of
perpendicularly, and conſequent-
ing perpendicularly upon the Reti-
the Bottom of the Eye. Thus a
the

the Image of the Object A w
painted most lively and distinct
the Retina. The Image *b* and
the Objects B and G will be paint
lively and distinct than the former
a, as being made by oblique Rays
alike lively and distinct in respect
another, as being made by the R
and Gg equally oblique: and more
and distinct than the Images *c* a
the Objects C and F, because th
Bb and *Gg* are less oblique than th
Cc and *Ff*.

Schol. Agreeably hereto we f
Experience, that we see most di
such Objects, as (*cæteris paribus*)
actly before our Eyes. And up
Account it is, that, in all the
Parts of Opticks largely so called
cipal Regard is to be had to such
as enter the Eye perpendicularly.

THEOREM III.

Fig. 2. An Object appears greater or
cording as it is seen under a greate
Angle.

Demonst. Let the Lines AB and
they are of equal Length, so) re
two Objects of equal Bigness. I
dent that the Angle AOB, unde
is seen the Object AB, is only a
the Angle COD, under which is

CD. It is alſo evident, that the
AO and BO being extended be-
O, the Pupil of the Eye ODC
fall reſpectively upon the Points
b of the Retina in the Bot-
the Eye; and likewiſe the Rays
DO being extended beyond O,
ll upon the Points *c* and *d* of the
. Whence it follows, that *ab* the
of the Object AB will be ſo much
portion leſs than the Image *cd* of
ject CD, that is, the Object AB
pear to a Spectator ſo much leſs
e Object CD, (though they be
an equal Bigneſs in reality,) as the
OB is leſs than the Angle COD.
D.

. Hence it appears, why the Diſ- *Fig.* 4.
tween the ſame two Objects A
ppears to be greater or leſs, ac-
as the Diſtance of the Spectator
ſaid Objects is greater or leſs;
becauſe the Diſtance AB between
two Objects is ſeen under an An-
portionably greater or leſs, as the
of the Eye from the ſaid Objects
r greater; that is, the Angle
proportionably greater than the
EB, as the Diſtance OD is leſs
Diſtance ED. And conſequent-
ding to this *Theorem* III.) the
AB will appear ſo much in pro-
portion

portion greater to a Spectator at O
at E.

THEOREM IV.

Fig. 5. The Rays of an Object being ad
through a narrow Hole, do there
one the other, so as that the Imag
by the said cross Rays will be inv
respect of its Object.

Demonst. The Ray *Aa* proceedi
the Point A of the Object or Arr
being (in direct Vision) a righ
and so likewise the Ray B*b*, th
cross one the other, being exten
yond the Hole O. And there
upper Point *A* of the Arrow AB
come the lower Point *a* of the Im
and the lower Point B of the Ar
will become the upper Point *b* of i
ab. And the like will hold as to all t
mediate Points, (except the ver
Point C, which being perpendicu
a right Line with O, its Image *c*
wise be the middle Point of th
ab) and consequently the Imag
be inverted in respect of its Ob
Q. E. D.

Schol. 1. Agreeable hereto a
ments of this Nature. Namely,
being darken'd by putting to
Window-shuts, and a small H
made with a common Gimlet

in one of the Window-shuts, and
Glass of the Window being remo-
from before the Hole by open-
the Casement or the like; such
as are without, and so situated
that their Rays will pass through
Hole, will appear upon a Sheet of
Paper, held on the Inside overa-
the Hole, in an inverted Manner
ing to this Theorem. And the
the Paper is held to the Hole, so
less will the Image be; because as
less as the Rays A*a* and B*b* are ex-
beyond O the Hole or Point of
leeting, so much less is the Dis-
etween them: Namely, the Dis-
d is less than the Distance *ab*, and
ently the Image of the Object
e the Arrow) AB will appear less
Paper held at the Distance *cd* from
le, than if the Paper be held at the
e *ab*.

nd the same has been found, by
ments made for this Purpose, to
od in respect of the Eye it self,
ng as it were a small dark Cham-
to which are let pass through the
the Rays of outward Objects,
aint their Images on the Retina in
om of the Eye, agreeably to this
m, that is, in an inverted Man-
respect to its Object. As is il-

luſtrated *Fig.* 6, which repreſents
having on its hinder Part a little
each of its Coats or Skins cut aw
you come to the vitreous Hum
Candle being held before the Ey
Piece of white Paper being apply
hinder Part prepared as aforeme
the Image of the Candle has bee
to be made on the ſaid Paper i
verted Manner; All other Rays
being kept from falling on the
dark'ning the Room where the
ment was made. Concerning wh
ſhall be ſaid in Chapter IV, fo
as the Rays paſſing through the
of the Eye are refracted, and co
ly it is requiſite to underſtand
of Refraction, before we can u
aright, how Viſion is caus'd in
And for the ſame Reaſon I o
ſaying any thing of the Uſe of
Glaſſes in the Hole to a da
ber, for rendring the Images m
and diſtinct.

3. I ſhall add here (as being
Uſe for the clearer Underſtandi
has been or ſhall be ſaid in thi
of Opticks) a Deſcription of t
Parts of an Eye, repreſente
where A denotes the *Chryſtallin*
B the *Vitreous*, C the *Aqueou*
Cornea Tunica; E the *Retin*

call'd *Choroides*; G the Coat call'd *nea*, being a Part of the *Dura Mater* ned about the Ball of the Eye on its Part; H the Optick Nerve without ll of the Eye; I the Coat or Skin the *Uvea*; K the Hole of the *U.* call'd in one Word the *Pupil*.

remains only to obſerve further, that if not all the Particulars taken no- in this Chapter, belong not to di- ſion alone, but are applicable alſo lected and refracted Viſion; which oceed to ſpeak of in their Order.

CHAP. III.

Catoptricks, *or* Reflex Viſion.

THEOREM I.

Ray AB of Light be reflected by a *Fig.* 8. n Surface GC, the Angle PBE of ion will be equal to the Angle PBA dence.

onſt. Draw BP, and alſo AC, per- ular to GC. The Medium GC*de* uppos'd homogeneous to (*i. e.* of me Nature with) the Medium

EBCD, it follows that all the Ray
prehended between AB and AC
which take up or ſpread all over th
Line BC, were they not hinder'd
the reflecting Plane) from paſſin
the Medium GC*de*, would therein
themſelves after the ſame Manner
the other Medium EBCD, that i
right Line AB being continued to
AC to *d*,) at the Diſtance Be they
diffuſe or ſpread themſelves all o
And whereas the reflecting Plane B
neither increaſe nor diminiſh the
ing of the Rays, but only turn
back into the ſame Medium they
afore; hence it is obvious, that a
Reflection the Rays will ſpread
ſelves in the ſame Manner, as they
have ſpread, had they not fell up
Surface GC, but (that being re
gone on in the ſame homogeneou
um. But in that Caſe, at the D
B*e*, the Rays would (as is afore
ſpread themſelves all over *ed*.
fore in the Diſtance BE=B*e*, th
ſpread themſelves all over DE=D
conſequently the Figure BCDE wi
all Reſpects like and equal to the
BC*de*; and indeed is no other t
Figure BC*de* reflected, or turn'd a
it were) half round upon BC.
fore, if from the equal Angles E

here be taken away the right Angles
nd *p*BC, there will remain the An-
BP and *e*B*p* equal one to the other.
ABP=*e*B*p*, therefore ABP=EBP.
D.

1. Hence the Angles EBG and
which, as has been *Defin.* 6 and 7
d, are more properly call'd the
of Incidence and Reflection) are
one to the other. Namely, be-
PBG=PBC, and PBE=PBA,
re (PBG—PBE=)EBG=(PBC
=)ABC.

2. If the reflex Ray BE be taken
Incident, then the incident Ray
be its Reflex.

3. If the Angle PBE=PBA, or
CBA, then BE is the Reflex of

What has been prov'd as to a
g Plane GC, the ſame holds good
reſpect of any reflecting *Curve* Sur-
. For the Inclination or Directi-
he Curve, at B the Point of In-
, is the ſame with the Inclination
Plane DE that touches it there.
herefore the Reflection following
the Inclination, will be the ſame,
it be ſuppos'd to be made by B
rticle of the Curve Surface PBQ,
he Plain DBE. And the ſame
ood as to refracting Surfaces.

Fig. 9.

THEOREM II.

Fig. 10. At what Diſtance CA an Object placed before a reflecting Plane CF (ther of Metal or Glaſs, *&c.*) a ſame Diſtance C*a* will *a* the Ima the Object appear behind the ſaid ting Plane, each Diſtance CA and ing meaſur'd upon one and the ſam pendicular to the ſaid Plane, *viz.* A*a*.

Demonſt. Let AD, AB, and A Rays proceeding from the Object ny Point of the Object) A; and l BO, and FH be their reſpective Rays. It is evident, that theſe be tended beyond CF (*i. e.* behind flecting Plane) will all meet in *a*, therefore will be the Focus of t Rays, and conſequently the Place the Image of *a* will appear. But n Angle *a*DC=EDF, and EDF= by *Theorem* I. of this Chapter; fore *a*DC=ADC. And becauſe Triangles ACD and *a*CD, the ADC=*a*DC, and alſo ACD=*a*C the Side CD is common to both gles, therefore CA=C*a*. And a very ſame Manner may it be that in the other Triangles, AC *a*CB, as alſo ACF and *a*CF, t

CA of (any Point in the Object,) consequently of all) the Object ual to the Distance Ca of the Image E. D.

1. Hence it follows, that suppo- e reflex Ray OB to be that which to the Eye plac'd at O, though all of the Looking-glass or other re- Plane, CF be cover'd (or taken but the Part B, yet the Image a seen by the Eye; whereas if the be hid, and all the rest uncove- e Image can't be seen by the Eye

2. After the same Manner as Ob- e inclin'd towards the reflecting will also their Images be inclin'd, onsequently in such a Plane or g-glass plac'd horizontally, Objects pear inverted, or with their upper ownward.

3. Because aB=AB, therefore B+BO, that is, the Distance of age from the Eye is equal to the

---

ely, as the visible Object is conceiv'd to con- ral Radiant Points, so the Image of the whole ade up of all the Images of the several Radi- taken together; as is more fully illustrated and 12, of which see more in the following

incident Ray and reflected Ray ta
gether.

Cor. 4. Whatever has been f
the Image of any Object or prin
diant, the ſame holds true alſo
Image of an Image. Whence
the multiplying of Images by Me
two or more reflecting Planes du
ced. Wherein this is moſt rema
the Diſtance of every Image is e
the Ray propagated from the prin
diant to the Eye, through all the
mediate Reflexions.

Schol. 1. It is obſervable, that
Theorem, the Focus A of Rays
AB and AF falling on the ref
Plane CF being given, thereby
found the Focus *a* after Reflection;
ly, by drawing AC perpendicular
and extending it to *a* ſo, as that
be equal to CA. For then *a* will
Focus ſought.

Schol. 2. For the Sake of ſuch
not ſo readily apprehend from
how the Image of a whole Object
by Reflection, I have therefore add
11 and 12. In each of which
notes a Croſs, and *ab* its Image
the incident Rays reflected to the
E by the reflecting Plane CF. Name
11 ſhews expreſsly, how the Ra

that come from the two Extremi-
B of the Object are reflected to the
and ſo cauſe the Image of A and B
pear at *a* and *b*; and *Fig.* 12 ſhews
in like Manner the Images of the
D and H of the Object appear at
b; namely, in both Figures the I-
appear, where the reflex Rays EG
being extended, meet with the
diculars A*a*, B*b*, H*h*, and D*d*; and
C*a*=CA, F*b*=FB, C*b*=CH, and
D. And the ſame may be ſhewn
ny other Points.

THEOREM III.

reflecting Glaſs (or other Surface) *Fig.* 13.
ſpherical, either concave or con-
nd the Object E be far diſtant
he Glaſs; its Image will, to the
laced in the Axis AB of Radiation,
at C the Middle of the Semidia-
AB of the Glaſs. But if the Ob- *Fig.* 14,
be not far diſtant, then its Image 15, 16.
o the Eye in the Axis, appear at
Place C, where (BC : CA : : BE :
hat is, where) the Diſtance of the
from B the Vertex of the Glaſs will
its Diſtance from A the Center of
laſs, as the Diſtance of the Object
he ſaid Vertex is to its Diſtance
he ſaid Center.

I have choſen to comprehend all relates to reflecting ſpherical Glaſſes, reſpect to the Eye placed in the A Radiation, under one Theorem, thereby may the more eaſily be diſc the uniform Manner, whereby Vi perform'd in ſuch Caſes.

In reference to which it is to be ſerv'd, that (although in the Figur longing to this Theorem, the Point placed at ſome conſiderable Diſtance B, for the more diſtinctly perceivi Demonſtrations, yet) D is to be ſu very near to B, becauſe otherwiſe t flex CD of the Incident Ray ED wi affect the Eye placed in AB. For t flex Rays of all ſuch Rays as fall remote from B, will paſs by on on of the Pupil of the Eye, and ſo bute nothing to the Sight of the Add hereto, what has been obſer the *Corollary* of *Theorem* 2, Ch Hence Notice is here taken only o Rays as fall upon the Glaſs very n its Vertex B. Theſe Particulars premis'd, I come now to the Demo tion of this Theorem, in reſpect four principal Caſes following.

Fig. 13. *Caſe* 1. When the Object is far from the Glaſs, ſo that the Rays c from any one Point thereof may b ceiv'd parallel one to the other, a

the Axis AB, as the Ray ED. In this Case the Image will appear, to the Eye, to proceed somewhere in the Axis, at C the middle of the Semidiameter of the Glass. For biſect AB in C, and draw the Line CD, and produce them on the other Side of BD the Glaſs. Because the Point D is look'd to be ſo very near to B, as in a Manner to coincide with it, therefore we may alſo ſuppoſe CD=CB; and by Conſtruction CA=CB, therefore CA=CD, and conſequently the Angle CAD=CDA. But the Angle CAD=EDA the Angle of Incidence of the Ray ED, (for AD is perpendicular to the Surface of the ſpherical Glaſs) wherefore the Angle CDA is equal to the Angle of Incidence of ED, and therefore (by *Cor.* 2, *Theorem* I. *Chap.* I.) C is the Reflex of the Incident ED in the Concave Surface of the Glaſs. And the like may be demonſtrated of the Convex Surface of the Glaſs thus; for, becauſe the Angle EDA=eDO, and CDA=NDO, therefore the Line eD will be the Reflex of the Ray ED, and being produced backwards (or on the other Side of the Glaſs) will touch the Axis alſo at C. And what has been demonſtrated of any Ray ED taken at pleaſure, the ſame holds true of all Rays in the ſame Circumſtances. Wherefore Rays

parallel

parallel to AB, and whose reflex
contribute to the Sight of the Ima
they fall upon a Concave Glass,
their Reflexion will concur at C
from thence diverging again, will
the Image visible at C, to the Eye
somewhere in AB. And the reflex
of such parallel Rays, as fall u
Convex Glass, do diverge from C
consequently render the Image vis
C, to the Eye plac'd in AB produc
the convex Side of the Glass, that
eD be the incident parallel Ray, N
be the Convex, which will form
mage at C. Q. E. D.

Corol. Hence, and from *Corol.* 2.
rem I. of this Chapter, it follows,
Rays diverging from C and reflect
a Concave Surface, or converging
and reflected by a Convex Surface
parallel to AB; that is, if CD be t
cident, DE is the Reflex; or if
the Incident, De is the Reflex.

Fig. 14. *Case* 2. If the Object E be not f
tant from BD the Glass, and so the
proceeding from any single Point t
are (not parallel, but) diverging
the Image will, to the Eye in the
appear at that Place C, where BC :
BE : EA. And this shall be demo
ted first in respect of Concave G
and when the Distance of the Ob

r than half the Semidiameter of the
For let the Point C be ſo taken in
s is requir'd; and draw AD and
Becauſe (for the Reaſon above-
n'd) ED=EB, and CD=CB,
re by Conſtruction CD (=CB)
: DE(=BE) : EA. Wherefore al-
ly CD : DE : : CA : EA. And ſo
mp. 3. *Euclid.* 6.) the Angle ADE
C But ADE is the Angle of In-
e of the Ray ED; therefore (by
2. *Theorem* I. of this Chapter) DC
Reflex of the Ray ED. And ED
aken at Pleaſure, it follows that
cus of all other Rays diverging
will, after their Reflection by a
e Glaſs, be at C, and conſequent-
Image of the Object will appear
to the Eye placed in the Axis
oduced, or) EB. Q. E. D.

1. Hence C will be the Focus of
D converging to E in the like Cir-
nces, and reflected by a ſpherical
Reflecter.

2. Hence and from *Cor.* 2. *Theo-*
f this Chapter, if CD be the Inci-
DE will be the Reflex; and if ND
Incident, De will be the Re-

3. And the ſame holds true, when *Fig.* 15.
ſtance of the Object is leſs than
e Semidiameter of the Glaſs. For
in

in AB extended, let C be taken
ing to the Prop. be requir'd;
AD and CD be drawn and pro
as alſo let ER be drawn parallel
The Arch BD being as Nothing,
fore ED=EB, and CD=CB.
ED : EA : : CD : CA; and ſo (
ſon of DC being parallel to ER
EA : : ER : EA. Wherefore ER
and the Angle ERD=EDR. E
is the Angle of Incidence of E
ERD=NDA its Altern, theref
Cor. 2. *Theorem* I. of this Chapt
is the Reflex of the Incident ED
ED being taken at pleaſure, it is
that all Rays proceeding from E,
flected by a concave Glaſs, being
ed backwards, will concur in
conſequently the Image of the Ob
appear at C, to the Eye placed in
is. Q. E. D.

Corol. 1. Hence if CD be the
on a ſpherical Convex Reflecter,
be the Reflex, and ſo the Image
pear at E.

Cor. 2. Hence and from *Cor.* 2
I. of this Chapter, if ND be the
DE will be the Reflex; and if
Incident, DC will be the Reflex

Fig. 16. *Caſe* 4. The like holds good
of convex Glaſſes. For in AE
taken according to the Propo

d in this *Theorem*, or (which comes
e ſame) ſo as that AC : CB :: AE :
and let CD be drawn and extended;
parallel thereto let ER be drawn,
meets in R with AD produced. The
BD being as Nothing, therefore
CB, and ED=EB. And therefore
CD :: AE : ED. But becauſe the
gles ACD and AER are equiangu-
herefore AC : CD :: AE : ER; and
: ED :: AE : ER; and therefore
ED. And conſequently the Angle
, or its Altern, and ſo Equal)
=EDR the Angle of Incidence of
Wherefore DN is the Reflex of the
nt ED. But ED being taken at
re, it is evident, that all the Rays
ing from E, and entring the Eye
l in the Axis, after they are reflec-
the convex Surface BD, will di-
from the Focus C; and conſe-
y the Image of the Object E will
r at C to the Eye ſo placed. Q.

ll. Hence if ND be the Incident,
ill be the Reflex; and if in a con-
Reflecter *e*D be the Incident, DC will
Reflex; or if CD be the Incident,
l be the Reflex.

l. From this Theorem is deduced
y of finding the Focus of paral-

lel

lel or other Rays, falling upon
cal Surfaces, after their Reflection

The Focus C of ſuch paralle
ED after Reflection is found by
ing the Semidiameter AB of the
in C.

And the Focus C of other Rays
upon a ſpherical Surface, is foun
Reflexion, by taking C ſo, as th
AC : : BE : AE.

And thus it has been ſhewn, b
Way of Theorem and Problem,
the Image of an Object will appe
ny ſpherical Glaſs, (or other like
ing Plane) to the Eye placed in t
of Radiation. And this is ſuffi
our preſent Purpoſe, not only
moſt optical Inſtruments are mad
this manner, but alſo becauſe the
only in this Situation of the Eye,
ſo lively and diſtinct, as to dese
Name of an Image. Hitherto both
and Image have been denoted only
gle Points, as being ſufficient to il
what has been afore deliver'd.
proceed to ſpeak of Object and
conſider'd as Lines or Surfaces.

THEOREM II.

Image CT of a radiant plain Sur- *Fig.* 17
, made by a ſpherical Glaſs BD, & 18.
plain Surface.

A the Center of the Glaſs, draw
endicular to FE, and meeting the
its Vertex B. And (according
em III, of this Chapter) let C be
ge of the Point E, through which
rallel to FE the Plane CT. In
ne CT will appear the Image of
e FE made by the Glaſs.

let AF be the Ray, drawn from
r Point F of the radiant Plane
e Center of the Glaſs, and meet-
the Glaſs in D, and the Plane
. And by Conſtruction let CD
eflex of the Incident ED; and
eflex of the Incident FB be BH,
the Ray FD in H. Now, be-
ſuppoſe in the preſent Caſe the
AE to be ſmall, therefore the
will be alſo ſmall, and in effect
than a little or ſhort right Line.
Circumference of the Circle de-
with the Diameter BF will paſs
the Points D and E, becauſe the
EB and FDB are right. Whence
es BFD and BED being in the
gment, are equal. And when
s at A are Verticals or the ſame,

it will follow that FBA=EDA. (becauſe by *Theorem* 1, of this C ABH=FBA, and ADC=EDA, fore ABH=EDC. Whence the gles ABH and ADC are equi and ſo ſimilar. Wherefore AB AD : AC; and whereas AB=A fore AH=AC. But by reaſon Smalneſs of the Angle CAT, look upon AT=AC, and there may alſo look on AH=AT. But Image of the Point F in the radia whence the ſaid Image is plac Plane CT. And the ſame may the ſame Way as to the Image o ther Point in the Plane FE. W the Image of the Plane FE is p the Plane CT. Q. E. D.

If the radiant Plane be far diſt the Images of ſeveral radian will be (by *Theorem* III, of this in the Mid-points of the ſevera ameters of the Glaſs tending to radiant Points, that is, the Image ant Plane far diſtant will have th of a ſpherical Surface concentri Glaſs. But becauſe the ſaid Radi cording to our Suppoſition) ſee ſmall Angle, that ſmall Portion rical Surface, which is taken up b Image, will ſcarcely differ fro Surface, to which AB is perp

e this Theorem holds good in all
for the Demonſtration will ſerve
other Caſe, as well as thoſe two
ated in *Fig.* 17 and 18.

If the Angle EAF be too great,
ht Line AH will be ſenſibly leſs
T. Whence an Object being ſeen
ſuch an Angle, its Image, if made
ncave Glaſs, will appear concave;
by a convex Glaſs, will appear

THEOREM V.

Object HF, and its Image *bf*, are* *Fig.* 19
der equal Angles HBF and *b*B*f* & 20.
Vertex B of the reflecting Curve
ther convex (as *Fig.* 19) or con-
Fig. 20.)

f. Draw AE perpendicular to
ich by *Theorem* IV, of this Chap-
conſequently be perpendicular to
meeting the reflecting Curve BD
raw alſo BF, BH, B*f*, B*b*, and
h laſt will paſs through the Cen-
foraſmuch as according to our
on the Image of every Point is
the Axis of Radiation. In like
he Points H, A, and *b* will be
ne Axis. Now by *Theorem* III,
Chapter, BE : EA :: BC : CA.
e BE : BC :: EA : CA. But (by
the equiangular Triangles AEF
EA : CA :: EF : C*f*. Where-

fore BE : EF : : BC : *Cf*. And
the Angles BEF and BC*f* are right
fore the Angle EBF = CB*f*. And
Manner the Angle EBH = CB*h*. W
HBF = *h*BF. Q. E. D

Corol. The Object FH (if it be
and its Image *fh*, are one to the
their Distances from the Vertex,
BC. But if the Object be a Surfac
its Image will be one to the othe
Squares of their Distances a
Whence, the Distance of the O
the Image from the Reflecter bei
thereby is given also the Ratio or
on of the Image to the Object. A
Manner may be found the last
an Object, when it is propagate
ny Reflecters. Lastly, it is
that the same will hold true i
spects, if *fh* be the Object, an
Image.

Schol. By what has been said
fie to account for and illustrate
culars relating to Images made
rical Reflecters. Of which the
markable are these that follow

1. If the Reflecter be con
the Distance BC of the Object
the Vertex of the Reflecter, b
half the Semidiameter, then Its
will appear beyond or behind
ter, and in a right Position, (as

er, namely the upper Part of the will appear the upper Part of the the lower Part of that, the low- of this. *&c.)* and as the Object is nearer or farther from the Reflec- thin the Distance of the Semidia- ſo will the Image be nearer or far- m the ſame, and appear leſs or agnify'd. All which Particulars illuſtrated by comparing together and 21 and 22. Namely in *Fig.* Object *hf* being placed a fourth of the Semidiameter BA from cting Concave BD, and in *Fig.* Object *hf* being placed a third of the Semidiameter BA from the ncave Reflecter, it is evident that rmer the Image HF (as well as ct *hf*) is nearer to the Reflecter, proportionably, than in the lat- nd in both Figures the Image ap- a due Poſition, *viz.* F is on the e of the Axis AB as *f*, and H as therefore if *f* be the upper and wer Part of the Object, F will iſe the upper, and H the lower he Image. Laſtly, it is evident, oth Figures *hf* and HF, *i. e.* the nd its Image, are one to the o her, Diſtances from the Vertex BC and in *Fig.* 19, BC : BE : : 1 : 3 : : And in *Fig.* 21. BC : BE : : 1 :

2. It is evident from comparin
ther *Fig.* 19 and 22, that if the
ter be convex, and ſo HF the
and *hf* its Image, then likewiſe th
will appear in its due Poſition;
as the Object is brought neare
Reflecter, ſo will the Image com
But in this a convex Reflecter diff
a concave, *viz.* that (whereas i
cave, the nearer the Object *hf*
Vertex B, the leſs will the Image
pear, as is evident from compa
has been ſaid, *Fig.* 19 and 21,
contrary) in a convex, the neare
ject HF is to B the Vertex, th
will the Image *hf* appear, as i
from comparing *Fig.* 19 and 22.
ſo remarkable, that be the Obje
moved never ſo far diſtant fro
flecting Convex beyond the half
meter, yet its Image will fix it ſ
half Semidiameter.

3. If the Reflecter be a Conc
the Object HF be more diſtant
than the length of the Semidiam
(as *Fig.* 20 and 23) then the Ima
be more diſtant from it than the
diameter, and yet leſs than the wh
diameter, that is, it will appear i
between the Object and the Refle
alſo it will appear in an inverted
that is, the Point F and its Image

on contrary Sides of AB the Axis, as
he Point H and its Image *h*; (and
any other Point of the Object, and
age,) whence if F be the upper or
Hand Part of the Object, *f* will be
wer or left hand Part of the Image,
Likewise it is evident from compa-
g. 20 and 23, that in this Cafe, as
bject HF is remov'd farther from
flecter, ſo its Image *hf* will come
and appear leſs; and on the con-
as the Object comes nearer, the
will go off farther, (and appear
) 'till they meet one the other at
nter A. In like Manner if *fh* be
ject, then HF will be the Image;
ections whereof are eaſie to be ac-
t for from what has been ſaid.
ſo obſervable that if the Object HF
ov'd ſo far from the Reflecter, as
e Rays of it falling near the Vertex
e Reflecter, may be eſteem'd pa-
o AB, then (according to the for-
art of *Theorem* III, of this Chapter)
age *hf* will appear at the Mid point
Semidiameter of the Reflecter.
f the Sun be the ſaid far diſtant
, then the Place of the Image (or
oint of the Semidiameter) will be
ming-point of the Reflecter, if it be
rably larger than the Image of the
But if a lucid Body, as a Candle,

be plac'd in the ſaid Mid-point
Semidiameter, it will caſt a Ligh
great Diſtance. Add as in this Ca
thing but Light will appear on th
face of the Reflecter; ſo when oth
jects are placed at the half Semidia
nothing but their Colour will app
the Surface of the concave Reflecte

4. It is obſervable, that ſuppoſi
Object to ſtand ſtill, and the
change its Place, the ſame Alteratio
be produc'd in reſpect to the Imag
the Object had chang'd its Place in
reſpondent Manner.

5. It is evident from *Fig.* 19
that a reflecting Concave magnifie
a reflecting Convex leſſens an
Namely, if in the ſaid Figures B
conſider'd as a Concave, then *bf*
the Object, and HF the Image, w
in all the ſaid Figures much large
bf the Object. But if DBG be co
as a Convex, then HF will be t
ject, and *bf* the Image, which is
the ſaid Figures much leſs than H

6 It is obſervable, that cylindri
flecters, whether concave or conve
render the Image deform'd. For
linder being generated by the Mo
a right Line round two parallel
hence ſuch a Reflecter muſt partak
ly of the Properties of a plain Re

y of a ſpherical. Of that, in re-
of its right Lines, in Longitude; of
in reſpect of its circular Lines,
nd-ways. Hence the Longitude or
ht of an Object being repreſented
, as by a reflecting plain Surface; but
atitude magnify'd or diminiſhed, ac-
ing as the Reflecter is a Concave, or
ex; the Image appears diſproporti-
le. But on the ſame Account, if a
re be drawn proportionably bigger
adth than in Length, its Image will
preſented by a reflecting Cylinder in
t Dimenſions. Which is illuſtrated
4, 25, 26. Namely, in *Fig.* 24, is
the Picture in its juſt Dimenſi-
and divided into Parallelograms.
g. 25 are drawn as many circular
as there are (in *Fig.* 24) horizon-
nes parallel one to the other, *viz.*
l the ſaid circular Lines being con-
cal to the circular Baſis of the Cy-
C; and diſtinguiſh'd proportiona-
right Lines drawn a-croſs them in-
many Diviſions, as the Picture (*Fig.*
divided into Parallelograms: So
of the Picture, as is contain'd in each
elogram being to be drawn in each
foremention'd Diviſions anſwering
nto. Which being done, and the
g Cylinder being plac'd on C, the
of the Picture will appear in the

ſaid Cylinder according to its juſt Di
ſions, but leſs than the Picture, if th
flecting Cylinder be convex; which i
nerally us'd in this Caſe, and is repre
ed *Fig.* 26.

Laſtly, It remains only to obſerve,
by the foregoing Theorems may be f
ſuch a Poſition of the Radiant, a
the ſaid Radiant may have a given
portion to its Image made by the R
ter. For Inſtance, let BD (*Fig.* 2
the Reflecter, and the given Propo
be that of BA to AM. If the Ob
a Line, biſect BM in E, and by *T*
III, of this Chapter find the Focu
ſwering to the Focus E, which ſ
the Place of the Radiant. For
BC : CA : : BE : EA ; therefore
BC + CA : : BE : BE + EA ; that
cauſe BE = EM) BC : BA : : BE
and therefore BC : BE : : BA : AM
by the *Corollary* of this *Theorem* V
to BE, as the radiant Line in C i
Image in E ; therefore the Radia
in C, is to its Image made by BD
as BA to AM, *i. e.* in the Propor
ven. If the Radiant be a Surfac
inſtead of AM, a right Line muſt b
to which BA has a ſubdulicate R
that which it has to AM. And
ſufficient to our Deſign, concern
toptricks or reflex Viſion.

C

CHAP. IV.

Of Dioptricks, or Refracted *Vision.*

THEOREM I.

If a Ray AB of Light be refracted by the plain Surface BD of a Medium of different Thickness, the right Sine of ABK the Angle of Incidence will, in respect of the same different Mediums, always have the same (Ratio or) Proportion to the right Sine of GBF the Angle of Refraction, however the Ray AB be inclin'd to the Surface BD. *Fig.* 28, 29.

Demonst. Let BG be the Refract of the Ray AB, whether AB passes out of a rarer Medium into a thicker, (as *Fig.* 28.) or out of a thicker into a thinner, (*Fig.* 29.) Now as the Light proceeding from A, and contain'd between AB and AD, will be diffus'd within the Medium DK all over BD; so after it has entred the new Medium DG, it will at the Line BG diffuse it self all over GN: Whereas had the same Medium DK been con-

continued on each Side of BD, the L
at the Length of BC (=BG) would h
been diffus'd all over CE. But in both
ſes (namely, whether the Medium DC
different from or the ſame with the
dium DK) FN=ME, and conſeque
both the ſaid Lines may be look'd u
as not at all depending on the (*) Fa
ties of the Medium's, and ſo of no C
deration in the preſent Caſe. Where
GF (*i. e.* GN—FN) and CM (*i. e.*
—ME) are to be look'd on, as the g
ine Effects of the Facilities of the two
diums DG and (DC, or which is
ſame) DK; that is, GF is to CM, a
Facility of the Medium DG is to the
cility of the Medium DC or DK.
CM is the right Sine of the A
(CBM=) ABK, which is the Ang
Incidence of the Ray AB; and acco
to the ſame Semidiameter, GF is
right Sine of GBF the Angle of Ref
on of the ſame Ray AB. Where
ſince the Angle DAB (=ABK) is
at pleaſure, the like will hold good
ny other Angle; that is, in the Re
tion of any Ray AB, let it fall h
will on the Plain whereby it is refra
the Sine of the Angle of Incidence

(*) This Word is us'd to denote the ſeveral
which different Mediums have to diffuſe the Light,

ine of the Angle of Refraction, as Facility of the Medium DK is to acility of the Medium DG, whereby Ray AB is refracted into BG. But in eſt of the ſame Mediums, the Thick- s of the Mediums, and the Facilities ce ariſing, and their Ratio will abide ame. Wherefore in the Refraction ny Ray, the Sine of the Angle of In- nce always keeps, with reſpect to the Mediums, the ſame Ratio or Pro- ion to the Sine of the Angle of Re- ion. Q. E. D.

rol. 1. If BG be the refracted Ray of ncident AB, the reſt remaining the , BA will be the refracted Ray of the dent GB.

r. 2. The refracted Ray BG of any dent AB is rightly aſſign'd, when the CM (of the Angle CBM=) ABK the le of Incidence, is to the Sine GF of Angle GBF contain'd under the aſ- d Ray BG, and the right Line BM endicular to the refracting Surface as the Facilities of the given Medi- are one to the other. Namely, this ollary is no other than the Converſe his firſt Theorem.

chol. 1. In a Ray's paſſing out of a ker Medium into a thinner (as *Fig.*) be ſo inclin'd to the refracting Sur- , that GF (which is to CM, as the Faci-

Facilities of the given Mediums) doe
ceed the Semidiameter BC; then th
cident Ray AB will have no refra
Ray, (after the Manner of an impo
Case in a geometrical Problem;)
which ought to be the refracted Ra
entring the thinner Medium DG, bu
ing reflected by its Surface, accordi
Theorem I. of Catoptricks. And this
is illustrated *Fig.* 30.

Schol. 2. When a Ray of Light
out of the Air into Glass, it is obs
that the Sine of the Angle of Incide
to the Sine of the Angle of Refra
as 3 to 2 in whole Numbers; and in
sing from the Air into Water, I is
(by which Symbols henceforward i
noted the Ratio of the Sine of the An
Incidence to the Sine of the refracte
gle, which measures the Refraction) a
3. And on the other hand, in the pass
the Ray out of Glass into Air, I w
to R as 2 to 3; and in its passing o
Water into the Air, I will be to R a
4. Whence the Reason is manifest,
a Ray passing out of Glass, and falli
the Surface of the Air so, as to ma
Angle of Incidence more than abo
Degrees, will not enter into the Ai
is reflected by its Surface. Namely
cause 2 : 3 :: 42 : 63; that is, th
tio sesquialtera (as it is called) of

nd consequently the right Line GF,
h is to denote the ſaid Ratio of the
of 42 Degrees muſt exceed the Ray,
being the Sine only of 60 Degrees.
hich Caſe the Ray is reflected accord-
Schol. 1.

ol. 3. It is demonſtrable that if
diverging (*Fig.* 30 and 31) from
d refracted by the Surface BD, do
ge from C, the Mediums being tranſ-
but the ſame intermediate Surface
maining, the Rays converging to
ll after Refraction converge to C.
efore what ſhall be demonſtrated
following Theorems concerning di-
g Rays, may alſo be apply'd to
rging Rays.

THEOREM II.

n Object E be ſeen through diffe- *Fig.* 30,
ediums BDA and BDO, the Image & 31.
ſaid Object will appear, at that
C in the right Line EB (perpendi-
o the plain Surface BD of the re-
g Medium, whether thicker or
r) where CB : EB : : I : R, that is,
the Diſtance of the Image from the
Surface of the refracting Medium
e Diſtance of the Object from the
rface, as the Sine of the Angle of
Inci-

Incidence is to the Sine of the An
Refraction.

Demonst. Let ED be any Ray o
Object E falling upon the Surfac
Thro' D draw AO parallel to EB.
C and D, and produce CD to F.
Rays only being here (as well as
foregoing Chapter) consider'd, th
near EB the Axis of Radiation,
wherein the Eye is suppos'd to be
it follows (for the Reasons mentio
the foregoing Chapter) that we
look on ED=EB, and CD=CB. W
fore CD : ED :: I : R. But CD
ED, as the Sine of the Angle BED
Equal EDA, to the Sine of the
BCD or its Equal ODF. Therefo
Sine of the Angle EDA is to the S
the Angle ODF, as I to R. But E
the Angle of Incidence of the Ray
therefore ODF is the refracted Ang
swering thereto; that is, DF is t
fracted Ray of the incident Ray ED
the same may be shewn of any
Ray diverging from E. Wherefo
will be the Focus of all Rays div
from E and refracted by BD, that
Image of the Object E will appea
Refraction at C. Q. E. D.

1. Hence and from *Cor.* 1, *Theo-* of this Chapter it follows that *paribus*) Rays in the Medium onverging to C, will after Refrac- nverge to E. And if the Medi- transpos'd, then from hence and , *Theorem* I, of this Chapter it fol- that Rays in the Medium BDO ging to E, will, after Refraction, ge to C; and that parallel Rays after Refraction at the plain Sur- any Medium, ſtill continue pa-

2. If BDA be Water, and BDO en CB : EB : : 3 : 4. Whence it to paſs, that Water appears not deep, as it really is, by a fourth But if BDA be Glaſs, then CB : : 3.

3. From this Theorem is deduced thod of finding the Focus C of efracted by a plain Surface, *viz.* wing EB perpendicular to the re- Surface DD, and therein taking s that CB : : EB : : I : R.

THEOREM III.

e Surface BD of the Medium of *Fig.* 32, t Thickneſs be ſpherical, and the 33. E be very far diſtant, then its I- will, to the Eye plac'd in AB the

Axis of Radiation, and parallel
incident Rays, (as ED,) appear
Place C of the Axis AB prod
there is Occaſion, where BC : CA
Fig. 34, 35, 36, 37. But if the Object E be not very
ſtant, then its Image will, to
placed in AB produced if there be
on, appear in that Place C of AB
the Ratio compounded of the R
EA to AC and of CB to BE is
the Ratio of I to R. (*)

Demonſt. The firſt Part of thi
rem (as it relates to Objects at
diſtance from the refracting Surfac
that the Rays proceeding from ea
Point therein, may, conſider'd b
ſelves, be eſteem'd parallel one t
ther, ſo it) is thus demonſtrated.
denote any Ray falling on B
draw AD and CD, and produc
The Sine of the Angle BAD or i
EDO is to the Sine of the Ang
(or its Complement to two Right
to AC, or (by reaſon of the Sma
the Arch BD) as CB to AC, tha

(*) I have comprehended all here relating t
ſpherical Surfaces under this one Theorem
Reaſon, as I comprehended all relating to ref
rical Surfaces under one Theorem; and alſo t
mony between reflected and refracted Viſion
ter appear.

ruction, as I to R. But EDO is the
of Incidence of the Ray ED, there-
DC is its refracted Ray. In like
may it be ſhewn, that any o-
f the parallel Rays proceeding
any Point of) the Object E after
tion will paſs through C. There-
will be the Focus of all ſuch Rays,
ch is the ſame, at C will appear
Caſe the Image of an Object very
ant. Q. E. D.

latter Part of this Theorem is thus
ſtrated. Let ED be the incident
draw AD and DC, and through
CH parallel to AD, and meeting
H. By reaſon of the ſuppos'd
ſs of the Arch BD, we may look
=CB, and ED=EB. Wherefore
io of I to R is equal to the (Ratio
to AC and the Ratio of CD and
ether; that is, by reaſon of AD
arallel to CH, is equal to the Ra-
CD to ED and the Ratio of ED
ogether, or which is the ſame, to
tio of CD to DH. But CD is to
the Sine of the Angle (DHC, or
plement to two right, *viz.* ADH=)
to the Sine of the Angle (DCH
C, or in ſome Caſes to the Sine
omplement of ADC to two Right.
ore EDO being the Angle of In-
of the Ray ED, it follows that I

is to R, as the Sine of the Angle
cidence of ED to the Sine of th
ADC. Wherefore (by *Theorem* I
Chapter) DC is the refracted Ra
incident Ray ED. And ED bein
at pleaſure, it is evident that C
the Focus of all Rays proceeding
after their aforeſaid Refraction
conſequently at C will appear th
in ſuch Caſes. Q. E. D.

From the firſt Part of this T
and the Corollary of *Theorem* I,
veral Corollaries; among wh
ſhall here take notice of theſe fo
viz.

Fig. 32. *Corol.* 1. Rays ED in the Air
to AB the Axis, after their Refra
the ſpherical convex Surface DB
do converge to that Point, C, wh
ſtance BC from the Vertex B is
3 AB, three Semidiameters of the

Corol. 2. Rays ED in Glaſs pa
the Axis, after their Refraction
ſpherical convex Surface of the
diverge from C that Point, wh
ſtance BC from the Vertex, is
(3 AB or) the Diameter of the Sp
Fig. 33.

Corol. 3. Rays *e*D in the Air pa
the Axis, after Refraction at
concave Surface of the Glaſs,
verge from C a Point three

3AB distant from the Vertex: as
32.

l. 4. Rays *e*D in Glaſs parallel to
xis, after Refraction at a ſpherical
ve Surface of Air, do converge to
oint diſtant from the Vertex B the
eter of the Sphere (*): that is, BC=
as *Fig.* 33.

l. 5. In all the four Caſes, which
e Converſe of thoſe mention'd in
four Corollaries, the Rays after Re-
n will become parallel to (the Ax-
to) a right Line drawn through
diant Point and the Center of the
.

o the latter Part of this Theorem,
are eight principal Caſes relating
, which may all be demonſtrated
he ſame Manner, as thoſe four of
which are here mention'd, as be-
ficient to our Purpoſe; and are ex-
by *Fig.* 34, 35, 36, 37. Of
Fig. 34 and 35 relate to Rays pro-
g out of a thinner into a thicker
m, the other two, *Fig.* 36 and 37,
s proceeding out of a thicker Me-
into a thinner. And in both Caſes
rmer Figure relates to concave Re-
nts, the latter to convex. And in
, is repreſented the Caſe, wherein
cident Ray ED is not refracted,
flected into DC, according to what

was obſerv'd above, in *Schol.* 1 an
Theorem I, of this Chapter.

Schol. 1. From the former Part
Theorem is deducible the Way, h
find the Focus of parallel Rays falli
ſpherical Glaſſes, *viz.* by taking
AB ſo, as that BC : CA : : I : R. An
the latter Part of this Theorem is
ced the Way, how to find the Fo
diverging Rays falling upon a ſp
Glaſs, *viz.* by taking C in AB ſo,
the Ratio compounded of the Ra
EA to AC, and CB to BE may b
to the Ratio of I to R.

Schol. 2. If the Mediums prop
Air and Glaſs, and the Ray falls out
Air upon the Glaſs (as *Fig.* 34 and 3
the Focus C may be very eaſily
namely, by making $\frac{1}{3}$EA : AB : : E
For by tripling the Antecedents
(AB=) AD : : 3EC : (BC=) DC
EA is to AD, as the Sine of the
EDO is to the Sine of the Angle
and 3EC is to DC, as the Triple
Sine of the Angle EDC or HDC i
Sine of the Angle DEA. Wherefo
Sine of the Angle EDO is the Tr
the Sine of the Angle HDC, and
quently is to the Sine of the Angle
as 3 to 2. Therefore DC, or De
the ſame direct Poſition, is the Re
the Ray ED falling out of the Ai
Glaſs.

n the Ray paſſes from Glaſs into
r (as *Fig.* 36 and 37,) then the
may be found, according to the
Principles, by taking C ſo, as that
AB : : EC : BC.

. 3. By what has been ſaid un- *Fig.* 36.
foregoing Theorems, there being
one Focus of a given Lens, the
may be found. Where it is to be
that by a *Lens* is denoted a (dia-
s or) tranſparent Body (ſuch as
les, Burning-glaſſes, *&c.*) of a
t Thickneſs from the ambient
arent, terminated by two ſurfaces,
re either both ſpherical, or the
ain and the other ſpherical. A
ine perpendicular to both its Sur-
call'd the *Axis* of the Lens. The
where the Axis interſects the Sur-
are call'd the *Vertex's*; one the
of *Incidence* or *Immerſion*, the o-
Emerſion, accordingly as it is in
rface, on which the Rays firſt fall,
ugh which they paſs out again.
ickneſs of the Lens, is the Diſtance
n its Vertex's. To which add,
ere more than one Lens is us'd,
ich is next to the Object, is thence
e *Object-Lens*; and that which is
the Eye, is thence diſtinguiſh'd
Name of the *Eye-Lens*.

Fig. 38. Now if the Focus before Incide
Immerſion be given, and the Focu
Immerſion be ſought; let there be
firſt the Focus of the Rays after th
fraction at that Surface of the L
Fig. 38. which they firſt fall; and this
by *Theorem* II, if the Surface of t
be plain; but if it be ſpherical b
rem III, namely, by the former
it, if the Rays are parallel; and
latter Part, if diverging or (*) c
ing. Having thus found the Focu
Rays after Refraction at the firſt
of the Lens, (that is, while th
through the Lens, whence the
cus is call'd the *Focus of Tranſit*
like Manner may be found the
after Refraction at the ſecond Su
the Lens, or rather at that Su
the ambient Medium, which is
ous to this ſecond Surface; tha
Focus after Emerſion.

If there be more than one L
pos'd, after the like Manner is
peration to be carried on in re
each.

(*) Namely, according to what has been
converging Rays in *Schol* 3. of *Theorem* I. c
ter.

n like Manner, there being given the
us of one given Lens or more, there-
may be found the Focus before Inci-
ce, that is, the optical Inſtrument
g given, thereby may be determin'd
Diſtance of the viſible Object.

THEOREM IV.

The Image CT of a radiant plain Sur- *Fig.* 39.
EF, made by a refracting Surface
whether plain or ſpherical, will be
a plain Surface.

Demonſt. Let A be the Center of BD
refracting Surface, and AE be perpen-
lar to EF, meeting with BD in B.
C be the Focus of the Rays diverging
E and refracted by BD. From F,
Point taken at Pleaſure in the radi-
Plain draw, FA meeting with BD in
and the Plain CT in T. The Ray
will be the Refract of ED; and let
be the Refract of FB. Becauſe the
le EAF is ſuppos'd to be ſmall, there-
the Arch BD may be look'd on
ſhort right Line; and a Circle drawn
the Diameter BF will paſs through
Points D and E, becauſe of the right
les BEF and BDF. Whence the An-
EBF and EDF, (*viz.* the Angles of
ence of the Rays FB and ED) being
e ſame Segment, are equal; and con-
ſequently

ſequently their refracted Angles ABH
ADC will be equal. Therefore by
ſon of the equal Angles at A, the Tr
gles BAH and DAC are equiangular.
BA is to AH as DA to AC; and, wh
as BA=DA, therefore AH=AC.
becauſe the Angle EAF or TAC is
ſmall, therefore (in a manner) AT=
and therefore AH may be look'd on
qual to AC, that is, the Focus of th
diant Point F, is placed as near as
be in the Plane CT. And the Poi
being taken at pleaſure, it holds goo
ſo in all other Points of the Plane EF,
their Focus's, are in the Plane CT; th
that the Image of the radiant Plan
will in this Caſe be alſo a plain Su
Q. E. D.

Corol. 1. Hence it follows, that t
mage of the radiant Plane EF, wh
the Axis of the Lens is perpendi
will alſo be a Plane parallel to th
mer. For the Image CT is to be
ceived as the plain Surface, which
forth Rays upon the ſecond Surface
Lens.

Corol. 2. If the Angle EAF b
great, ſo that AT is ſo much longe
AC, as not to be capable of bei
ſteem'd equal thereto, then the Ima
the Plane EF will be concave towar
if BD be the convex refracting Surf

a

cker Medium, or the concave refrac-
Surface of a thinner Medium; and
e contrary.

THEOREM V.

e Angle *fGh*, under which the Image *Fig.* 38.
ppears from G the Vertex of Emerſi-
n a Lens, is equal to the Angle FBH
which the Object appears from B
ertex of Incidence in a Lens.

monſt. Let GB be the Axis of the
and perpendicular to the viſible
or Object FEH at E, and conſe-
y (by *Corol.* 1, *Theorem* IV,) per-
ular to its Image *fCh*. Draw BF,
f, and G*h*. Of the innumerable
lowing from the Point F, and af-
fraction at the Lens again united
e Image of the Point F, let two
ſen, whereof one FB meets the
t B its Vertex of Incidence, and
there refracted tends to *p* the Fo-
Tranſition in reſpect of F; and be-
ain refracted at L tends towards *f*.
her Ray FD being firſt refracted at
ds directly to *p*, 'till paſſing out of
ns at G the Vertex of Emerſion, it
refracted again, and thence pro-
irectly to *f*. The Angle of Inci-
of the Ray DG is DGB, and its
is CG*f*; and the Angle of Inci-
dence

dence of the Ray LB, which would (by the *Corollary* of *Theorem* I,) be refr into BF, is the Angle LBG, and its fract EBF. Now becauſe the right *p*B and *p*G are to be eſteem'd equal, reaſon that the Thickneſs of the L not to be conſider'd,) therefore the of the Angles DBG and LBG being portional to the ſaid Lines *p*B an and conſequently the Angles them DGB and LBG, and therefore the fracted Angles CG*f* and EBF will qual. In like manner it may be p that the Angles CG*b* and EBH are Therefore the Angles FBH and *f*C equal. (And the like may be ſhewn other Caſe.) Q. E. D.

Corol. Hence it follows, that a r Line is to its Image made by a Le the Diſtance of That from the Ve Incidence is to the Diſtance of Th the Vertex of Emerſion, or (the neſs of the Lens being not regard their Diſtances from the Lens. But Radiant be a Surface (the homo Lines will keep in the ſame Ratio, b ſaid Surface will be to its Image (in Ratio duplicated, or) as the Squ their Diſtances from the Lens. may be determin'd the Ratio, wh laſt Image, (*i. e.* that which is i ately ſeen by the Eye) made by th

ne Lens or more, has to the visible
ct, whose Image it is.

bol. From what has been said, may
ccounted for and illustrated the Affec-
of (or several Particulars relating
refracting Surfaces, whether plain
pherical. Among which Affections
more remarkable are these that fol-
viz.

If the prime Radiant or Object be *Fig.* 40.
istant, and the Surface PCQ of the
be plain, the Surface PDQ be of
and convex, and the ambient Bo-
the Air, then (by *Corol.* 4, *Theorem*
Db will be equal to the Diameter of
phere PDQ; (for by *Axiom* 1, the
parallel to BC pass through the
Surface PCQ, whereon they fall
ndicularly, without being refrac-
and consequently by *Theorem* IV
fbe will be the Image of the far
t Object, from which proceed the
FBE; and therefore the said Image
ppear in an inverted Situation with
to its Object: Namely, the Point
he Object, and the correspondent
f of the Image, will be on different
of B*p* the Axis; and so of the o-
Points B and *b*, E and *e*. And
ore, if F be the lower Part of the
, *f* will be the upper Part of the
; and so as to the rest agreeably.

2. But

Fig. 41. 2. But if the Surface PCQ be co
and PDQ plain, then (by *Corol.* 1,
rem II, and *Corol.* 1, *Theorem* III of
tricks) C*b* will be equal to the Dia
of the Sphere PCQ, and a third P
the Thickneſs of the Lens over. O
lecting the Thickneſs of the Lens,
uſual in the Object Lens of a
cope,) in this as well as the former
and conſequently in general of any
convex Lens, it may be ſaid, that
mage of an Object far diſtant, ma
ſuch a Lens, will be equal to the D
ter of the Sphere.

Fig. 42. 3. If the Lens PQ be of Glaſs, a
qually convex on both Sides, the
Corol. 1. *Theorem* III, and *Theorem*
Dioptricks) D*b* will be equal to th
midiameter of either Sphere. Laſt

Fig. 43. a whole Glaſs Sphere, the Image
Object far diſtant will appear at th
ſtance of half the Semidiameter fro
Sphere: For in this Caſe Regard
be had to the Thickneſs of the Len

Of Burning-glaſſes. 4. In all theſe or the like Caſes,
Sun be the far diſtant Radiant or O
and the Lens be conſiderably large
the Sun's Image, in the Place of
mage will be the *Burning-point*:
ny ſuch convex Lens will burn ſt
than a concave Reflecter (*cæteri*

by reaſon more Rays are loſt by the
than by the former.

If in the forementiond Place of the
there be placed a lucid Body it ſelf;
the Image of the ſaid Body will ap-
at a great Diſtance; that is, the
lucid or ſhining Body will caſt a
at a great Diſtance, ſo as that
gs ſo far diſtant may be clearly ſeen
ſaid Light. And this is the Manner,
by a Candle, put into a Dark-Lan-
with a convex Chryſtal, gives a
at ſo great a Diſtance as uſual

If the Radiant or Object be not (as
) far diſtant, but yet is more diſtant
the Lens than is the Place where
appear the Image of a far diſtant
; beſides the Affection aforemen-
(*viz.* that the Image will appear
ed,) there will be in this Caſe alſo
other Affections. Namely, as the
nt comes to the Lens, the Image
from it, and contrariwiſe; 'till
diant being come into the Place,
would appear the Image of a far
Radiant, its own Image becomes
ant. All which Affections may be
'd in a Dark-Chamber, into which
s at ſeveral Diſtances ſend their
hrough a convex Lens or Glaſs.
lace of the Image of each Radiant
may be diſtinctly known, by ob-
ſerving

Of Dark-Lanterns *with* Convex *Glaſſes.*

ſerving *where* the Image of the ſaid
does appear moſt diſtinctly on a Sh
White Paper or the like.

Of a Magick Lantern.

7. And on theſe ſame Principl
pend the Mechaniſm of (what is
a *Magick-Lantern*; namely, w
the Images of little Pictures are rep
ed on a white Wall or the like.

8. If the Radiant be nearer to
vex Lens, than the Place where
appear the Image of a far diſtant
ant or Object, then its Image will
on the ſame Side of the Lens wi
Object it ſelf; and its Place may
termin'd from the given Place of t
diant, as afore. The Image in th
will always appear in the like Si
with the Object, and greater than
if the Object comes to the Lens,
mage will alſo; if that goes fr
Lens, this will alſo; only the Ima
in both Caſes move quicker.

Fig. 44, & 45.

9. If the Lens be concave, th
the reſpective Corollaries of *The*
of Dioptricks) the Image *ebf* ma
plain concave Lens, PQ, will be
the Diameter of the Sphere PD
will be erect or in the like Poſitio
the Object, and on the ſame Si
the Object reckoning from the V
If the Surface PCQ be alſo conca
the Object not far diſtant, it

e determin'd after the like Method,
ding (according to *Theorem* IV of
ricks) the Image of that Point of
bject which is ſituated in the Axis
Lens. In reſpect to which Image,
s what has been already ſaid, it is
obſerv'd, that it will together with
bject come to or from the Lens, but

Further, it is here obſervable, that
foremention'd Theorems depends
echaniſm or Contrivance of the ſe-
optick Inſtruments, made uſe of to
he Eye, or to ſupply the ſeveral
s of our Sight. For, as the Eye is
ral Organ or Inſtrument, deſign'd
End, namely, that on the Bottom
f may be painted the diſtant Ima-
outward Objects; ſo it is manifeſt
e and the ſame Eye can ſee diſtinct-
at a certain Diſtance from the vi-
bject. Could the Eye always place
t ſuch a Diſtance from the Object,
eceſſary to diſtinct Viſion, then
ſing the other Requiſites to Sight
iven, ſuch as a due Degree of Light,
he Eye might always ſee diſtinct-
s true indeed that the Diſtance re-
to diſtinct Viſion is not rigidly de-
d to one ſingle Point, but admits
e Latitude; foraſmuch as the ſame
n ſo alter its Figure according to

of Teleſcopes, Microſcopes, *and* Spectacles.

the various Diſtance given, that i
be look'd on not as one Eye but ſe
But then, ſince this Latitude of Di
is contain'd within certain Bound
yond which the Eye can by no n
Means ſee diſtinctly, hence ariſ
Need of artificial Inſtruments. An
Lens, or ſpherical reflecting Glaſs,
cient to remedy this; foraſmuch as
by the Image of ſuch an Object,
can't come to as we would, ma
Help of the Theorems above dem
ted) be brought near us, ſo as t
may have a diſtinct View thereof.
of this Kind are thoſe ſingle Glaſſe
by ſuch as can't ſee well at a co
Diſtance, and apply'd by them to
their Eyes, when they would diſc
Face or Picture of a Perſon (or th
that is farther diſtant from them
they can diſtinctly ſee with the ba
As the Word *Teleſcope* does in the
Tongue denote an Inſtrument whic
us to *ſee far off*, ſo theſe ſingle
may be call'd *ſingle Teleſcopes*, to
guiſh them from ſuch Teleſcopes
made of two or more Glaſſes.

11. But becauſe (beſide diſtin
on) it is often requiſite in humane
that the more minute Parts of an
ſhould be diſcern'd; and becau
found by Experience, that an

h appears under a leſs Angle than of Minute, appears but as a Point, or that its Parts can't be diſtinguiſh'd rom the other: Hence it often comes ſs, that the Object being brought ſo to the Eye, as that its Particles to amin'd may be ſeen under a ſenſible e, thereby the Object it ſelf becomes ear to the Eye, ſo as not to be with- Bounds requiſite to diſtinct Viſion. Inconveniency, when alone, may al- remedied by any given Lens or re- g Glaſs; foraſmuch as the Image e ſaid Object (*) may by the ſaid or Glaſs be repreſented (in any gi- Meaſure, *i. e.*) ſo as to have any gi- Ratio or Proportion to its Object.

---

ow this may be done in reſpect of a reflecting ſhewn above in *Schol* 7, to *Theorem* V, or laſt of cks. How the ſame may be done in reſpect of a refracting Glaſs, ſhall be here ſhewn. Let a Glaſs iven (*Fig.* 46) the Semidiameter of whoſe fore- *Fig.* 46. ace is AB, the Semidiameter of the hindermoſt B. Draw CM as you pleaſe; and let the Ratio e ſuch as ought to be between the homologous he Radiant and its Image, and be denoted by the MD to DC. Draw AM, whereto through B draw el BE, meeting CM in E; ME doubled will be the ſtance of the Radiant from the Lens. And after Manner may be found, in reſpect of any given Poſition, or Place where any Radiant is to be (its Image made by the Lens may be equal to a ure, which is ſimilar or like to the Radiant, *i. e.* hat) the Radiant may have a given Ratio to made by the Lens.

And of this Kind are thoſe ſingle G
us'd to ſee the Shape or Parts of
Inſects, and the like, and thence
Microſcopes, the Word importing
Greek Language an Inſtrument, w
by we *ſee ſmall* Things. To theſe
be referr'd Spectacles, of which mo
and by.

But if both the foremention'd Inc
niencies concur, then they are not
remov'd but by the joynt Help, eit
a Lens and Reflecter together, or
more than one Lens, or laſtly of
than one Reflecter. And how
ments conſiſting of two or more of
are made, ſhall be next illuſtrated.

Fig. 46. 13. Let R denote the Object giv
the given Angle under which the
is to be repreſented, D the give
ſtance of the Sight, and L the giv
ſtance of the Eye from the Lens.
the Triangle AOB, ſo as that the v
Angle at O may be equal to the
Angle S, and OE the Perpendicula
the Vertex to the Baſis may be e
the given Line D. If you'd ha
middle Point of the Object be
Axis of the Lens or Reflecter, (w
moſt convenient for Practice) the
angle AOB muſt be made an I
Take OV=L, and at V place a

cter ſo, as that its Axis may fall
th OE.

en having E thus given for one Fo-
f the Lens or Reflecter, by the re-
ive *Theorem* III of Catoptricks or
ricks or their Scholium, find the o-
ocus *e*; namely, that ſo taking *e* for
ocus before Incidence, E may be the
after Inflection at the Lens or Re-
. Through *e* draw *aeb* parallel to
meeting with V*a* and V*b* (drawn
gh V, and parallel to VA and VB)
nd *b*.

ollows from *Theorem* V, of Catop-
or Dioptricks (reſpectively, as you
ate for a reflecting or refracting
) that if *aeb* be the Radiant, AEB
its Image. Wherefore if the Ob-
ven R, (which is here ſuppos'd to
r at hand) and a ſecond Lens or
ter be ſo plac'd, according to *Schol.*
orem V of Catoptricks, or accord-
the Note before added to *Schol.* 11
Theorem V of Dioptricks,) that the
of the Object made by this ſecond
r Reflecter, may appear juſt in the
Situation, and Bigneſs of *aeb*;
will be thus made ſuch a Microſ-
s is requir'd. For *aeb* will be the
of the given Object R, the Image
ch Image, and that ſeen by the
will be AEB. And this will appear

under the Angle AOB equal to the
Angle S; and in the Distance OE
to the given Distance D, and consec
ly will appear distinct; and lastly
Distance of the Eye from the Lens
flecter next to it in V, will be equal
the given Distance in this respect.
so the Instrument will be in all Re
such as is requir'd.

14. If the Instrument be made f
Eye of an aged Person, then the
Line OE becomes infinite, and
quently the Point e is to be fou
Theorem III of Catoptricks and Diop
As for the rest of the Work, it is th
as afore.

If the Object propos'd to be di
seen be far distant, then having al
fore) given, but the Angle S, a
cope may be made fit for the Purpo
In a right Line extended toward t
mote Object propos'd, take from
ward OE and OV equal to the g
and L; and in V place a Reflec
Lens having its Axis in VE. The
E of the Lens or Reflecter plac'd a
ing thus given, find the other F
namely, that this being taken
Focus before Inflection, E may
Focus after Inflection. In the righ
OE so place a second Lens or Re
having also its Axis in OE, tha

of the propos'd Object made there-
ay appear in the right Line *aeb*
dicular to OF ; and ſo there will
de ſuch a Teleſcope as is requir'd.
e firſt Image of the far diſtant Object
'd will be in *aeb* ; and the Image
Image will be in AEB, the Diſtance
ich AEB from O is by Conſtructi-
al to the given D, and conſequent-
inct to the Eye ; and alſo the Di-
of the Eye from V is equal to the
L. And from hence may be found
gle, under which the laſt Image is

By the Help of three or more Lens's
lecters, an Inſtrument may be made
reſent even a far diſtant Object un-
given Angle. Namely, let any one
r Reflecter form the Image of the
propos'd ; and then with the o-
wo make (as afore) a Microſcope,
nting the firſt Image (as being a
Radiant nigh at hand) according to
nditions propos'd ; and ſo there
e made ſuch a Teleſcope as is re-
In like Manner, a Microſcope be-
ade of three Lens's or Reflecters, by
a fourth it will become a Teleſ-
In all which Caſes is known the
or Proportion, which the Image
y the Eye has to the Object, or
comes to the ſame) the Proporti-

on, which the Angle under whic
ſaid Image is ſeen, has to the Ang
der which the Object is ſeen witho
Inſtrument, (which, in reſpect
diſtant Objects, is as the Diſtances
Lens's or Reflecters from the comm
cus *e*.) And conſequently from
(according to the Corollary of T
IV, *Chap.* III and IV,) may be com
how much the Inſtrument does he
Sight.

Fig. 48, & 49. 17. I ſhall next adjoin ſome Ob
tions, in reference to a dark Chamb
was abovemention'd in *Schol.* 2, T
IV, *Chap.* II. Namely, it was the
ſerv'd, that Rays let into a dark
ber through a naked little Hole
paint the Images of their reſpecti
diants on a Sheet of white Paper h
gainſt the Hole. To which it is here
added, that the ſame will be don
a much larger Hole, if a Lens or
cal Glaſs be placed therein, with th
ference, that the Images of outwar
jects will be painted much more
and diſtinctly this Way than the
without a Lens; which is thus ac
ed for. Through the little Hole
48) muſt neceſſarily paſs fewer
from the outward Objects than thr
much larger Hole (*Fig.* 49.) And
ly ſo, but thoſe fewer Rays, whic

gh the bare Hole are diſpers'd;
as by a convex Lens or Glaſs all the
proceeding from any one Point of
bject, are united again at a certain
nce, and ſo render the Image
more lively and diſtinct, than the
Way. This is illuſtrated *Fig.* 48
9. Whence it is alſo obvious, why
h Caſes, the farther the Paper is
from the Hole, the bigger the Image
ppear; as alſo why the farther the
rd Objects are from the Hole, the
eir Images will be; and why the
made by a Glaſs will appear moſt
tly at the Diſtance *ac* (*Fig.* 49) with
ſuch like Particulars.

But it is obſervable, that in both
, either with or without a Glaſs,
nage will appear inverted in reſpect
e Object, as has been afore obſerv-
der *Theorem* IV, *Chap.* II, and is e-
t from *Fig.* 48 and 49. And from
8 it is evident, that when the Image
painted erect on the Bottom of the
then the immediate Radiant *abc* it
ſeen inverted by us; and on the
ry, when the Image *abc* is painted
ed on the Bottom of the Eye, then
e the immediate Radiant ABC e-
For as in the ſaid Figure the ſe-
mage $\alpha\beta\gamma$ is erect in the Eye, ſo if
e were put in the Place of the Pa-

per,

pet, the first Image *abc* would be
ted therein inverted, in respect o
Object ABC.

19. This being so, the Manner
that Sensation, which we call Vis
perform'd, seems to be the same, a
wherby a blind Man by the Thrust
End of several Sticks against his
can thereby distinguish, which
thrusts from above, which from
Namely, so by the Sensation ca
the Ends *a* and *b* of the Rays A*a*
and C*c*, touching the Retina, we
turally endued with a Faculty of
distinguishing, from whence eac
proceeds, *viz.* that the Ray A*a*, as
ing the lower Part of the Retina
proceed from the upper Point A
Object; and the Ray C*c* as tou
the upper Point of the Retina, do
ceed from the lower Point *c* of t
ject; and the Ray B*b* as touching
Middle of the Retina, does proce
B the Middle of the Object. And
quently we are enabled to judge
Situation of Things aright, by re
their Rays after such a Manner as
pears from the foresaid Experime
do: Which is the same as to sa
we see Things in their Erect or
tuation, by having their Images
on the Retina in an inverted M

ntrariwiſe, we ſee Things invert-
having their Images painted on
tina in an erect Manner.

I ſhall now, in the *laſt* Place, ex- *of Spectacles.*
he Mechaniſm of *Spectacles*, or how
ome ſo to help the Eye. It is then
from the Rules of Dioptricks,
nvex Glaſſes turn the Rays in-
or towards the Axis of the Glaſs;
more convex they are, ſo much
re inward the Ray is turn'd. It is al-
d by Obſervation or Experiments,
e Chryſtalline Humour has the
ffect as a convex Glaſs. Where-
when the Surface or Coat of the *Fig.* 56.
alline Humour is of ſuch a due
rity, as that it makes the Rays of
eral Points of an Object to unite
ſo many ſeveral Points juſt up-
Retina, then Viſion is duly per-
as *Fig.* 50.

But when the Chryſtalline Humour
onvex enough, but too flat, (as it *Fig.* 51.
hrough Age,) then the Rays paſ-
ough it are not united ſoon e-
i. e. are not united at the Retina,
ir Point of Union is further be-
e Retina, and conſequently then
can't ſee duly or diſtinctly, but
onfuſedly, the Objects, from
proceed the Rays. Wherfore to
s, convex Glaſſes, (call'd Spectacles)

cles) are us'd, whereby the Ra
made to unite in their proper Pla
at the Retina. In order thereto, th
flat the Chryſtalline Humour is, t
the more aged the Perſon is, the
convex muſt the Spectacles be.

22. On the other Hand, if the S
of the Chayſtalline Humour be to
Fig. 52. vex, (as is the Caſe of ſhort-ſighte
ſons:) then the Rays are thereby
too ſoon, or before they come to t
tina, and ſo Viſion is not duly per
Hence ſuch Perſons hold Thing
near to their Eyes, to ſee them the
becauſe (according to the Rules of
tricks) the nearer the Object is
(Lens, or in this Caſe) the Chry
Humour, the further will be the
of its Rays on the other Side. H
ſo concave Spectacles helps ſuch
Sight, by making the Rays unite
proper Place, *viz.* at the Retina.
laſtly, ſuch Perſons ſee better and
as they grow elder; becauſe the C
line Humour grows more and mo
or leſs convex; ſo as to come but
Convexity in their old Age: W
the way is the Reaſon, that ſuch
as are ſhort-ſighted when Young
not Spectacles when they are Old
this is ſufficient to our Purpoſe con
Dioptricks.

C

CHAP. V.

Of Perſpective.

erſpective the chief Things (beſide Eye and the Object) to be con- and therefore explain'd, are thoſe are contain'd in the following De- s.

1. *Definitions of* Terms *us'd in Perſpective*

(†) *Geometrical Plane* is a plain parallel to the Horizon, and pla- wer than the Eye, in which we the viſible Objects without any , (except it be reducing a great nto a ſmall one,) and upon which ation of the Object to be repre- n Perſpective is deſcrib'd.

2. *The* Geometrical Plane, *what.*

Situation (call'd alſo the *Ich-* or *Plan*) of an Object is its raphical Projection upon the

3. *The* Situation *of an Object, what.*

---

lſo called the Baſis. This and the following here denoted, are not repreſented in Figures y being not eaſie to be ſo apprehended by uts; but they may be more eaſily apprehended Imagination, or elſe by actually placing ſo ma- ſuch a Manner.

(*) geo-

(*) geometrical Plane, or, that which is describ'd upon the Geo Plane, when from all the Points Object right Lines are drawn perp lar to the said Geometrical Plane. the (Plan or) Situation of a right der is a Circle, and the Situatio right Cube is a Square.

4. *The* Picture, *what.* The *Picture* is a plain Surfac pos'd as transparent as Glass, a ways plac'd at some Distance betw Eye and the Object, to represen the Object in Perspective; whe Picture is also call'd the *Pe Plane.*

5. *The* Ground-line, *what.* The Picture stands (usually pendicular) upon the geometrica Whence the common Section of ture and the geometrical Plane, *the Base of the Picture*, or the *B* and also the *Ground-line.*

(*) If the Orthographical Projection be (no geometrical Plane, but) upon a Plane parallel ture, then it is call'd *Front*; and if it be upon rallel to the vertical Plane (describ'd §. 11) *Profil.*

(†) Sometimes the Picture is inclin'd to the Plane or Horizon, and has also a curve Surface the Surface of an arch'd Roof is to be drawn being not common, the Picture is look'd upon this Chapter as a Plane perpendicular to the Plane or Horizon.

Horizontal Plane is a plain Sur- rallel to the Horizon, and which giv'd to pass through the Eye, and perpendicular to the Plane of the

6. *The* Horizontal Plane, *what.*

Horizontal Line is that right Line, ch the horizontal Plane and the f the Picture intersect, and which re is parallel to the Ground-

7. *The* Horizontal Line, *what.*

Principal Ray is a right Line from the Eye, perpendicular to ne of the Picture, which there- st run along the horizontal Plane.

8. *The* Principal Ray, *what.*

Point of Sight, call'd also the *l Point*, is the Point where the is cut by the principal Ray, must necessarily be in the horizon- e.

9. *The* Point of Sight, *what.*

Point of Distance is a Point in the tal Line, whose Distance from nt of Sight, is equal to the Length principal Ray.

10. *The* Point of Distance, *what.*

Vertical Plane is a plain Surface, going along the principal Ray, is dicular to the Horizon, and so to ometrical Plane, and also to the : The common Section of the Pic- d vertical Plane being call'd the *Line*, and being also the Mea- the Height of the Eye; and the n Section of the geometrical and vertical

11. *The* Vertical Plane *and* Line, *&c. what.*

vertical Planes being call'd the *Station.*

12. *The* Appearance *of a Point, what.*

The *Repreſentation* or *Appearan* ny Point of an Object is that Point the Picture is cut by a right Line from the Eye to the Point propos'd Object.

13. *The* Accidental Point, *what.*

The *Accidental Point* of a right the Point, where the Picture is right Line drawn from the Eye pa the Line propos'd. Whence it is conclude, that all Lines, which ar lel to the Picture, have no ac Point; and that all other Lines are parallel one to another, h ſame accidental Point. It is alſo able, that all right Lines, which pendicular to the Picture, have t cipal Point for their accidental and that all thoſe, which make right Angle with the Picture, h of the Points of Diſtance for their tal Point.

14. Obſervables *from what has been ſaid.*

It is alſo obſervable from w been ſaid, that all the Parts of an which are lower than the Eye or tal Plane, ought to be repreſente Picture below the horizontal Lin on the contrary, all above the tal Plane, or higher than the Ey to be repreſented in the Picture a ſaid horizontal Line. In like Ma

Objects, which with reſpect to the
re on the right Hand of the verti-
ne, muſt in the Picture be repre-
on the right Side of the vertical
and ſuch as are on the Left, muſt
reſented on the left Side of the ſaid
l Line.

rder to perform any Operation, 15.
eet of Paper, which you would *How to de-*
ſcribe on
Paper the
Plane of
the Picture,
and the
Geometri-
cal *Plane.*
Fig. 53.
pon, muſt be divided into two
by the Line AB (*Fig.* 53) which
reſent the Ground-line. The up-
t of the Paper, *viz.* ABDD, muſt
n for the Plane of the Picture;
lower Part ABYZ of this Paper
geometrical Plane.

next Thing is to know, (under 16.
ngle the Picture ought to be ſeen, *How to*
find *or ſet*
off the
Points of
Diſtance.
atever is repreſented therein may
in juſt Proportion, and may eaſi-
ken in by the Eye at one View;
in ſhort,) how far the Points of
ought to be from the Point of
Let there be propos'd a Square,
akes in all that you would repre-
Perſpective. And let the Ground-
be of equal Length with one Side
quare propos'd. Wherefore, if
of the Spectator was at G, he
t ſee the two Ends A and B, be-
Eye could ſee, (†) at moſt, but

rding to *Theorem* II, of *Chap.* II.

ſo much as is comprehended und
right Angle GHI. But if the Eye
K, it could ſee the Ends A and
cauſe the Angle AKB is leſs than
viz. 80 Degrees. And the Eye
ſee the Ends A and B better at L
better yet at M, where the Angle
is 60 Degrees. And from ſuch a D
the Objects (to be repreſented in P
tive, and contain'd in the foreme
Square) may be ſeen in Perfectio
viſual Angle AMB being neither to
nor too little. The Angle ANB
alſo be very well. Wherefore
biſected AB in F, and from F dra
Line FV perpendicular to DD
vertical Line, and conſequently
the viſual Point or Point of Sigh
the ſaid Point V ſet off the D
((*) FK or rather FL, or) FM,
on each Side upon the horizont
DD, *viz.* to D and D, which wil
Points of Diſtance requir'd or fou

17. *A general Rule concerning the ſame.*

And this may be look'd on as
ral Maxim, *viz.* that the Diſta
twixt the Point of Sight V and th

(*) If the Diſtance VD were leſs than FK, t
ſpective Square ABCE (concerning which ſee
appear irregular, becauſe it would be ſeen u
tuſe, and ſo too great an Angle.

ſtance D and D muſt be (at leaſt)
to FA or FB. And it will not be
to let this Diſtance be ſomewhat
r. And, if in this Caſe the Plane
Picture be not wide enough to
e two Points of Diſtance, there
ouly one be mark'd; and the prin-
Point or Point of Sight V may be
on one Side, as *Fig.* 59, *&c.*

e Points of Diſtance D and D (*Fig.* 18.
eing found, draw the Lines AV *The* Per-
V, as alſo AD and BD. And the ſpective
e ABCE will be the Appearance (in Square, *what.*
icture) of the Square, propos'd to
reſented in Perſpective; which Ap-
nce of the ſaid Square is in ſhort
the *Perſpective Square.*

ceed we now to the Repreſentation 19.
e Picture) of Points, Lines, and *How to find*
. And firſt, how to find in the *in the Pic-*
e the Repreſentation or Appear- *ture the Ap-*
f a *Point* given in the geometrical *pearance of*
From C the given Point (*Fig.* *a* Point *gi-*
t fall upon the Ground-line AB the *ven in the*
ndicular CE; and from the Point *geometrical*
w to the principal Point V the Line *Plane.*
Upon AB ſet off EF (or EA) = *Fig.* 54.
and then draw FD (or AD either
hich will upon VE give at *c* the Ap-
nce of the given Point C.

20. *To find in the Picture the Appearance of a* right Line *given in the geometrical Plane.* *Fig.* 55, 56.

Secondly, how to find in the P the Appearance of a *right Line* giv on the geometrical Plane. Suppo (*Fig.* 55 and 56) to be the right L ven. Find (as afore) the Appear the Point M, *viz.* at *m*, and the Appearance of the Point N of the Line MN, *viz.* at *n*. Then join *n*, the Line *mn* will be in the P the Appearance of MN the Line sed (*).

21. *To find in the Picture the Appearance of a* curve *Line given on the geometrical Plane.*

When the Line given on the geo cal Plane is (not a right Line, curve Line, then from ſeveral Po it draw Perpendiculars to the G line; by means of which, and wh been ſaid, may be found in the P the Appearances of all thoſe Points, which being join'd by a Line, that Curve will be the Appe of the Line propos'd.

(*) That every Thing you deſign to repreſe ſpective, may appear in juſt Proportion, it is ſometimes to have the Eye at a great Diſtance Picture, which may hinder from marking eithe Diſtance upon the horizontal Line. In which muſt mark only half of the Diſtance of the Eye Picture upon the horizontal Line, as (*Fig.* 57) Q, which will repreſent one of the Points of And then you muſt only ſet off half the Perpend upon the Ground-line AB, from I to P. Then and the Point *b* will be the Appearance of H the pos'd.

T

irdly, To find in the Picture the rance of a plain Figure given on ometrical Plane, ſuppoſe the Square H *Fig.* 58. Find the Appearance s been afore ſhewn) of its four an- Points C, E, G and H; which will e, *g* and *h*. Join the ſaid (laſt) and the Perſpective Square *cegh* will Appearance of the given Square I, when ſeen foreright, or when ne KL, of the given Square falls in VF the vertical Line. If the given be all on one Side of the vertical VF, (as *Fig.* 59) then the Perſpec- uare *cegh* (in *Fig.* 59) will be the rance of the given Square CEGH.

22. *To find in the Picture the Appearance of a plain Figure given in the geometrical Plane. Fig.* 58, 59.

ere alſo it is to be noted, that the rance *g* of the Point G may be alſo after this Method. Namely, Ground-line AB take the Point B ſure, and draw BV. Then ſet off ual to the Perpendicular IG, and MD, which upon the Ray VB will e Point N. Through which NC drawn parallel to AB, will upon y VI give the Point requir'd *g*. is Method may be uſed, when the dicular IG is ſo long, that the thereof can't be ſet off upon the d-line AB within the Picture.

23. *A Remark. Fig.* 59.

e ſame Manner may be found in ure the Appearance of any other plain

24. *Another Remark.*

plain Figure, *viz.* Triangle, Pen Heptagon, *&c.* Namely, by dif finding the Appearance of each a Point of the plain Figure propos'd then joining the Appearances so fo right Lines.

25. *To find in a Picture the Appearance of a Circle given in the geometrical Plane.*

It shall here be only observ'd in reference to plain Figures, that in the Picture the Appearance of cle given in the geometrical Plane, must be describ'd about it a Squ (*Fig.* 60.) the Square EFGH ab Circle ILKM. One Side of the S *viz.* EF (or GH) must be parallel Ground-line AB, and consequently ther Side EH (or FG) must be pe dicular to AB. Draw the two Di EG and FH, which intersecting a Center of the Circle, will give upo M the Circumference of the Circl four Points, P, Q, R and N. Th in the Picture the Appearance Square EFGH, which will be t spective Square *efgh*, and also the A ances of the Points L, Q, K, R, I and P; which will be in the P tive Square the Points *l*, *q*, *k*, *r* *i* and *p*. A Curve drawn throug last Points will determine the App of IKLM the Circle propos'd. note, that the Appearance of th will also be a true Circle, if th

Fig. 60.

'd be ſeen fore-right, that is, if its (*) LM, being perpendicular to make one right Line with VF the l Line. But if the Circle propos'd be ſituated (as *Fig.* 61,) then its Apnce will be (as there) elliptical.

ceed we now to ſhew, how to re- 26.
t in Perſpective a *Floor of equal* *To repreſent a Floor of equal Squares ſeen fore-right.*
ſeen fore-right, without a geol Plane. Divide the Ground-line (*Fig.* 62) into as many equal Parts pleaſe, each of which will repre- *Fig.* 62.

---

have the Appearance of a Circle in the Picture, ewiſe a Circle, the Diſtance between the Eye and e muſt be of a determinate Length, which may thus. Having produc'd the Side of the circumquare EFGH, 'till it meets at right Angles with ontal Line DD in a Point as S, draw SL, and om it SC=SV, and the remaining Part LC (of SL) e Diſtance between the Eye and the Picture, or to the ſaid Diſtance. If VD be given, and you nd VE (or the Diſtance between the horizontal and the Ground-line AB, that is) the Height of equiſite, that the Appearance may be alſo a true the rectangular Triangle SVL ſhews, that the the Line EL (or of the Ray of the given Circle uſt be taken from the Square of the Line LS, and ES of the remaining Square will be the Height re- Note further, that the Perſpective Square *efgh* nd, the Mid-point X of the Line *m*L (or *ml*) will nter of the Perſpective Circle; upon which there- r the ſaid Circle may be drawn without any , or without finding the Points *l*, *q*, *k*, &c. Laſtly, in *Fig.* 60, the Perpendiculars EI and TN are on the Ground-line AB, partly to avoid Confuſion bing the Arch of many Circles, partly and chiefe it is evident (from the ſetting off FK and QR) =EI and TF=TR.

ſent

ſent the Side of a Square. Draw th
each Point of Diviſion to the pri
Point V, as many right Lines or
the two laſt of which will be V
VB. Then draw the Ray AD (or
which will cut the foregoing R
Points, through which you muſt
Parallels to the Ground-line AB.
laſt of theſe Parallels will be EF,
will have its Diviſions equal betw
and H. And the like equal Di
may be continued from G to E
from H to F, in order to draw th
theſe new Points of Diviſion, fro
principal Point V, other Rays
will upon the Lines DA and DB me
foregoing Parallels reſpectively belo
to them. Thus you'll have in P
tive all the Squares, which can be
prehended in the Space ABFE.
you would have more, draw DE,
will cut all the Rays that proceed
V in Points, thro' which (as afore
muſt draw Parallels to AB, the
which will be KL. And ſo, go o
fore.

27. *To repreſent a Floor of* Equal Squares *ſeen* Corner-wiſe. *Fig.* 63.

If you would repreſent in Perſpe
Floor ſeen fore-right, and contai
qual Squares ſeen *Cornerwiſe*, wor
The Side AB of the ſquare Floor
determin'd upon the Ground-line A
ſcribe the Appearance of the ſquare

drawing from the two Ends A and B
he Side given, to the principal Point
he Rays VA and VB; and to the
Points of Diſtance the Rays DA and
These Rays will cut the former in
Points E and F, which being join'd,
Line EF will be parallel to the Ground-
; and ABEF will be the Perſpective
re. Having divided AB (or the
of the Perſpective Square) into e-
Parts, to each Point of Diſtance D,
Rays through each Point of Divi-
; which Rays by their common In-
ions, will form the Appearances of
Squares ſeen Corner-wiſe. With
Squares it will be eaſie to fill the
ctive Square ABFE, becauſe all
Rays that proceed from the two
s of Diſtance, divide equally in
ame Points the Side EF parallel
B; which is alſo divided equal-
d in the ſame Points, by the ſaid
which go from the two Points of
nce.

28. *To repreſent a Floor of Squares ſeen fore-right, and encompaſſed with a Border.* *Fig.* 64.

represent in Perſpective a Floor
of equal Squares ſeen fore-right,
ncompaſs'd *with a Border* or Frame,
nt a geometrical Plane; work thus.
e the Ground-line AB into as many
alternatively equal and unequal,
would have Squares and Borders,
Points 1, 2, 3, 4, 5, 6, 7, 8.

From all these Points draw Rays to
the first Ray will be VA, the last
Then draw the Ray DA, which will
those that go from V in Points, thro
which you must draw Parallels to
the last of these Parallels will be
which terminates the Perspective Sq
ABFE. And so you will have the
presentation of the Floor requir'd.
you may continue it, if you draw
ther Ray through the Point E and
Point D; and then proceed to wo
afore.

29. *To represent a Floor of equal Squares seen Corner-wise, and with a Border, &c. Fig. 65.*

To represent such a Floor as is
mention'd, but seen corner-wise,
thus. Divide the Ground-line AB
Parts as afore, *viz.* alternatively equal
unequal. Then draw thro' the Point
Division, to the Points of Distance,
or Lines; which will (as afore) te
nate in the respective Square, this
being to be describ'd as is afore taug

30. *An useful Way to represent in Perspective one or more Figures.*

Lastly, It is observable, that if
describe Squares given in a geome
Plane, then the Ground-line AB m
divided into Parts equal to the Sid
the given Squares. And the little Per
tive Squares, not only will be the
pearances of those in the geome
Plane, but also may be very useful
present in Perspective one or mo
gures made up of several Lines;

Inst

nce, a fortified Polygon. Namely, aid Polygon (or Wall of a fortified n) being deſcrib'd on the geometri- lane with the Squares, it will not be ult to deſcribe the ſame likewiſe in pective with the Squares of the Pic- drawing ſuch a Part of it in each e of the Picture, as is deſcrib'd in orreſpondent Square of the geome- Plane. All which will be ſuffici- illuſtrated by a bare Sight of *Fig.*

d in like Manner, *Fig.* 63, if up- 31. B a Square be drawn in the geome- *A like Me-* Plane, as ABGH, whoſe Repreſen- *thod.* is the perſpective Square ABEF; hat Square ABGH on the geometri- lane be divided into other little es by Lines parallel to the Diago- G and BH, the ſaid Square ABGH e made uſe of, to put or repreſent ſpective ſeveral Things at once, the ht of the ſaid Things being de- upon the ſaid Square.

d thus I have taken notice of ſo 32. of Perſpective, as is agreeable to *The Con-* eſign of this Treatiſe. And there- *cluſion.* all here put an End to this Trea- Opticks.

F I N I S.

ERRATA to *Trigonometry.*

PAGE 7. Line 33. For *fit for Use.* Read *not fit for Use.* p. 9. l. 3[illegible] r. $\frac{1}{10}$*ax.* p. 19. l. 5. f. $\frac{B-D}{2}=B$. r. $\frac{B-D}{2}=D$. p. 30. l. 1[illegible] *EA.* p. 35. l. 22. f. *B*=350. r. *B*=35 *degr.* p. 42. l. 13. f. *C*[illegible] *CA*=793. p. 44. l. 6. f. *Arch CBA.* r. *Angle CBA.* p. 51. l. [illegible] *BA.* r. *Line BC.* p. 54. l. 16. f. *Angles meet.* r. *Angles, 'till* [illegible] 55. l. 13. f. *at M.* r. *at N.* ibid. l. 19. f. *at N.* r. *at M.* p. 5[illegible] *at the End* m. r. *at the End* n. ibid. l. 4. f. *at the End* n. r. *at* [illegible] p. 65. l. 1. f. *so that.* r. *so shall.* p. 82. l. 22. f. *to* 99. r. *to* 8[illegible]

ERRATA to *Mechanicks.*

PAGE 6. Line 14. For *contriv'd.* Read *conceiv'd.* p. 10. l. 21. f. [illegible] r. *little Body may.* p. 13. l. 12. f. *Laver.* r. *Leaver.* ibid. [illegible] *but (the.* r. *but) the.* p. 51. l. 21. f. *said Squares.* r. *said* [illegible] penult. f. *is equidistant from C to Aa.* r. *is six times as distant from* [illegible] p. 17. l. 1. f. *to b.* r. *to β.* ibid. l. 3. f. *bC.* r. *βC.* ibid. l. 7. f. [illegible] r. *this being.* p. 20. ult. f. *G2.* r. *C2.* p. 30. l. 10. f. *were.* [illegible] ibid l. 11. f. *at C.* r. *at G.* ibid. l. 12. f. *at* r. *at B.* p. 31. l. [illegible] *Mark.* r. *or Hook.* p. 34. l. 23. f. *as Fig.* r. *as Fig.* 28, 29, 30. [illegible] f. *Pulley)* r. *Pulley as Fig.* 27.) p. 35. l. 19. f. *by TS.* r. *by FS.* [illegible] f. *d, e, f, E. By.* r. *d, c, f. By.* p. 50. l. 6 f. *CF.* r. *CG.* p. 5[illegible] *aqual.* r. *equal.* p. 57. l. 18. f. *EFAB.* r. *EFHB.* p. 65. l. 17. [illegible] r. *Now the.*

ERRATA to *Opticks.*

PAGE 87. Line 14. For *Fig.* 11. Read *Fig.* 10 p. 88. ult. f. [illegible] *(Fig.* 28.) p. 89. l. 5, 7, 17, 25. f. *(Fig.*) r. *(Fig.* 28.) ibid. [illegible] f. *(Fig.*) r. *(Fig.* 8) ibid. l. 16. f. *Surface BD.* r. *BC.* p. 10[illegible] *AH.* r. *AF.* p. 107. l. 8. f. *is look'd to be.* r. *is look'd on to be.* p. 10[illegible] *Convex.* r. *the Reflex.* p. 110. l. 2. f. *to the Prop. be requir'd.* r. *to* [illegible] *here required.* p. 113. l. 1. f. *Theor. II.* r. *Theor. IV.* ibid. l. penult. f. [illegible] *And whereas.* p. 114. l. 4. f. *EDG.* r. *ADC.* p. 116. l. 5. f. *bBF.* r. *bB*[illegible] ult. f. 2 : *bf*, r. 4 : *bf.* p. 122. l. 29. *subdulicate.* r. *subduplicate.* p. 1[illegible] *(of the Angle CBM=)*.. r. *of the Angle (CBM=).* ibid. l. ante [illegible] *Fig.*) r. *(as Fig.* 29) p. 126. l. 10. f. *Fig.* 30. r. *Fig* 36. p. 1[illegible] 3 *AB.* r. 2 *AB.* p. 133. l. 28. f. *Concave,* r. *Convex.* ibid. l. 29. [illegible] *Concave.* ibid. l. 30. f. *Fig.* 36. r. *Fig.* 35, and 36. p. 141. l. 28. [illegible] p. 148. in Margin, f. *Fig.* 46. r. *Fig.* 47. p. 151. l. 3. f. *OF* r. [illegible] l. 8. f. *Chaystaline,* r. *Chrystaline.*

In *Astronomy,* Page 63. Line 15. Read, *Three Diameters* [illegible]

In *Chronology,* Page 33. After these Words, (in the last [illegible] of the Text,) viz. *and in the Calendar adjoining to this Tr*[illegible] as follows: *Or else the Golden Numbers are call'd the Prime,* [illegible] *Luna Prima or the First Day of the New Moon; according to* [illegible] *of speaking the Full Moon is frequently stil'd Luna Quarto* [illegible] *falling on the fourteenth Day after the New Moon inclusively.*

Fig. 1.

Fig. 1.

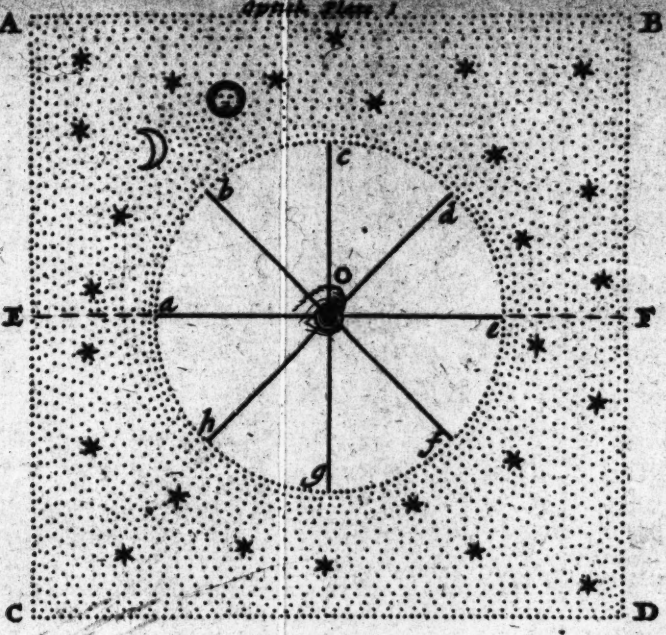


Fig. 2

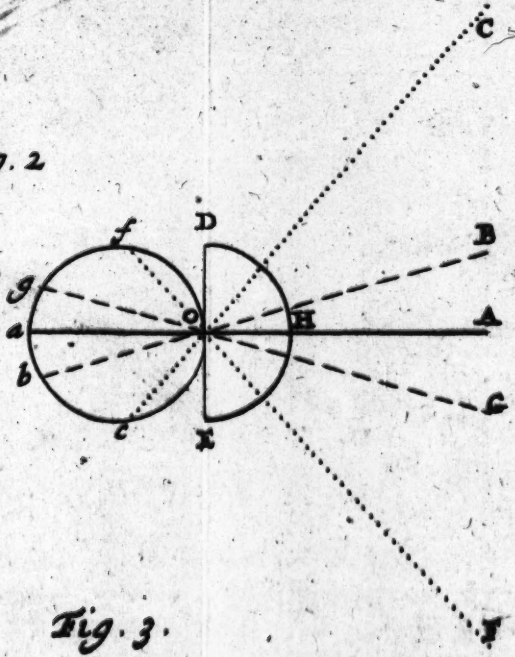
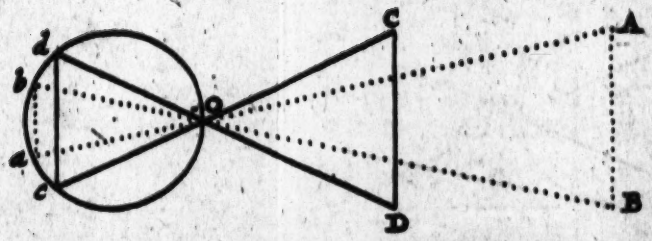


Fig. 3.



b
c
a

Fig. 5.

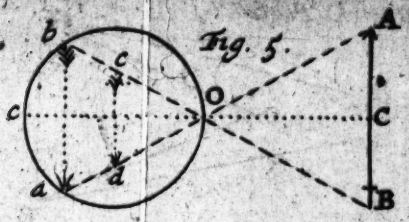


Fig. 4

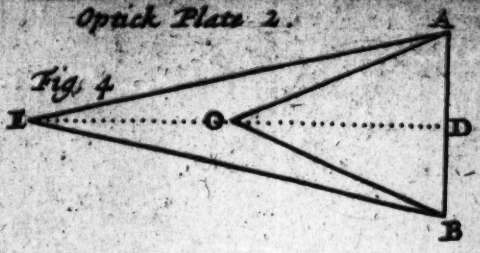


Fig. 6

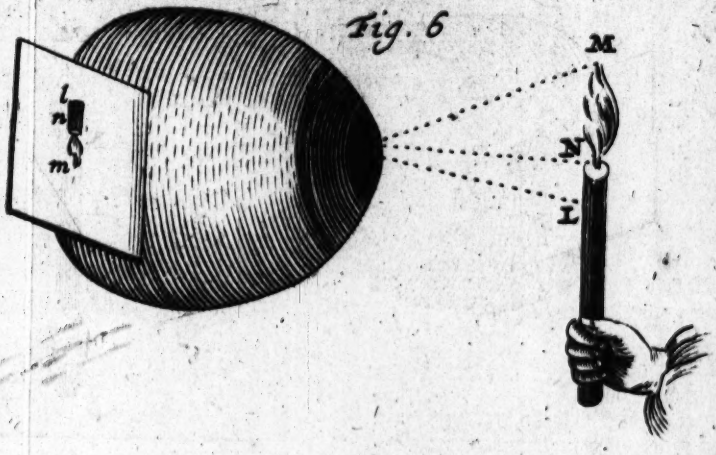


Fig. 7

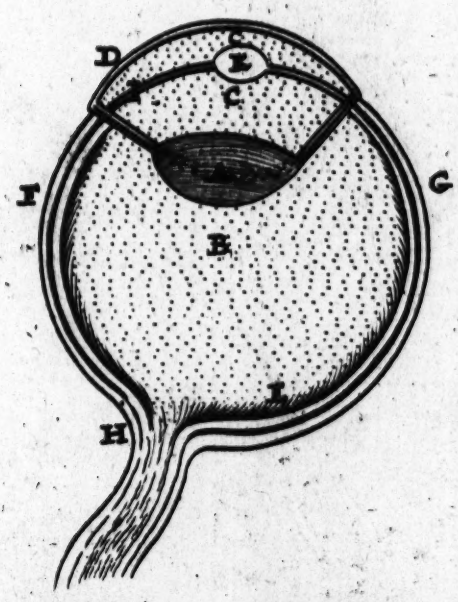


Fig. 8
E
G
e
A
D
P
A
C
a

Fig. 8

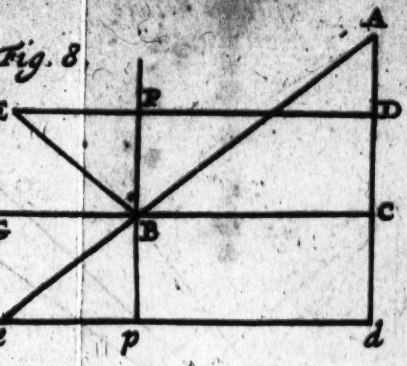


Fig. 9.

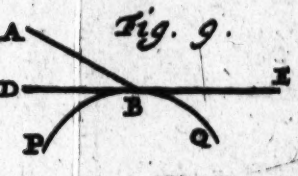
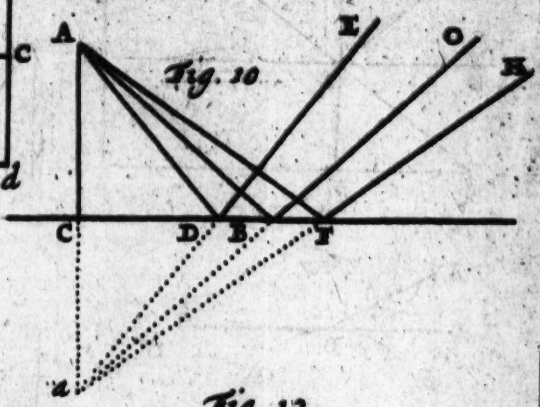


Fig. 10



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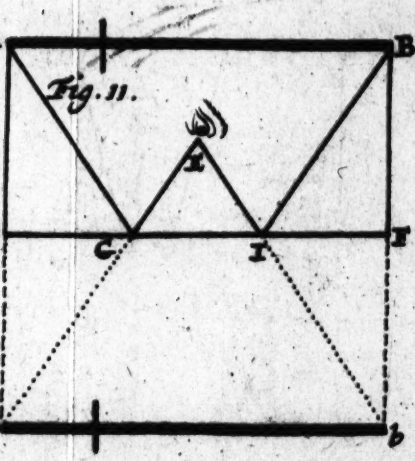


Fig. 12

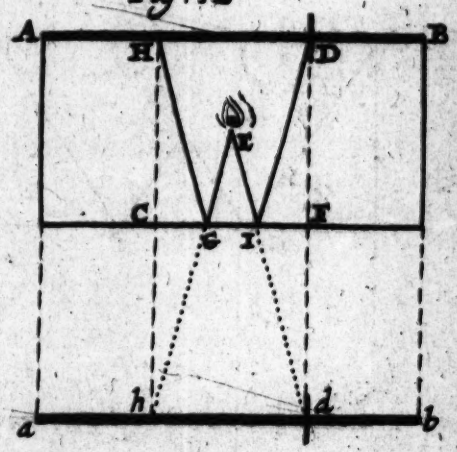
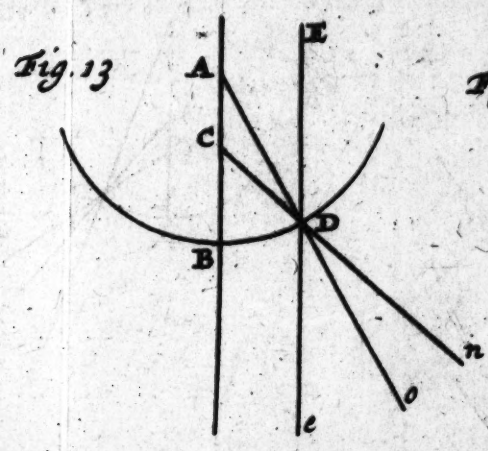


Fig. 13



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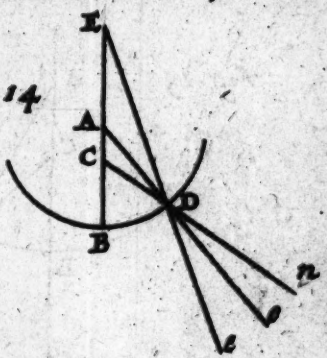


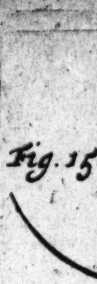
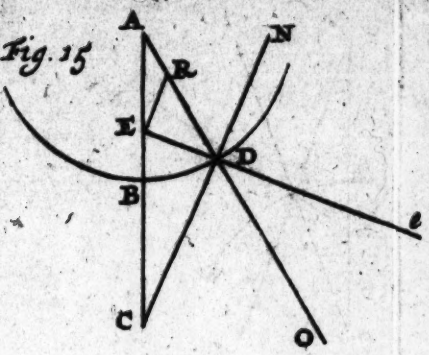


Fig. 15

Fig.

H

Fig. 15



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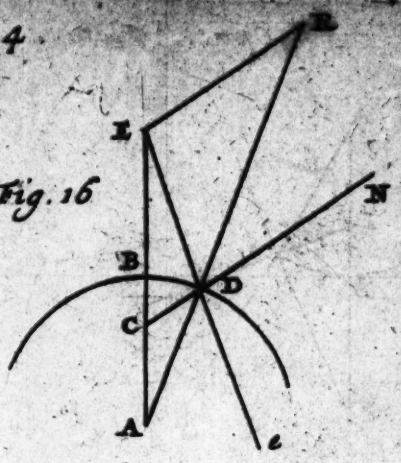

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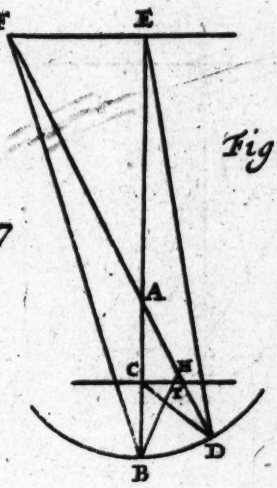

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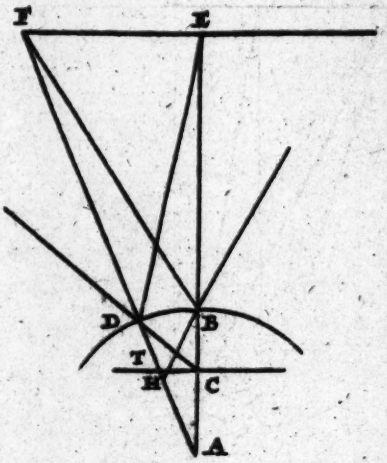

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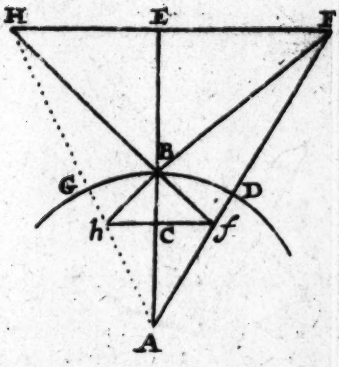


Fig. 20

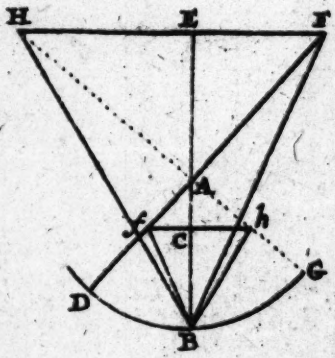


Fig. 2

Fig. 2

Fig.

Fig. 21

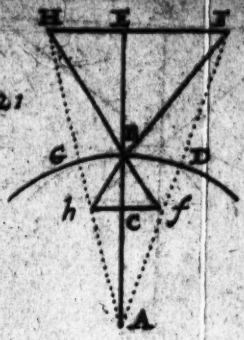
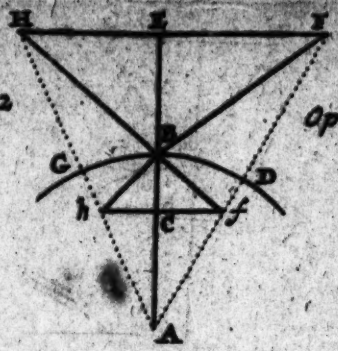


Fig. 22



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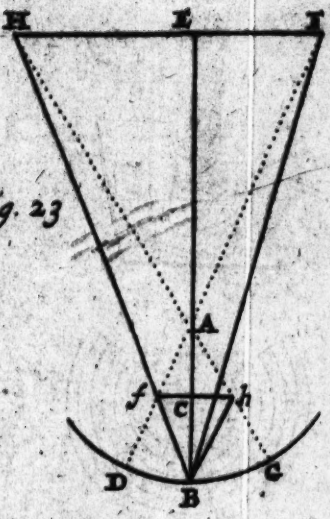
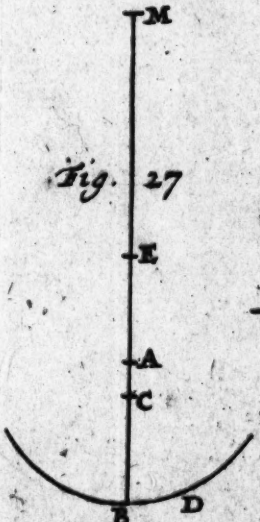


Fig. 26

Fig. 25

Fig. 24

Fig. 27



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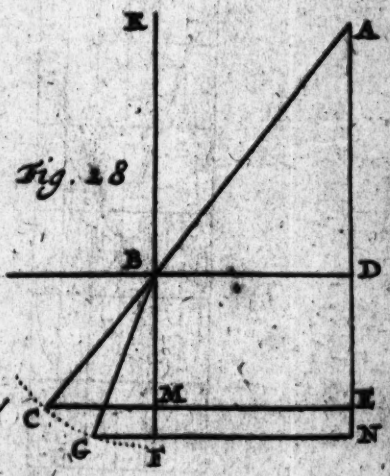


Fig. 29

Fig. 37

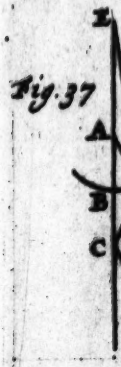


Fig. 4

Fig. 29

Fig. 30

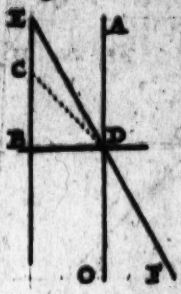


Fig. 31.

Fig. 32

Fig. 33

Fig. 34

Fig. 35

Fig. 36

Fig. 37

Fig. 38

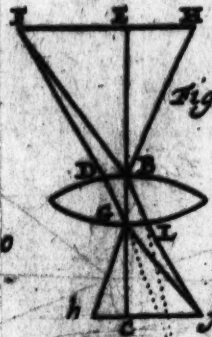
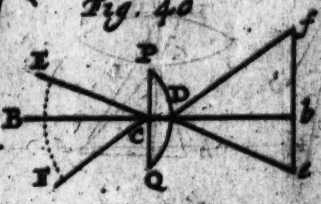


Fig. 39

Fig. 40



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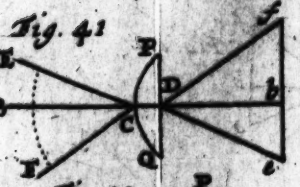

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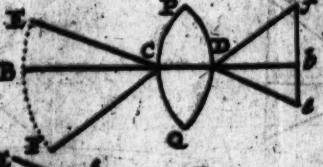
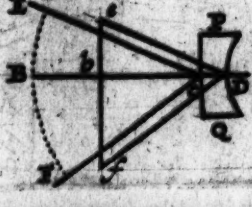
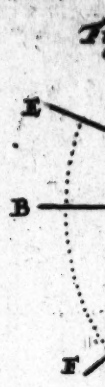
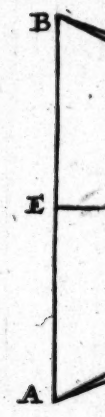


Fig. 43

Fig. 44




E
B
F


B
E
A

A
B
C

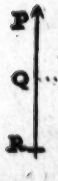
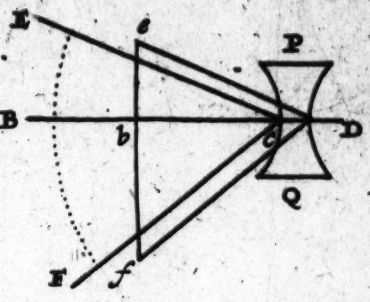

P
Q
R

Fig. 45



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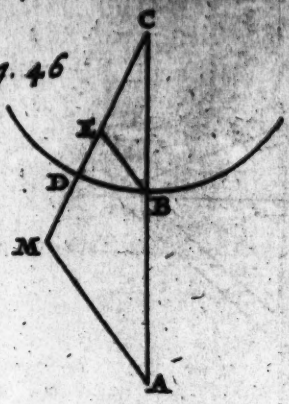


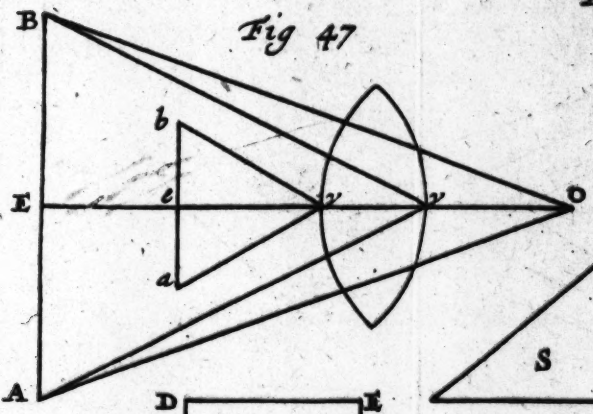


Fig 47

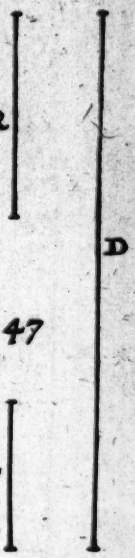
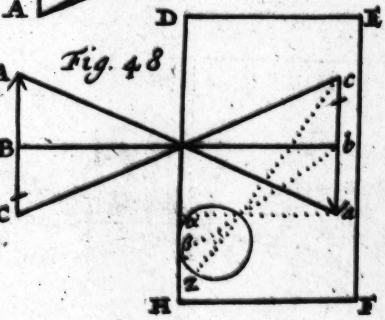


Fig. 47



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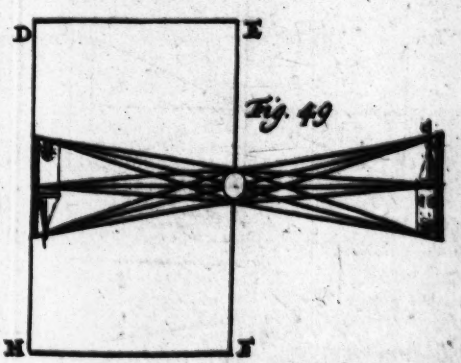


Fig. 49

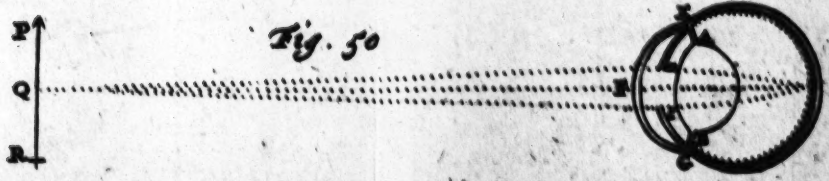
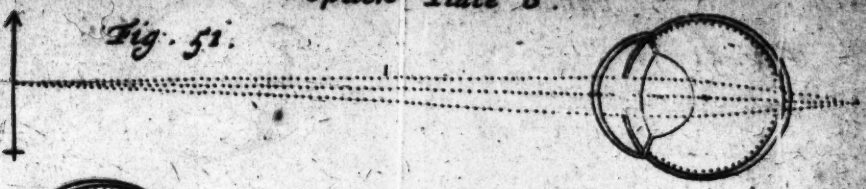


Fig. 50

Fig.
D
H
A
Y

Fig. 51.



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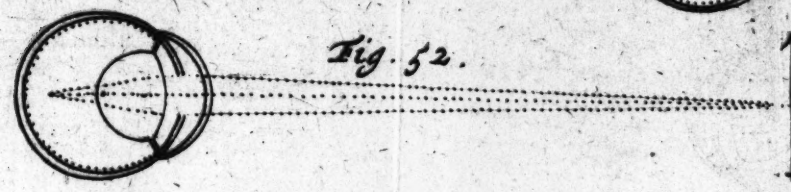

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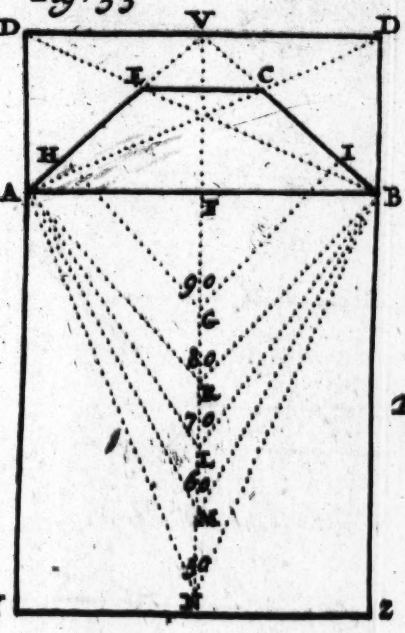

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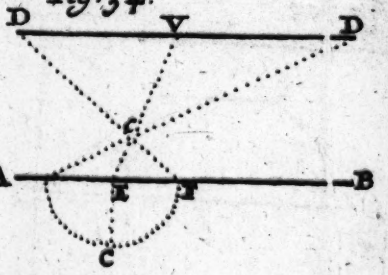


Fig 55

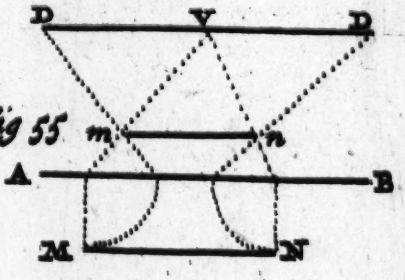
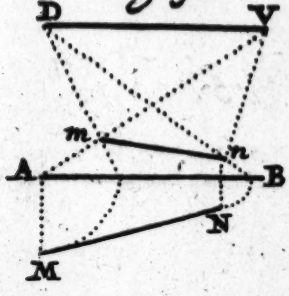


Fig. 56



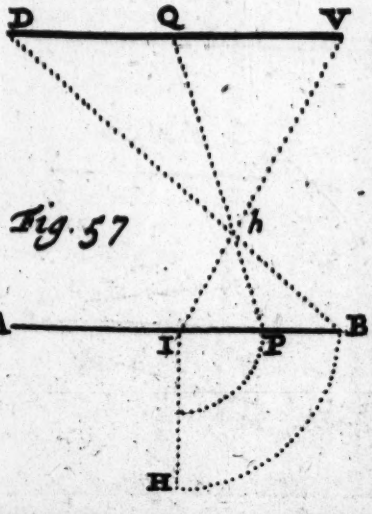
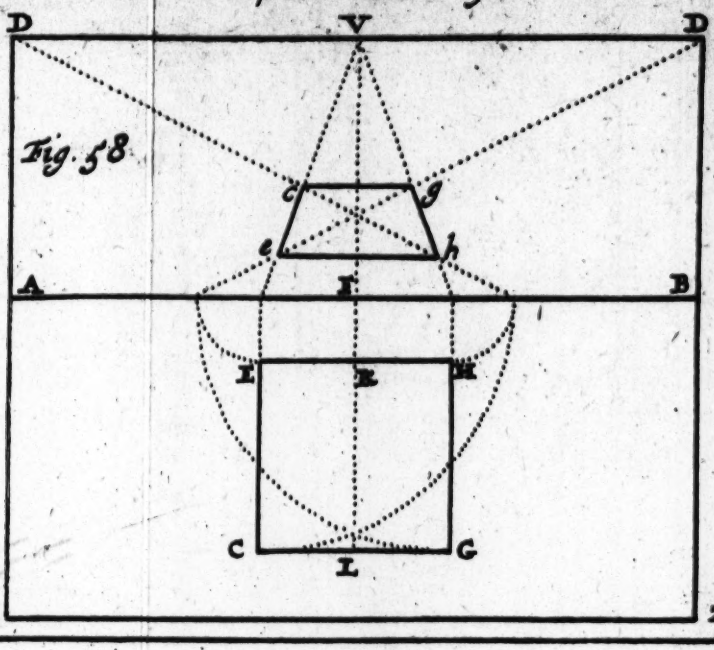


Fig. 57

V
H
A
Y

Fig. 58.



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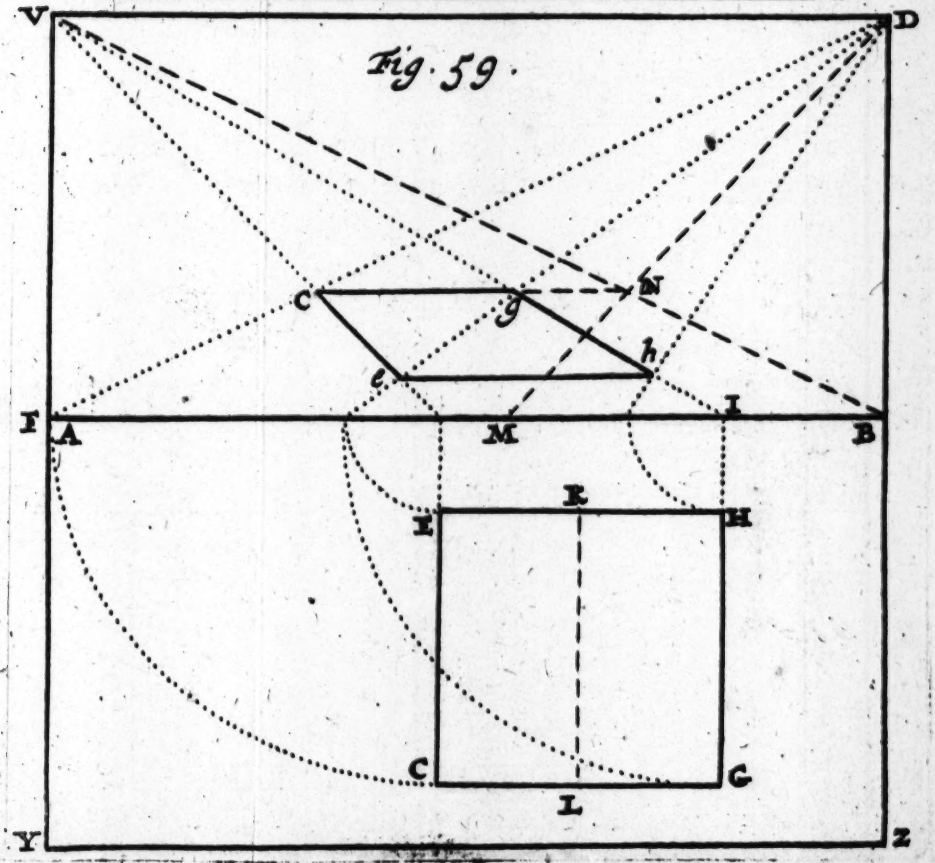

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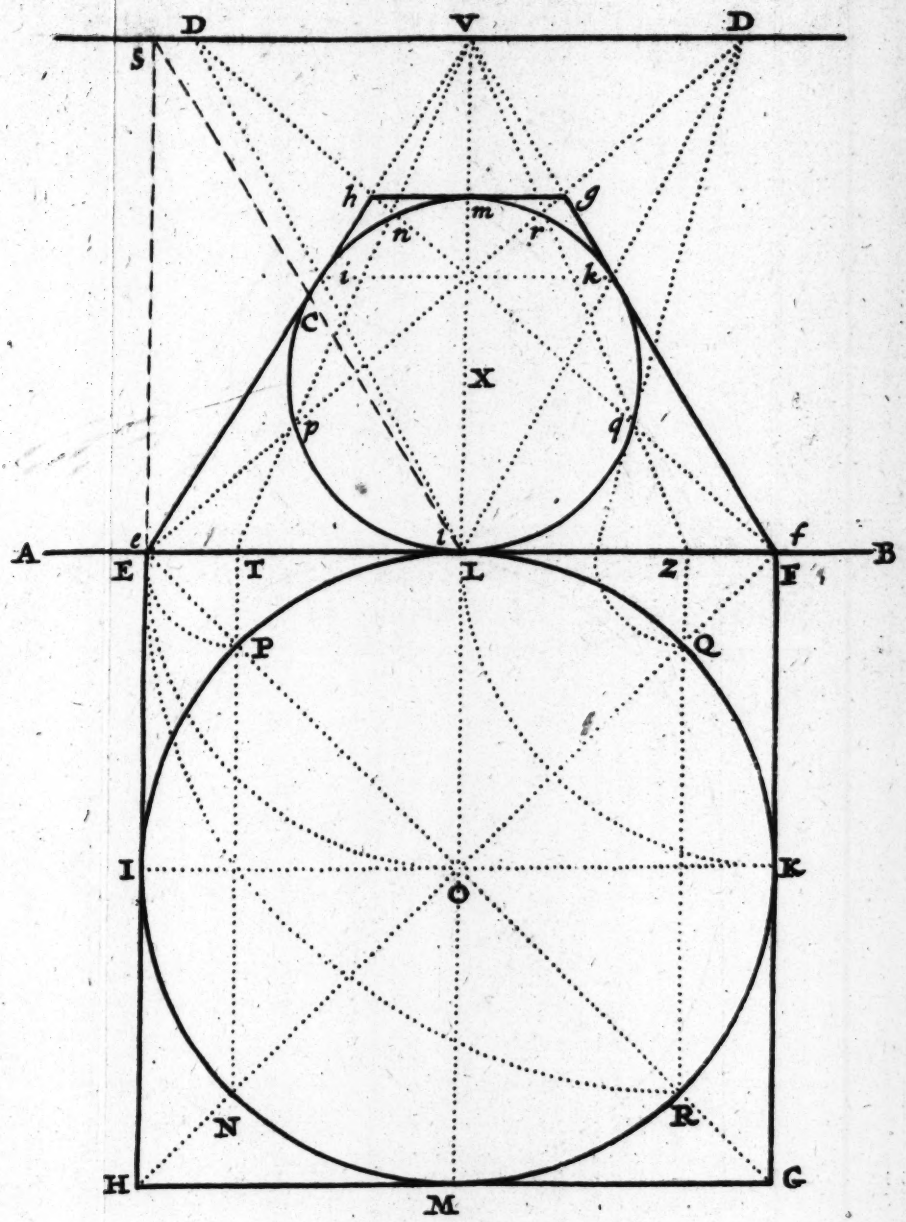
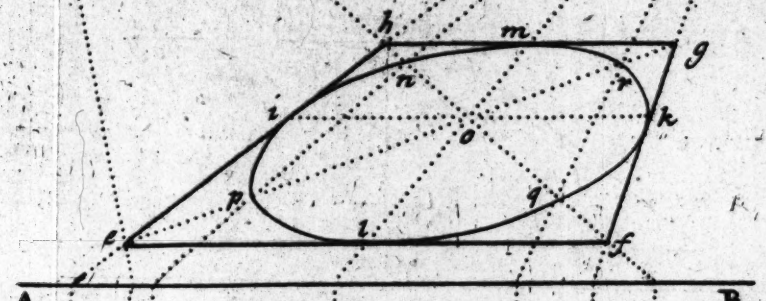
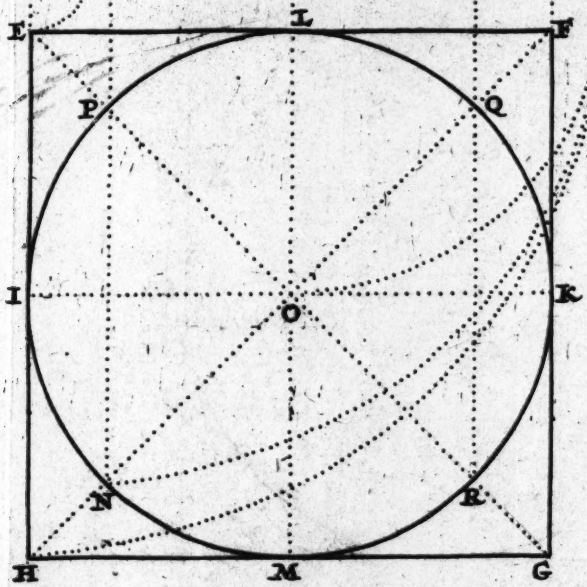


A
D
K
E
A

Fig. 61.





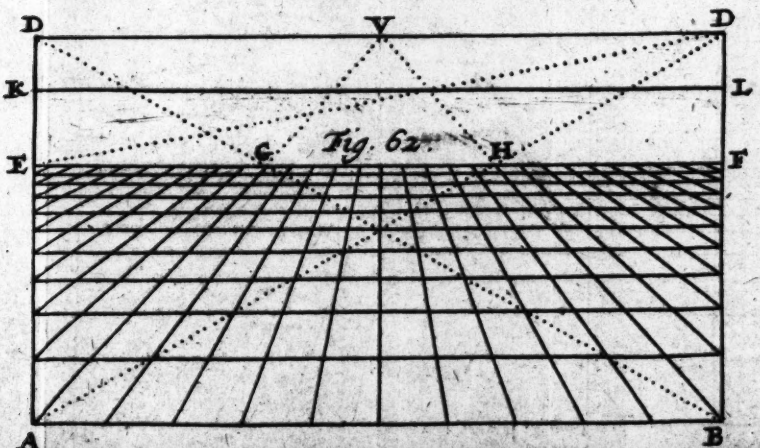
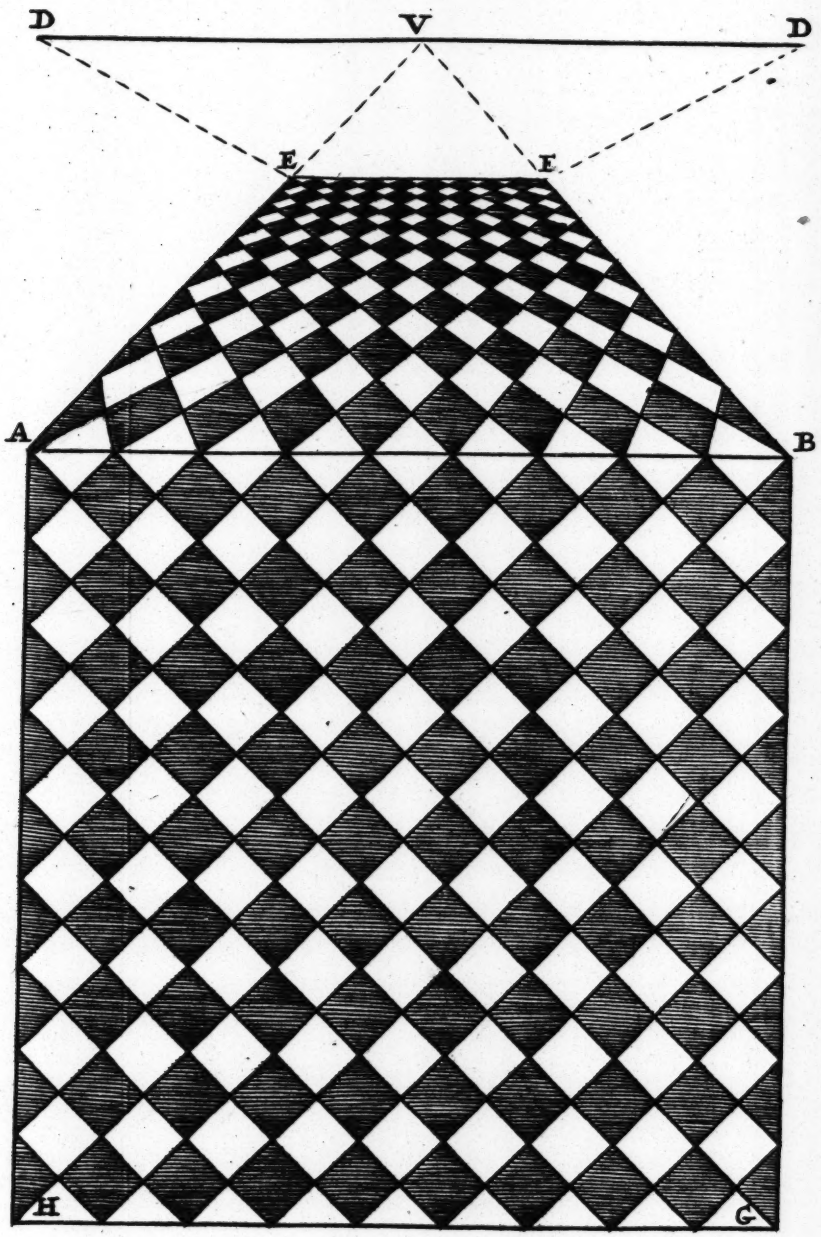


Fig. 62.

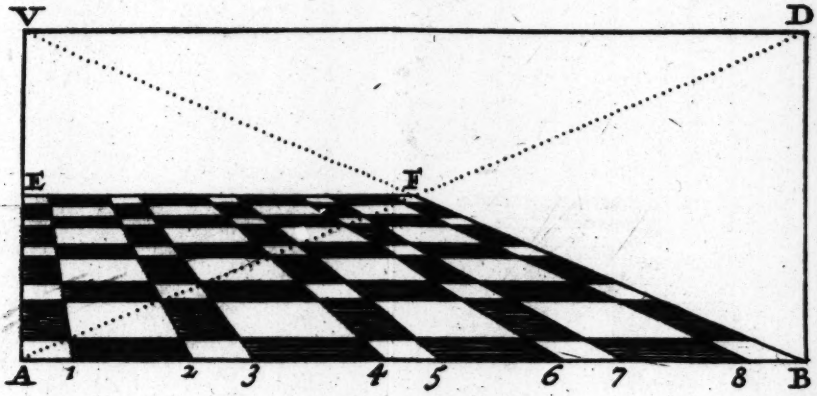
D
A
H

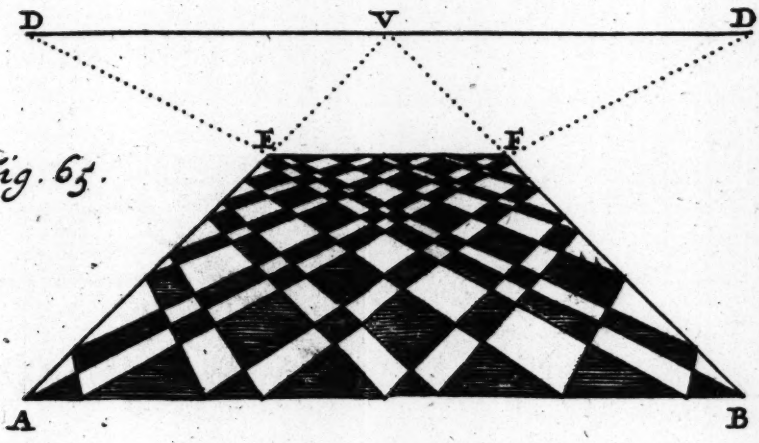
Fig. 63.



V
A

Fig. 64





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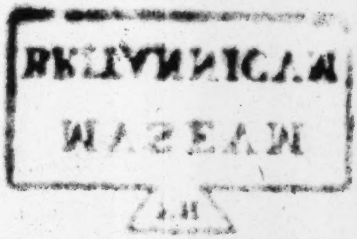

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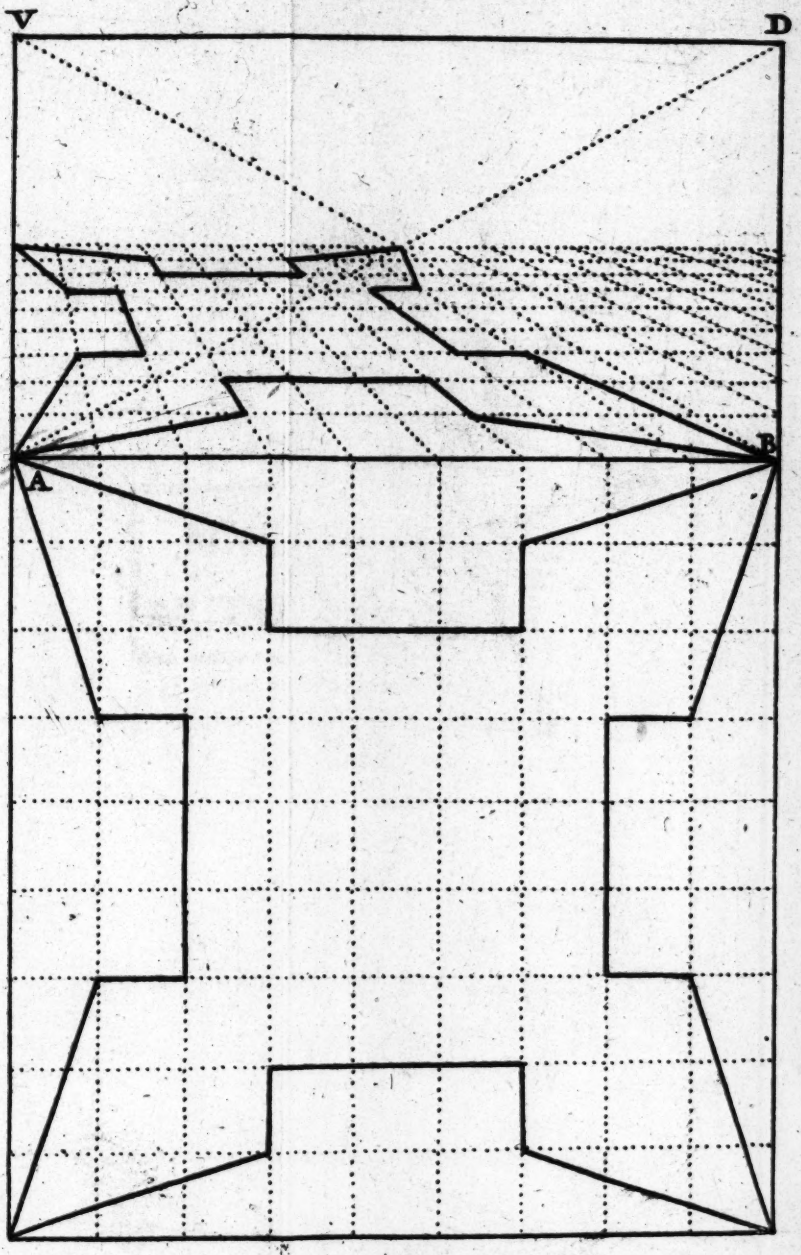

FH
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